WORKSHOP - Local and global aspects of function spaces and operators

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16 – 20 November, 2026

Contributions to the program

Dear participants, please check in the following program that the slot allocated to you suits you. Then, in Section 3 are given the titles and summaries of your talks (slots of 45 min including questions for SOFTA members). Each of you has an associated file in the contributions subfolder (see on the left hand side menu). Please check the file associated to your name and update it accordingly if necessary.

As you can see, the schedule includes sessions on open problems. Topics for open problem discussions are expected to be proposed by participants, please feel free to add items to the list, so that sessions can be planned

1 Program at a glance

	Monday 16/11	Tuesday 17/11	Wednesday 18/11	Thursday 19/11	Friday 20/11
9h-10h		S. Jaffard	B. Pascal	H. Feichtinger	Open problems
10h-10h30	10h15 : welcome	Break	Break	Break	Break
10h30-12h	M. Dörfler	H. Turcinova D. Stoeva	B. Torresani L. Neyt	D. Kubicek J. Malonzo	Discussions
12h-14h	Meal	Meal	Meal	Meal	Meal
14h-15h30	A. Meynard N. Teofanov	Z. Mihula M. Zigic		L. D. Abreu T. Pogány	
15h30-16h	Break	Break		Break	
16h-17h30	D. Jankov Maširević Open problems	C. Melot Open problems		Open problems	

2 Contributions to open problems

Enter a list of problems or questions you would like to discuss during the workshop.

1. Problem 1

Abstracts

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3 Titles and summaries (alphabetical order)

3.1 Abreu, Luis Daniel

Title Abreu: Energy concentration and isoperimetric inequalities

Abstract TBA

3.2 Dörfler, Monika

Title Detecting Structure in Data Sets using QHA

Abstract TBA

3.3 Feichtinger, Hans

Title Some Harmonic Analysis Tools

Abstract The Banach Gelfand Triple has developed into a universal tool for Fourier and Time-Frequency Analysis. By studying particular finite dimensional subspaces of the space of mild distributions (the space of signals or operators) we propose constructive methods which allow to realize important operations in the field computationally using linear algebra methods.

There are several paths leading to a common set of ideas in modern Fourier and time-frequency analysis. From the mathematical side, one needs suitable notions of "generalized functions" in order to describe objects such as Dirac measures and Dirac combs, which occur naturally in engineering and physics. Convolution describes linear time-invariant systems, while on the Fourier transform side the same systems are represented by multiplication with a transfer function. Gabor analysis provides another point of view: functions and distributions are represented by time-frequency shifted building blocks, typically well-localized windows such as Gaussians, resulting in a description which may be compared heuristically with a musical score.

From the applied side, signals arise through measurements. An audio signal is recorded by a microphone, while an image represents a scene captured by a digital camera. In both cases only a finite region of interest can be observed, and only up to a certain resolution. At the same time, the uncertainty principle places an intrinsic limitation on simultaneous localization in time and frequency.

This suggests the use of function and distribution spaces which are flexible enough for applications while retaining a robust mathematical structure.

Against this background, the presentation will focus on the Banach Gelfand Triple $(S_0(\mathbb{R}^d), L^2(\mathbb{R}^d), S_0^*(\mathbb{R}^d))$, consisting of the Feichtinger algebra $S_0(\mathbb{R}^d)$, the Hilbert space $L^2(\mathbb{R}^d)$, and its surrounding dual space $S_0^*(\mathbb{R}^d)$ of mild distributions. The continuous inclusions $S_0(\mathbb{R}^d) \subset L^2(\mathbb{R}^d) \subset S_0^*(\mathbb{R}^d)$ provide a setting in which well-localized functions, finite-energy signals, measures, and more general distributions can be treated within one framework.

A central feature is the close connection with Gabor analysis. For suitable Gabor systems the three spaces are characterized by coefficient sequences in ℓ^1, ℓ^2, and ℓ^∞, respectively. Using Wilson bases, this correspondence becomes particularly transparent and provides natural sequence-space models for the three levels of the triple.

