

Research School

Representation Theory and Quantization on Homogeneous Spaces
Théorie des représentations, géométrie et quantification sur les espaces homogènes
CIRM, 21 September – 25 September 2026

Titles and Abstracts

I. Courses

Angela Pasquale (Metz)

Title: The Howe correspondence over the reals

Abstract: The Howe correspondence, also called theta correspondence, is one of the cornerstones of the modern Representation Theory over local and finite fields, with important applications in Harmonic Analysis, Number Theory and Quantum Mechanics. In these lectures, we will be concerned with the Howe correspondence over the field of real numbers. After constructing the metaplectic group, that is the unique connected double covering group of the symplectic group, we will define the Weil representation (aka as Segal-Shale-Weil or oscillatory representation) in its Schrödinger and Fock models, following the explicit approach initiated by Aubert and Przebinda. We then study the Weil representation through its distribution character and matrix coefficients. The Howe correspondence is based on Howe's notion of reductive dual pairs in a symplectic group. We will present their classification and their realization as Lie supergroups. We then introduce Howe's correspondence, first for the special case of dual pairs with a compact member and then in general. Examples will illustrate the general theory.

Dominique Manchon (Clermont)

Title: Deformation quantization and representations

Abstract: These lectures will address some applications of deformation quantization to representation theory, focusing on nilpotent and solvable Lie groups. Prerequisites are basics on Lie groups and Lie algebras, as well as elementary notions of differential geometry.

1. Poisson algebras, Poisson manifolds, deformation quantization.
2. A brief survey of M. Kontsevich's formality theorem, implying that any Poisson manifold is deformation quantizable.
3. Linear Poisson manifolds. Symplectic leaves and coadjoint orbits.
4. Kirillov orbit method (nilpotent and solvable case). Kirillov and Dixmier maps.
5. Characteristic variety and Poisson characteristic variety.

Fanny Kassel (IHES)

Title: Proper actions on reductive homogeneous spaces

Abstract: Proper actions of discrete groups on homogeneous spaces G/H play an important role in several areas of mathematics, in particular in geometry. If G and H are both noncompact, then not all discrete subgroups of G act properly on G/H , and it is a very difficult problem in general to understand which discrete subgroups of G can reach the delicate balance of being small enough to act properly on G/H , but large enough for the quotient to be interesting, for instance compact. This problem has given rise to a rich literature, at the interface of group theory, Lie theory, representation theory, geometry, topology, and dynamics. We will survey some of the key developments in this area, from the late 1980s to the present. Applications to spectral analysis on pseudo-Riemannian locally symmetric spaces will be presented by T. Kobayashi in the same week.

Toshiyuki Kobayashi (Tokyo)

Title: Spectral analysis on locally symmetric spaces

Abstract: In this series of expository lectures, I will provide an introductory overview of recent advancements in the spectral analysis of locally symmetric spaces, going beyond the traditional Riemannian framework. This topic is built upon the theory of proper action on reductive homogeneous spaces, which will be presented by F. Kassel in the same week, alongside the theory of global analysis on symmetric spaces and a rapidly developing theory of branching in infinite-dimensional representations of reductive groups. My aim is to clarify key concepts in a non-technical manner, using several elementary examples to make the fundamental ideas more accessible.

II. Young Sessions

Khouloud Abid (Sfax)

Title: Uncertainty Principles for the Gabor Transform on Certain Lie Groups

Abstract: In this work, we study uncertainty principles for the Gabor transform on non-commutative Lie groups. We establish Heisenberg-type uncertainty inequalities for several classes of nilpotent Lie groups and compact extensions of $\mathbb{R}^n$. We also prove a Hardy-type uncertainty principle on connected simply connected nilpotent Lie groups. These results extend classical uncertainty principles to a non-commutative setting.

Ibtissem Ben Chenni (Monastir)

Title: Autour de la topologie duale de certains groupes de Lie à nilradical co-compact

Abstract:

Ce travail s'inscrit dans l'étude de la théorie des représentations des groupes de Lie compacts ainsi que de la topologie duale de certains groupes de Lie de la forme

$$G = K \ltimes N,$$

où K est un sous-groupe de Lie compact de $\text{Aut}(N)$ et N un groupe de Lie nilpotent connexe et simplement connexe.

La méthode des orbites de Kirillov–Lipsman permet de paramétrer le dual unitaire de certains groupes de Lie G par un objet géométrique, l’espace des orbites coadjointes $\mathfrak{g}^*/G$, où $\mathfrak{g} = \text{Lie}(G)$.

Cette procédure est appelée quantification. L’étude de la convergence des orbites coadjointes admissibles de certains groupes de Lie à nilradical co-compact permet alors de donner une description explicite de la topologie de Fell sur le dual unitaire du groupe considéré.

