Long Talks

- **Fanny Augeri**: *Large deviations of the empirical spectral measure of supercritical sparse Erdös-Rényi graphs*

  What is the large deviation behaviour of the spectrum of an Erdös-Rényi graph? Owing to the non-integrability of the model, this question has been challenging the classical large deviation theory for decades. Motivated by this question, we examine a sparse version of this problem, by considering supercritical sparse Erdos-Renyi graphs, meaning that the connection probability goes to zero and that the mean degree diverges at least as the logarithm of the number of vertices. In this sparsity regime, we obtain a large deviations principle for the empirical spectral measure with a rate function solution of a certain variational problem. The rate function reveals in particular that the only possible deviations are around symmetric measures coming from Quadratic Vector Equation.

- **Tim Austin**: *Annealed almost periodic entropy*

  This talk will be an introduction to "almost periodic entropy" and its "annealed" version. Almost periodic entropy is an analog of Lewis Bowen's "sofic entropy" from ergodic theory, but defined for positive definite functions on groups rather than measure-preserving systems. The analogy covers many of their basic properties, but some important differences also appear.

  I will introduce these quantities by sketching a specific application to large deviations for uniform random tuples of unitary matrices. The outcomes include a new proof of the Collins—Male theorem on strong convergence of such tuples and a large deviations principle for operator norms to accompany that theorem.

- **Anirban Basak**: *Large deviations in sparse random graphs*

- **Benoit Collins**: *Old and new results around the minimum output entropy*

  A crucial — and unsolved — question in quantum information theory is to check whether the classical capacity of quantum channels is additive. A non-regularized version of this problem is the problem of the additivity of the Holevo capacity, and of the minimum output entropy. This problem was at the heart of mathematical quantum information theory in the last two decades, and was solved with large deviation techniques on random matrices, and then refined with techniques of free probability, operator algebras and strong convergence. We will recall the problem and its stakes, revisit the mathematical aspects of the proofs, and explore potential applications of recent results on strong convergence of random matrices to "multi-shot" versions of entropies and capacities.

- **Nicholas Cook**: *Limiting spectral distribution for Brownian motions on the general linear group*

- **Djalil Chafai**: *Monotonicity of the logarithmic energy for random matrices*

  It is well-known that the semi-circle law, which is the limiting distribution in the Wigner theorem, is the minimizer of the logarithmic energy penalized by the second moment. A very similar fact holds for the Girko and Marchenko–Pastur theorems. In this work, we shed the light on an intriguing phenomenon suggesting that this functional is monotonic along the mean empirical spectral distribution in terms of the matrix dimension. This is reminiscent of the monotonicity of the Boltzmann entropy along the Boltzmann equation, the monotonicity of the free energy along ergodic Markov processes, and the Shannon monotonicity of entropy or free entropy along the classical or free central limit theorem. While we only verify this monotonicity phenomenon for the Gaussian unitary ensemble, the complex Ginibre ensemble, and the square Laguerre unitary ensemble, numerical simulations suggest that it is actually more universal. We obtain along the way explicit formulas of the logarithmic energy of the mentioned models which can be of independent interest. Joint work with Benjamin Dadoun and Pierre Youssef. Published in RMTA (2024). See also http://arxiv.org/abs/2212.06090

- **Stéphane Dartois - Benjamin McKenna**: *The injective norm of random tensors*

  The injective norm is a natural generalization to tensors of the operator norm of a matrix. In quantum information, the injective norm is one important measure of genuine multipartite entanglement of quantum states, where it is known as the geometric entanglement. We present two methods to give upper bounds on the injective norm of real and complex random tensors, corresponding to lower bounds on the geometric entanglement of random quantum states. The first method is based on spin-glass methods, the Kac-Rice formula, and recent progress coming from random matrices. The second method, more recent, is a kind of moment method for random tensors.

