# GEOMETRIZATION OF THE LOCAL LANGLANDS CORRESPONDENCE: TITLES AND ABSTRACTS

## 1. Mini-courses

- Johannes Anschütz and Arthur-César Le Bras

  **Title:** Geometric aspects of the $p$-adic locally analytic Langlands correspondence

  **Abstract:** In this series of 4 talks, we will outline how various aspects of the geometrization of the classical local Langlands correspondence by Fargues-Scholze admit meaningful analogs in the p-adic Langlands program for locally analytic representations of p-adic reductive groups. Based on the crucial notion of a thickened Fargues–Fontaine curve we will explain how to define 1) a stack $\mathrm{Bun}_G^{la}$ geometrizing locally analytic representations, and 2) a moduli space $\mathrm{Div}^1$ of degree 1 divisors geometrizing $(\varphi, \Gamma)$-modules. Then we will discuss aspects of a Hecke action linking 1) and 2). This is work (very much) in progress with Juan Esteban Rodriguez Camargo, Peter Scholze, and partially Guido Bosco.

- Mingjia Zhang

  **Title:** Igusa stacks and applications

  **Abstract:** Following a conjecture of Scholze, Igusa stacks have been introduced to study the geometry and cohomology of Shimura varieties, leading to several interesting results, including a new proof of the Eichler-Shimura relations and Yang's proof of generalized Ihara's lemma. In these talks, I will explain the following two topics:
    1. the construction of Igusa stacks for abelian type Shimura varieties and applications;
    2. intersection complexes on Igusa stacks and applications to the intersection cohomology of Shimura varieties.
    These results are based on joint work in progress with Daniels-van Hoften-Kim and Caraiani-Hamann.

## 2. Talks

- Lucas Gerth

  **Title:** Local Shimura varieties from analytic $p$-divisible groups

  **Abstract:** We prove a equivalence between families of analytic $p$-divisible groups on adic spaces $S$, and Hodge–Tate triples, generalizing a theorem of Fargues. From this, for $S$ a perfectoid space, we construct a Dieudonné functor, with values in mixed characteristic shtukas over the relative Fargues–Fontaine disc. As applications, we realize the local Shimura varieties of EL and PEL type of Scholze–Weinstein as moduli spaces of analytic p-divisible groups with framed universal cover, and we reinterpret the Hodge–Tate period map of Scholze–Cariani in terms of topological p-torsion subgroups of abelian varieties.

- David Helm

  **Title:** A generic categorical local Langlands correspondence for unramified classical groups

  **Abstract:** For general linear groups, one can prove a weak form of the categorical local Langlands correspondence by a "reduction to the unipotent block". This was carried out in

joint work of the speaker and Ben-Zvi, Chen, and Nadler, and is a key input in the forthcoming proof of the full Fargues-Scholze correspondence for $GL_n$ by Hansen and Mann. We will show how, for a large class of quasi-split groups including unramified classical groups $G$, one can adapt this construction to obtain a categorical Langlands correspondence for "generic" blocks of $G$. This is joint work with Yujie Xu.

- Sean Howe

  **Title:** Differential aspects of $p$-adic moduli spaces

  **Abstract:** We will explain some differential computations on the moduli spaces arising in Fargues–Scholze's geometrization of the local Langlands correspondence and related moduli spaces in $p$-adic and real Hodge theory, discuss some applications of these computations to the geometry of period maps, and then pose some questions about the relation between these computations, geometric representation theory, and the $p$-adic and real local Langlands correspondences.

- Naoki Imai

  **Title:** Eisenstein series functors and categorical local Langlands correspondence

  **Abstract:** In geometric Langlands, it is known that the geometric Langlands functor is compatible with Eisenstein series functors, which is a key ingredient in the proof of the geometric Langlands conjecture. In this talk, we discuss how to prove this compatibility for $GL_2$ in the setting of Fargues–Scholze. This is based on joint work in progress with Linus Hamann.

- Tasho Kaletha

  **Title:** Uniqueness of the local Langlands correspondence

  **Abstract:** I will discuss joint work with Chi-Heng Lo and Cheng-Chiang Tsai towards establishing a list of conditions that uniquely characterizes the classical local Langlands correspondence for general connected reductive groups over $p$-adic fields and complex coefficients.