The natural convergence notion in S_0^* is weak-* convergence, also called mild convergence. It provides a convenient framework for approximation and limiting procedures and allows many familiar operators to be treated consistently throughout the triple. The Fourier transform is a basic example, and the same viewpoint also applies to fractional Fourier transforms and related transformations.

As time permits, applications in modern Gabor and time-frequency analysis, as well as connections with classical Fourier analysis, will be presented. The aim is to illustrate how the Banach Gelfand Triple provides a flexible bridge between classical harmonic analysis, generalized functions, and signal analysis.

3.4 Jaffard, Stéphane

Title Characterization of function spaces using non-smooth wavelet bases or frames

Abstract A classical assumption concerning the wavelet characterization of function spaces is that the uniform regularity of the wavelets has to be larger than the regularity index of the function space considered. On other hand the numerical determination of multifractality parameters indicates that the relevant parameter seems to be the number of vanishing moments. If mathematically confirmed, this would lead to more performant numerical methods. We give several partial results which to confirm this intuition.

3.5 Jankov Maširević, Dragana

From Kaiser–Bessel window functions to four-parameter probability distributions

Abstract The Kaiser–Bessel window function, defined in terms of the modified Bessel function of the first kind, appears frequently in signal processing and related applications. By normalizing its symmetric form one obtains the Kaiser–Bessel probability distribution, which has been studied recently from the viewpoint of probability theory and special functions [1]. In this talk we study a four-parameter extension of the Kaiser–Bessel distribution [2]. Its density involves the modified Bessel function of the first kind and has compact support on a symmetric interval. We discuss several analytic aspects of this distribution, including hypergeometric representations for the distribution function and the moments. We also analyse the characteristic function and examine further probabilistic properties, such as unimodality. Finally, we return to the window side and present recent results on the smoothness and the Fourier transform of the corresponding four-parameter Kaiser–Bessel window.

References

[1] Á. Baricz, T. K. Pogány, Probabilistic and analytical aspects of the symmetric and generalized Kaiser–Bessel window function, Constr. Approx. 58 (2023), 713–783.

[2] Á. Baricz, D. Jankov Maširević, T. K. Pogány, Lj. Teofanov, N. Ujić, Certain analytical properties of the four-parameter generalized Kaiser–Bessel distribution, Constr. Math. Anal., submitted.

3.6 Kubíček, David

Title Fine properties of Orlicz-Sobolev functions

Abstract This talk discusses the fine properties of functions of (fractional) Orlicz-Sobolev spaces $W^s L^A$ over n-dimensional Euclidean space. A key role in this endeavour will be played by the Orlicz capacity, a generalisation of a well known Sobolev p-capacity. We first investigate the basic properties and behaviour of the Orlicz capacity as well as its relations with other outer measures. We then explore some capacitary-type estimates involving Hardy-Littlewood maximal operator and precise representatives of functions in Orlicz-Sobolev spaces. Finally, we address the approximation of Orlicz-Sobolev functions via functions of a given modulus of continuity. This is a joint work with Ladislav Drážný and Franz Gmeineder.

3.7 Malonzo, Jerielle

Title Generalized Inversion of Multipliers

Abstract Multipliers are operators on a Hilbert space consisting of three steps: analysis, (term-wise) multiplication, and synthesis. They have various applications, for example in signal processing. In particular, multipliers are tools to modify a given signal in certain desired ways. With this application, it is crucial to ask if one can retrieve the original signal after applying a multiplier – to which the answer is not always affirmative. Nevertheless, noninvertible multipliers might still be useful in some applications and in this case, generalized inverses will replace the role of inverses. One important result shown in the case of invertible multipliers is that the inverse takes a specific form and involves some sequences satisfying some duality conditions. In this work, we ask if it is possible to obtain a similar form for a generalized inverse of a noninvertible multiplier. We showed that this is possible after solving the two main challenges, namely, determining an appropriate generalized inverse to be considered, and determining an appropriate duality condition to be later satisfied. The talk is based on a joint work with Diana T. Stoeva. This work is supported by the Austrian Science Fund (FWF) through Project P 35846-N "Challenges in Frame Multiplier Theory", grant DOI 10.55776/P35846.