- **Raphael Ducatez, Jonathan Husson, Jana Reker** Large deviations for random matrices: The spherical integral method

  Over the last few years, the method of tilting by spherical integrals (or 'spherical integral method' for short) has been established as a versatile tool to prove large deviations principles for the largest eigenvalue of various Hermitian random matrix models. In this talk, we present the base method, as well as recent generalizations, e.g., to matrices with a variance profile or a tensor product structure. Based on joint works with Fanny Augeri, Nicholas Cook, Alice Guionnet, and Benjamin McKenna.

- **Shirshendu Ganguly** :*Universality of Gaussian Multiplicative Chaos via a high dimensional central limit theorem with low rank increments*

  A log-correlated field (LCF) arises often in probability and statistical mechanics. Particularly important examples are log-correlated Gaussian fields (LCGF) with the two dimensional Gaussian free field being a canonical example expected to arise as the scaling limit of many critical models. A one dimensional example is obtained by a random Fourier series/wave model with i.i.d. standard Gaussian coefficients which is known to be the scaling limit of the log-characteristic polynomial of Haar Unitary matrices. Exponentiating such an LCGF yields a random fractal measure known as the Gaussian multiplicative chaos constructed first by Kahane. This is a central building block of quantum field theories and Liouville quantum gravity.

  In this talk we will explore the question of anMonotonicity of the logarithmic energy for random matrices invariance principle. For instance, the universality of the Gaussian multiplicative chaos as the underlying coefficients in a random Fourier series form an arbitrary i.i.d. sequence.

  A key ingredient will be a new high dimensional central limit theorem where the increments are low rank. Our approach relies on a path-wise analysis via Skorokhod embeddings. Along the way, we also develop a new perturbative bound for the matrix square-root, which may be of independent interest.

- **Jiaoyang Huang**: *Asymptotics of Symmetric Polynomials: A Dynamical Point of view*

- **David Jekel**: *Entropy and stochastic control in free probability*

  We use stochastic control theory to understand the large-n behavior of entropy for invariant random multi-matrix models. We consider random matrix tuples $X^{(n)} = (X_1^{(n)}, \dots, X_m^{(n)})$ in $M_n(\mathbb{C})^m$ with probability density of the form $C \exp(-n^2 \operatorname{tr}_n(f(x)))$ where f is a non-commutative polynomial in $x_1, \dots, x_m$ and $x_1^*, \dots, x_m^*$. The large-n behavior of functions of $X^{(n)}$ is often described by a tuple $X = (X_1, \dots, X_m)$ from a von Neumann algebra, and large-n behavior the entropy of $X^{(n)}$ by Voiculescu's free microstate entropy χ of X. However, there are fundamental difficulties in determining whether the large-n limit of the entropy even exists (some of these difficulties result from the quantum complexity result MIP* = RE), and the related question of whether χ agrees with another quantity χ^* defined using free heat flow. Using the Bou'e-Dupuis formula, we express the entropy using a stochastic control problem, which motivates our development of stochastic control theory and related PDE theory in the free setting.

- **Justin Ko**: *One-sided large deviations for the ground-state energy of spin glasses*
- **Florent Krzakala**: *Open problem at the intersection between deep neural networks and random matrices*
- **Alex Little - Ronan Memin** : *Large deviations of the periodic Toda chain: approached by beta ensembles and by separated variables*

  We will introduce some key objects related to the so-called Toda chain, which is an integrable system whose dynamics are encoded by a pair of N by N matrices. One of these two matrices encodes the conserved quantities of the system, and it is of interest to study the convergence of its spectrum. In the first approach it will be explained how a comparison with the beta ensembles of random matrix theory allows one to establish large deviations of the spectrum, and hence to show its convergence. In the second approach, the method of classical separation of variables is used which allows one to deduce the rate function of this large deviation principle. This rate function has an interpretation in terms of scattering of solitons.
- **Myléne Maïda**: *Old and new : from transport-entropy inequalities to Monte Carlo integration with repulsive gases*