- Si-Ying Lee

  **Title:** $p$-Isogenies with G-structure

  **Abstract:** I will talk about defining the notion of $p$-isogenies using the theory of F-gauges, and how this allows us to construct local and global Hecke correspondences. I will also discuss some expected consequences of this, such as constructions of Rapoport-Zink spaces, and formal Igusa stacks. This is joint work in progress with Keerthi Madapusi.

- Joao Lourenço

  **Title:** Comparison of sheaf categories on $\mathrm{Isoc}_G$ and $\mathrm{Bun}_G$

  **Abstract:** In the geometrization of local Langlands for p-adic fields, 2 different categories of étale sheaves have appeared on the geometric/automorphic/constructible side in the works of Fargues–Scholze and Zhu, namely those coming from the moduli stack of G-torsors on the Fargues–Fontaine curve, and those coming from the moduli stack of G-isocrystals. They offer somewhat complementary advanteges, the first being more suited for studying Hecke operators and Eisenstein series, whereas the second is more suited for taking categorical traces. We will explain how to construct an equivalence between these categories with torsion coefficients. This is joint work with Gleason, Hamann, Ivanov and Zou.

- Lucas Mann

  **Title:** Towards a proof of the $\ell$-adic categorical local Langlands correspondence

  **Abstract:** In joint work with David Hansen we leverage classical results on the $\ell$-adic local Langlands correspondence in order to prove the categorical correspondence (in the sense of Fargues–Scholze) for $GL_n$ and potentially all classical groups – conditional on a work-in-progress compatibility of Fargues–Scholze's construction with parabolic induction. Along the way we derive new results on ind-coherent sheaves on the stack of L-parameters and on ind-coherent sheaves on algebraic stacks in general.

- Yutaro Mikami

  **Title:** The $p$-adic monodromy theorem for families over relatively discrete algebras

  **Abstract:** The analytic Emerton-Gee stack is the rigid analytic moduli stack of $(\varphi, \Gamma)$-modules over the Robba ring and plays a central role in the formulation of the categorical p-adic local Langlands correspondence proposed by Emerton, Gee, and Hellmann. The result of Rodrigues Jacinto and Rodríguez Camargo in the case of $GL_1$ suggests that, as test objects for the analytic Emerton–Gee stack, one should consider algebraic-affinoid $\mathbb{Q}_p$-algebras, which are "combinations of discrete algebras and affinoid $\mathbb{Q}_p$-algebras". Motivated by this observation, I introduce the families of $(\varphi, \Gamma)$-modules parametrized by algebraic-affinoid $\mathbb{Q}_p$-algebras. I then explain how the p-adic monodromy theorem for families proved by Berger and Colmez can be generalized to this setting.

- Jared Weinstein

  **Title:** Excursion functions on $p$-adic groups

  **Abstract:** I present material from my student Jacksyn Bakeberg's thesis. The Bernstein center of a $p$-adic group is its ring of conjugation-invariant distributions; this ring controls the representations of the group. Fargues-Scholze gives a geometric construction of a subring of the Bernstein center, consisting of excursion operators, which are labeled by elements of the Galois group. One can think of these operators as an encoding of the Langlands correspondence. It would be interesting to give a completely explicit description of these excursion operators. We do exactly this in the case of $G = \mathrm{SL}_2$, where we show that the excursion operator is represented by a function (an "excursion function") on a dense open locus.

- Konrad Zou

  **Title:** The spectral action at bad primes

  **Abstract:** In seminal work of Fargues and Scholze, an action of the category $\mathrm{IndPerf}(\mathrm{LS}_{\check{G}}$ on the category of sheaves on $\mathrm{Bun}_G$ is constructed, pinned down by the fact that it is compatible with Hecke operators. In positive characteristic, working with $\overline{\mathbb{F}_\ell}$, we need to assume that $\ell \nmid \pi_0(Z(G))$. This result is sharp, as proven by a no-go theorem there. Thus the question arises, what additional datum other than Hecke operators might one require to go beyond $\ell \nmid \pi_0(Z(G))$? In this talk, we propose the answer to be a compatibility datum for extensions of the center. Up to a small modification of the category $\mathrm{IndPerf}(\mathrm{LS}_{\check{G}}$, we can descend spectral actions along central extensions on $\check{G}$, we may thus always assume that $G$ has connected center. We will explain how one can construct a descent datum in the context of the Fargues-Scholze program. If time permits, we will discuss how one can show that all descent data are effective, which also applies to other situations in modular geometric representation theory.