Yasmin Ben Soula (Bizerte)

Title: Lattices in the Metabelian Nilpotent Lie Group

Abstract:

The main aim of this presentation is the description of a class of lattices in the nilpotent semidirect product group

$$G = \mathbb{R}^n \rtimes_{\eta} \mathbb{R}^m,$$

where η is the representation of the abelian group $\mathbb{R}^m$ in $GL_n(\mathbb{R})$ given by

$$\eta(t_1, \dots, t_m) = \exp \left(\sum_{i=1}^m t_i N_i \right),$$

with $(N_i)_{1 \leq i \leq m}$ a set of pairwise commuting nilpotent matrices in $M_n(\mathbb{Q})$.

Hassen Bourokba (Sfax)

Title: Visible actions on complex domains related to Euclidean motion groups

Abstract: Recent developments on visible actions for nilpotent Lie groups, particularly in the Heisenberg group case, motivate the study of the geometry of complex actions for the Euclidean motion group $G_n = O_n(\mathbb{R}) \ltimes \mathbb{R}^n$. We first prove that $\mathbb{C}^n$, as a complex homogeneous space, admits no strongly visible action of any compact subgroup of $O_n(\mathbb{R})$. In contrast, the natural action of $O_n(\mathbb{R})$ on the unit sphere in $\mathbb{C}^n$ is strongly visible. We provide a complete characterization of visible (equivalently, strongly visible) actions for semi-direct subgroups of the form $K \ltimes U$, where $K < O_n(\mathbb{R})$ and $U < \mathbb{R}^n$, acting on $\mathbb{C}^n$. Such an action is visible if and only if $U = \mathbb{R}^n$. We extend this classification to nilpotent subgroups of G_n , confirming in all cases that the translational component alone determines visibility.

This is joint work with Ali Baklouti.

Rafik Khalfi (Bizerte)

Title: The Benson-Ratcliff invariant of MD-algebras and central extensions of nilpotent Lie algebras

Abstract: This presentation focuses on the Benson–Ratcliff invariant in the context of central extensions of nilpotent Lie algebras. We successively analyze the abelian, metabelian and r -step cases, then those of Lie algebras with two-dimensional generic coadjoint orbits. We illustrate this result by explicit computation in the case of 6-dimensional nilpotent Lie algebras with 1-dimensional center and identify all counterexamples to the Benson–Ratcliff conjecture in this class. Furthermore, we study this invariant for MD-algebras.

References :

1. Benson, C., Ratcliff, G. : An invariant for unitary representations of nilpotent Lie groups. *Michigan Mathematical Journal* 34(1), 23–30 (1987).

2. Benson, C., Ratcliff, G. : Quantization and an invariant for unitary representations of nilpotent Lie groups. *Illinois Journal of Mathematics* 32(2), 304–322 (1988).
3. Baklouti, A., Tounsi, K. : On the Benson–Ratcliff invariant of coadjoint orbits on nilpotent Lie groups. *Osaka Journal of Mathematics* 44(2), 399–414 (2007).
4. Dali, B., Khalfi, R. : On Kirillov’s lemma and the Benson–Ratcliff invariant. *Journal of Mathematical Physics* 62(5) (2021). AIP Publishing.
5. Dali, B., Khalfi, R. : Benson–Ratcliff conjecture on solvable Lie groups. *Communications of the Korean Mathematical Society* 40(2), 333–355 (2025).
6. Khalfi, R. : Benson–Ratcliff invariant and central extensions of nilpotent Lie algebras. *Ricerche di Matematica* (2026).

Cheikh Khoule (Dakar)

Title: Structure de l’Espace des Origines Affines d’une Structure de Contact

Abstract:

Cet exposé explore l’interaction algébrique et géométrique entre les feuilletages (formes intégrables) et les structures de contact (formes maximale-ment non intégrables).

Nous étudierons l’espace des origines affines $A(\beta)$, constitué des formes de Pfaff intégrables pouvant être déformées vers une forme de contact β par un chemin affine dont les formes intermédiaires restent strictement de contact.

Nous montrerons que l’existence de telles déformations est gouvernée de manière rigide par un invariant géométrique d’interaction $I_\beta(a)$, qui doit être globalement positif ou nul sur toute la variété.

L’exposé s’articulera autour de trois idées principales :

- **Construction analytique :** introduction du chemin de déformation affine et démonstration de la formule exacte décrivant l’évolution du volume de contact.
- **Obstructions topologiques :** étude des contraintes fortes imposées par la condition $I_\beta(a) = 0$, notamment de son lien avec la dynamique du champ de Reeb.
- **Structure algébrique de l’espace :** mise en évidence de la non-linéarité inhérente aux formes intégrables, à travers le défaut d’intégrabilité mutuelle.