  I will present several results related to concentration and large deviations for particle systems such as beta ensembles or Coulomb and Riesz gases. I will then explain how one can apply those results and methods to design algorithms for numerical integration and quantify their performances. Based on old and new works with, in chronological order, E. Maurel-Segala, D. Chafaï, H. Hardy, R. Bardenet and M. Rouault.
- **Hariharan Narayanan**: *Large deviations for random hives and the spectrum of the sum of two random matrices.*

  Hives, as defined by Knutson and Tao, are discrete concave functions on a triangular grid on an equilateral triangle of side n. It is known through the work of Knutson and Tao that the probability distribution of the spectrum of the sum of two independent random Hermitian matrices with unitarily invariant distributions and given spectra can be expressed in terms of the distributions of the values of certain vertices on random hives. We prove the existence of a surface tension function depending on the Hessian, for continuum limits of random hives, and prove a large deviation principle for the large n limit of random hives. Through the aforementioned connection, we also obtain a large deviation principle for the spectrum of the sum of two random Hermitian matrices with given spectra. This is joint work with Scott Sheffield.
- **Vanessa Piccolo** : *Landscape complexity of spiked Gaussian random polynomials*

  High-dimensional random landscapes arise naturally in the modeling of complex systems, including energy landscapes in physics, fitness landscapes in biology, and loss landscapes in machine learning. They are typically highly non-convex, with many local minima, maxima, and saddles. A central question is how structured perturbations affect the abundance and organization of these stationary points. I will present results for Gaussian random polynomials with a finite-rank deterministic perturbation depending on fixed directions ("spikes"). Using the Kac-Rice formula, the expected number of critical points and local maxima can be expressed through conditional expectations of determinants of large random matrices, leading to variational formulas for their exponential growth rates. These formulas reveal a phase transition in the perturbation strength: beyond a critical threshold, new regions emerge with sub-exponentially many critical points strongly correlated with the spikes. The analysis connects landscape complexity with random matrix asymptotics and large deviation principles.

- **Kavita Ramanan**: *TBA*
- **Alain Rouault**: *Entropies, sum rules and large deviations.*

  The celebrated Szego-Verblunsky formula (1936) establishes an identity between two functionals of probability measure μ on the unit circle $\mathbb{T}$. The first one is the Burg entropy of μ, which is also the reversed KL relative entropy of μ with respect to the Lebesgue measure on $\mathbb{T}$. The second one is a functional of the Schur (or Verblunsky) coefficients of μ.

  With F. Gamboa and J. Nagel, I provided a probabilistic proof identifying both quantities as rate functions of some Large Deviations Principle of two different encodings of the spectral measure of a random matrix of the Circular Unitary Ensemble. This perspective is robust enough to be extended to other random matrix models such as the Hua-Pikrell and the Gross-Witten ensembles, which involve new references measures, possibly supported on a proper arc of $\mathbb{T}$, and new functionals of the coefficients. It also applies to matrix-valued measures.

  A nice abstract extension has recently be developed in the papers of Tim Austin, where the Toeplitz matrices of moments are replaced by positive definite functions on non-abelian groups.
- **Gregory Schehr**: *Large deviations and free cumulants in switching diffusion*

  We study a Brownian particle whose diffusion constant randomly switches between values drawn from a fixed distribution. Using a renewal approach, we derive exact finite?time expressions for its moments. In the long?time limit, the cumulants grow linearly with time and are directly related to the free cumulants of the underlying distribution. For certain distributions, the large deviations of the particle's position reveal dynamical transitions in the rate function. These results provide an interesting link between stochastic processes and free-probability concepts.
- **Dimitri Shlyakhtenko**: *TBA*
- **Kohei Suzuki**: *Dyson Brownian Motion as the Wasserstein Gradient Flow of Entropy*
- **Balasz Szegedy**: *Action limits of random matrices: an information theoretic approach*
- **Dan Voiculescu**: *Around topological free entropy*

  Topological free entropy is a notion of entropy adapted for C*-algebras. It is based on norm-microstates, which are defined using norms instead of traces. There is a certain similarity to the dynamical entropies: Kolmogorov-Sinai entropy versus topological dynamical entropy.