3.8 Melot, Clothilde

Title New Insights into Force Deconvolution in Non-Contact Atomic Force Microscopy

Abstract TBA

3.9 Meynard, Adrien

Title TBA

Abstract TBA

3.10 Mihula, Zdeněk

Title Compactness and uniform decay of functions

Abstract We discuss a new relation between function spaces over measure spaces of infinite measure, which is motived by a problem of establishing compactness in function spaces over infinite measure. It captures the uniform decay of function (quasi-)norms "at infinity". We investigate it in the scope of quasi-Banach function spaces, which need not be normable or rearrangement invariant. As a useful tool, we introduce the notion of extremal fundamental functions associated with a (quasi-)Banach function space. The relation first appeared implicitly in the speaker's work on the compactness of Sobolev embeddings of radially symmetric functions on the entire Euclidean space. Later, it was systematically studied in a joint paper with Maximilián Pándy (Technical University of Munich).

3.11 Neyt, Lenny

Title Optimal Beurling-Hörmander-type uncertainty principles and the non-triviality of Gelfand-Shilov spaces

Abstract Beurling proved that if both a function and its Fourier transform, weighted separately by the exponential of conjugate convex weight functions, give finite integrals, the function must be zero. This recovers many well-known classical and optimal uncertainty principles, such as those of Hardy and Morgan. Beurling argued that the result seems rather precize, though no exact formulation of this was given. In this talk, we show optimal uncertainty principles for a broad class of weight functions and characterize when it coincides with the one of Beurling. As an application, we characterize the non-triviality of Gelfand-Shilov spaces. Such spaces refine the classical Schwartz space of rapidly decreasing smooth functions by imposing bounds on the differentiability and decay of the functions. So far, characterizations of the non-triviality of the Gelfand-Shilov are known only in some specific cases. Using the fact that the Gelfand-Shilov spaces may be expressed through the decay in time and frequency of the function, we apply our optimal uncertainty principles to establish characterizing conditions on the defining weight sequences for non-triviality.

3.12 Pascal, Barbara

Title The Kravchuk transform: a novel covariant representation for discrete signals amenable to zero-based detection tests

Abstract Recent works in time-frequency analysis proposed to switch the focus from the maxima of the spectrogram toward its zeros, which form a random point pattern with a very stable structure. Several signal processing tasks, such as component disentanglement and signal detection procedures, have already been renewed by using modern spatial statistics on the pattern of zeros. Tough, they require cautious choice of both the discretization strategy and the observation window in the time-frequency plane. To overcome these limitations, we propose a generalized time-frequency representation: the Kravchuk transform, especially designed for discrete signals analysis, whose phase space is the unit sphere, particularly amenable to spatial statistics. We show that it has all desired properties for signal processing, among which covariance, invertibility and symmetry, and that the point process of the zeros of the Kravchuk transform of complex white Gaussian noise coincides with the zeros of the spherical Gaussian Analytic Function. Elaborating on this theorem, we finally develop a Monte Carlo envelope test procedure for signal detection based on the spatial statistics of the zeros of the Kravchuk spectrogram.

3.13 Pogány, Tibor

Title Truncation error upper bounds in irregular sampling restoration by multivariable Plancherel-Pólya inequality

Abstract In this talk we show that for the d–dimensional Bernstein functions class the Plancherel–Pólya inequality holds with the constant which equals to the product of the constants occuring in the one-dimensional cases. Related truncation error upper bounds are precised in the irregular sampling restoration of functions in several variables. The results are based on the paper [1].

[1] T. K. Pogány, On multivariable Plancherel-Pólya inequality and truncation error upper bounds in irregular sampling, *Sampl. Theory Signal Process. Data Anal.* **21** (2023), No. 1, Article No. 5, 13pp.

3.14 Stoeva, Diana

Title Inversion of Frame Multipliers on Hilbert, Banach, and Fréchet

Abstract TBA

3.15 Teofanov, Nenad

Title Regularity Classes and Related Transforms I

Abstract In this talk, we consider spaces of test functions given by suitable properties of the iterates of the Laguerre operator, [2]. These spaces extend the (G)-type spaces introduced by A. J. Duran in the 1990s as a natural counterpart of the global Gelfand–Shilov spaces on the half-line $(0, \infty)$, [1].