Jonathan Ditlevsen (Tokyo)

Title: Branching laws for Stein’s complementary series and Speh representations of $GL(2n, \mathbb{R})$

Abstract:

We obtain the explicit direct integral decomposition of Stein’s complementary series representations and Speh representations of $GL(2n, \mathbb{R})$ when restricted to the subgroup $GL(2n-1, \mathbb{R})$.

The main tools for this are two families of symmetry breaking operators between degenerate series representations of $GL(2n, \mathbb{R})$ and $GL(2n-1, \mathbb{R})$.

Abdou Ganiou Débayo Atindehou (Benin)

Title: Towards a Characterization of the Gabor Frame Set for Compactly Supported Continuous Functions

Abstract:

The determination of the Gabor frame set is a central problem in time–frequency analysis. Although the frame set is known for certain specific classes of window functions, its complete characterization remains largely open, in particular for compactly supported continuous functions.

In this talk, we consider the class $V_{N,a}$ of compactly supported continuous functions, which includes, in particular, the B-splines. We present a matrix-based approach relying on the Ron–Shen duality conditions. By exploiting the compact support of the window and an appropriate partition of the parameter domain, the infinite system arising from the Ron–Shen duality conditions can be reduced to a family of finite matrices $G_m(x)$.

This reduction provides a unified framework for studying the frame property through the invertibility of these matrices.

We recall several previously known results and present new regions in the parameter plane where the matrices $G_m(x)$ are uniformly invertible, thereby yielding new regions of the Gabor frame set for every $g \in V_{N,a}$. We also discuss the construction of alternative compactly supported dual windows associated with these regions.

We then address a more general question : to what extent can the matrices $G_m(x)$ characterize the Gabor frame set ? The positive results obtained so far suggest that the condition

$$\inf_{x \in [-a/2, a/2]} |\det G_m(x)| > 0$$

may play a characterizing role.

Imane Ghanimi (Kenitra)

Title: Characterization of the L^p range of a generalized Poisson transform of the quaternionic unit ball $B(H)$

Abstract:

In this talk, we give a characterization of the L^p -range ($p = 2$) of a generalized Poisson transform P_l on the quaternionic unit ball $B(H)$, associated with an irreducible representation τ_l of $Sp(1)$.

More precisely, since $B(H) = Sp(1)$, if f is a hyperfunction on $Sp(1)$, then, for $\text{Re}(i) > 0$, we prove that

$$f \in L^p(Sp(1)), \quad p = 2,$$

if and only if its generalized Poisson transform $P_l(f)$ satisfies a Hardy-type growth condition.

Jocelyn Gonessa (Bangui)

Title: Estimations L^p - L^q des opérateurs de type Forelli-Rudin dans la boule minimale

Abstract:

Nous donnons les conditions sur p et q pour que des opérateurs de type Forelli-Rudin dans la boule minimale soient bornés de L^p dans L^q .

Tatsuro Hikawa (Tokyo)

Title: The Schwartz Space for the (k, a) -Generalized Fourier Transform and the Minimal Representation of the Conformal Group

Abstract:

This talk discusses an analog of the classical Schwartz space $\mathcal{S}(\mathbb{R}^N)$ in the framework of (k, a) -deformed harmonic analysis associated with the (k, a) -generalized Fourier transform $\mathcal{F}_{k,a}$.

Motivated by the observation that $\mathcal{S}(\mathbb{R}^N)$ coincides with the space of smooth vectors for the Segal–Shale–Weil representation, we define the (k, a) -generalized Schwartz space $\mathcal{S}_{k,a}(\mathbb{R}^N)$ as the space of smooth vectors for the unitary representation associated with $\mathcal{F}_{k,a}$.

We will present our recent results on the explicit determination of $\mathcal{S}_{k,a}(\mathbb{R}^N)$.

Anoh Yannick Kraïdi (Abidjan)

Title: On some properties of Reproducing Kernel Hilbert C^* -modules

Abstract:

In this talk, we investigate the reproducing kernel theory in the framework of Hilbert C^* -modules and the linear transformation of Hilbert C^* -modules. We give an analogue of the inversion formula and the theorems of approximation in a reproducing kernel space.

Siwei Liang (Orsay)

Title: Tempered homogeneous spaces

Abstract: Following the work of Benoist–Kobayashi, we study the quasi-regular representations $L^2(G/H)$ of semisimple Lie groups. In a joint work with Y. Benoist, we establish a geometric criterion characterizing when such a representation is tempered, extending earlier results of Elstrodt, Patterson, Sullivan, Shalom, Benoist–Kobayashi, Edwards–Oh, and Lutsko–Weich–Wolf in various settings. If time permits, I will also discuss extensions to other locally compact groups.