Short talks

- **Remi Bonnin**: *The spectrum of the infinite regular hypertree*

  In this short talk, we introduce the infinite d-regular hypertree. We give a sense to a notion of spectrum and we compute it. We present ideas of Friedman that inspired this work and we make the link with the notion of eigenvalues of a tensor defined by Qi. We discuss some applications.
- **Thomas Buc d'Alché et Ella Hiesmayr** : *Formal LDP for the empirical measure of subGaussian matrices*

  We formulate a conjecture on the existence of a large deviation principle for the empirical eigenvalue distribution of a Wigner matrix plus a GUE matrix. We give a formal expression of the prospective rate function. Its coefficients are described in terms of the Laplace transform of the entries of the Wigner part, families of combinatorial objects, and the free cumulants of the deviation measure.

- **Charlie Dworaczek**: *Mesoscopic CLT for β-ensembles at intermediary temperature*

  We investigate β-ensembles with real-analytic potential V in the intermediate temperature regime $1 \gg \beta \gg 1/N$. Although the macroscopic description is still given by the usual equilibrium measure, the Gaussian fluctuations are only obtained when removing a tower of deterministic terms.

  We prove a mesoscopic CLT in the bulk after recentering by a refined equilibrium measure. Our result reveals a phase transition and the nature of the fluctuations depends on the scale β: in the random matrix phase, energy dominates entropy and the variance is given by the $H^{1/2}$-norm; in the Poisson phase, entropy dominates energy and the variance is given by an L^2-norm.

- **Kewei Pan**: *Boolean entropy*

  It is well-known that Boolean independence, along with classical independence and freeness are the only three universal independence relations. In the talk, I will present a Symmetric Block random matrix model that gives asymptotic Boolean independence and a notion for Boolean entropy in one dimension, which can be derived from a LDP for this model. Moreover, we are able to define the Boolean Fisher information via the de Bruijn identity and recover many analogous results between classical and free entropy in the Boolean setting.

- **Anda Skeja**: *Multivariate Dependencies in Multiplex Graphons*

  Pairwise interlayer dependence can miss genuinely multivariate structure in random multiplex graphs. A simple example is XOR: generate two layers independently and define a third by applying XOR edgewise; the first two layers are marginally independent, yet become dependent conditional on the third layer. We study this problem in exchangeable multiplex graphs, a framework that includes many standard random graph models. We introduce a joint graphon entropy and a family of relabeling-invariant, graphon-level information functionals derived from it. These capture conditional dependence structure and detect XOR-type synergy. We also discuss a nonparametric approach to estimating these quantities from a single observed n-vertex multiplex under standard smoothness assumptions.

- **Giulio Zucal** : *Probability graphons and large deviations for random weighted graphs*

  Graph limit theory provides a rigorous framework for the convergence of dense graph sequences toward analytic objects known as graphons. We extend the theory of dense graph limits to weighted graphs and multiplex networks by introducing probability graphons: symmetric, measurable functions from the unit square into the space of probability measures. We establish a Large Deviation Principle for random weighted graphs that generalizes the seminal framework of Chatterjee and Varadhan (2011) for the Erdös-Rényi model. A key component of our approach is the analysis of the cut-metric topology on these generalized graphon spaces and the characterization of the resulting rate functions via relative entropy (joint work with P. Dionigi). We conclude by applying these variational results to the study of edge-colored Exponential Random Graph Models (ERGMs), providing new insights into their limiting behavior (ongoing work with B.B. Bhattacharya, P. Dionigi, and A. Ganguly).