We proceed with continuity properties of the Laguerre operator propagator and provide an overview of results concerning extensions of the well-known representation of the fractional Fourier transform as the harmonic oscillator propagator, [4]. In particular, we present the connection between several fractional integral transforms and the propagator of the Laguerre operator, [3].

Acknowledgement: This work was supported by the Science Fund of the Republic of Serbia, #GRANT No. 2727, Global and local analysis of operators and distributions - GOALS, and SOFTA project within multilateral scientific and technological cooperation in the Danube region.

References

[1] A. J. Duran, Laguerre expansions of Gel'fand-Shilov spaces, J. Approx. Theory 74 (1993), 280-300.

[2] S. Jakšić, S. Pilipović, N. Teofanov, Dj. Vučković, Ultradifferentiable functions via the Laguerre operator. Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. 119, 89 (2025).

[3] S. Jakšić, N. Teofanov, Dj. Vučković, Continuity properties of the Laguerre operator and its propagator, preprint, https://arxiv.org/abs/2605.16939

[4] J. Toft, D. G. Bhimani, R. Manna, Fractional Fourier transforms, harmonic oscillator propagators and Strichartz estimates on Pilipović and modulation spaces, Appl. Comput. Harmon. Anal. 67 (2023) 101580

3.16 Torrésani, Bruno

Title Entropy-based criteria for adaptive time-frequency transforms

Abstract In [1] a general construction for adaptive, time-focused time-frequency/scale transform in $L^2(\mathbb{R})$ was introduced, in which the time-frequency trade-off depends on local behaviour (in time or frequency) of the analysed function. Adaptivity makes the corresponding transform non-linear. Adaptivity relies on a so-called time focus function. Time focus functions based upon regularised Rényi entropies were subsequently analysed in [2].

This talk will present new results on the entropy-based adaptive STFT, including sufficient conditions for invertibility of the transform, smoothness properties for regularised Rényi entropies and the corresponding transform. Generalisations to adaptive, frequency focused continuous wavelet transform will also be briefly discussed. That case requires the introduction of a so-called squeezing function defined in the frequency domain, and frequency focus functions for which similar results can be obtained.

This is a joint work with Pierre Warion.

References

[1] P. Warion and B. Torrésani, A class of non linear adaptive time-frequency transforms, *Sampling Theory, Signal Processing, and Data Analysis*, **22**:2 (2024), pp.18.

[2] P. Warion and B. Torrésani, Stability estimates for adaptive focused time-frequency transforms, Proceedings of the 2025 International Conference on *Sampling Theory and Applications* (SampTA), Vienna, Austria (2025).

3.17 Turcinova, Hana

Title On the properties of rearrangementinvariant quasi-Banach function spaces

Abstract In this talk, we will present several recent results on the theory of rearrangement-invariant quasi-Banach function spaces. In particular, we will discuss an analogue of the Luxemburg representation theorem for rearrangement-invariant quasi-Banach function spaces over resonant measure spaces, as well as the theory of fundamental functions and endpoint spaces, highlighting their main properties and recent developments.

3.18 Zigic, Milica

Title On Extended Gevrey Spaces

Abstract We consider spaces of smooth functions defined by relaxing Gevrey-type regularity and decay conditions. We show that these classes fit naturally within the general framework of the weighted matrices approach to ultradifferentiable functions. We examine equivalent definitions of Gelfand–Shilov-type spaces associated with extended Gevrey regularity and establish their nuclearity. In addition to invariance under the Fourier transform, we provide symmetric characterizations of these spaces. We also study time–frequency representations of the introduced classes. We further prove that controlling the derivatives of a function f only at orders d_n, by bounds characteristic of extended Gevrey classes, where $(d_n)_n$ is a divergent sequence of positive integers satisfying $d_{n+1} \leq Ld_n$, $n \in \mathbf{N}$, for some $L > 1$, already implies control of derivatives of all orders. Finally, we present a general construction of a smooth orthonormal wavelet which, together with its Fourier transform, belongs to the extended Gevrey class, yielding an example beyond all classical Gevrey classes.

This is a joint work with N. Teofanov, F. Tomić, D. Vučković, S. Tutić and J. Dimitrić.