Valentin Massicot (Reims)

Title: Symmetry Breaking Operators for $(GL(3, \mathbb{R}), GL(2, \mathbb{R}))$

Abstract:

Let $H \subset G$ be a pair of real reductive groups and π an irreducible representation of G . Branching problems consist in understanding how the restriction $\pi|_H$ behaves as a representation of H . One of the central questions in this direction is to explicitly construct and classify corresponding symmetry breaking operators (SBOs), i.e. elements of $\text{Hom}_H(\pi|_H, \tau)$, for given representations τ of H .

This approach advocated by T. Kobayashi in his ABC-program has been carried out in a handful of cases, e.g. for $(G, H) = (O(n+1, 1), O(n, 1))$.

In this work, we construct and classify all symmetry breaking operators between principal series for the pair

$$(G, H) = (GL(3, \mathbb{R}), GL(2, \mathbb{R})).$$

This reveals many interesting phenomena related to sporadic operators, hypergeometric polynomials and multiplicities.

Shunsuke Miyauchi (Tokyo)

Title: Classification of connected proper pairs in the affine transformation group

Abstract:

Consider a pair (L, H) of closed subgroups of a Lie group G . In the theory of transformation groups, it is important to find effective criteria for properness. We say that the pair (L, H) is proper if L acts properly on the homogeneous space G/H ; equivalently, H acts properly on G/L . The pair (L, H) is always proper if either L or H is compact. However, when both L and H are non-compact, determining whether (L, H) is proper becomes much more difficult. The systematic study of this problem was initiated by Kobayashi (1989).

Effective criteria are known in several important cases, for example when G is reductive or when G is a low-step nilpotent Lie group. Discontinuous groups for some affine homogeneous spaces, such as Margulis spacetimes, have also been studied.

In this talk, we consider the problem of determining properness for affine transformation groups. As a first test case, Kobayashi (1990) classified the connected subgroups L acting properly on

$$(GL(2, \mathbb{R}) \ltimes \mathbb{R}^2)/GL(2, \mathbb{R}) \simeq \mathbb{R}^2.$$

We give a classification of all connected proper pairs (L, H) in the affine transformation group

$$G = GL(2, \mathbb{R}) \ltimes \mathbb{R}^2.$$

Ryosuke Nakahama (Tokyo)

Title: Characterization of infinite-order differential holographic operators for symmetric pairs of holomorphic type by differential equations

Abstract:

Let (G, G_1) be a symmetric pair of holomorphic type, and we consider the space $\mathcal{O}(D, V)$ of vector-valued holomorphic functions on the bounded symmetric domain $D \simeq G/K$.

Then its restriction to the subgroup G_1 is generically decomposed into a direct sum of subrepresentations $\mathcal{O}(D_1, W)$ of G_1 , and there exist G_1 -intertwining operators between $\mathcal{O}(D, V)$ and $\mathcal{O}(D_1, W)$.

It is proved by Kobayashi–Pevzner (2016) that the intertwining operators from $\mathcal{O}(D, V)$ to $\mathcal{O}(D_1, W)$ are always given by differential operators, and their symbols are characterized as solutions of certain differential equations.

In this talk, we present a result that the intertwining operators in the opposite direction, called holographic operators, are given by infinite-order differential operators, and their symbols are again characterized as solutions of certain differential equations.

Gael Ngambali Ndzakima (Brazzaville)

Title: Polarisation and Irreducibility of Induced Representations of Nilpotent Lie Groups

Abstract:

Let G be a simply connected nilpotent Lie group and π a finite-dimensional representation of its Lie algebra $\mathfrak{g}$. To each Lie subalgebra $\mathfrak{h}$ subordinate to π , we associate an induced representation $\rho(\pi, \mathfrak{h}, \mathfrak{g})$.

We study the irreducibility of these representations and show, under a condition involving the center of $\mathfrak{g}$ and $\ker \pi$, that $\rho(\pi, \mathfrak{h}, \mathfrak{g})$ is irreducible if and only if $\mathfrak{h}$ is a polarization of π .

Olufemi Oyadare (Nigeria)

Title: Fourier and Helgason Fourier Transforms for Vector Bundle-valued Differential Forms on Homogeneous Spaces

Abstract:

The major aim of studying harmonic analysis on any group or symmetric space is to understand enough of its structure to prove the fundamental theorem via the Fourier transform. We derive the Helgason Fourier transform for vector bundle-valued differential forms on homogeneous spaces and describe its relation to the vector bundle-valued Poisson transform.

We also give explicit formulas for the Fourier and Helgason Fourier transforms and distinguish the cases of invertible and non-invertible vector bundle-valued Poisson transforms.

Chayma Sayari (Bizerte)

Title: Cortex of Lie algebras in particular cases

Abstract:

We investigate the cortex of 7-dimensional Lie algebras in several specific cases.