p-ADIC ASPECTS OF THE LANGLANDS PROGRAM: TITLES AND ABSTRACTS

1. Mini-courses

- Eugen Hellmann

Title: Moduli spaces of equivariant bundles on the Fargues-Fontaine curve and categorical p-adic Langlands

Abstract: In this series of two talk I will describe some older and some recent constructions and results about the geometry of stacks of equivariant $\check{G}$-bundles on the Fargues-Fontaine curve. These stacks (with $\check{G}$ being the dual group of a fixed reductive group over $\mathbb{Q}_p$) replace the stacks of L-parameters in a categorical approach to a locally analytic p-adic Langlands correspondence. I will describe some expected features of this correspondence, focussing on the counterpart on the Galois side of some "additional Hecke operators" in the locally analytic world and on the relation with (geometric) properties of eigenvarieties.

- Wiesława Nizioł

Title: p-adic local Langlands correspondence and the Drinfeld tower

Abstract: I will survey the geometrization of the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ in etale cohomology of the Drinfeld tower due to Colmez–Dospinescu–Niziol. In the first lecture I will focus on cohomological foundations; in the second one on computations of etale cohomology of the tower.

2. Talks

- Ana Caraiani

Title: Towards an Eichler-Shimura decomposition for ordinary p-adic Siegel modular forms

Abstract: There are two different ways to construct families of ordinary p-adic Siegel modular forms. One is by p-adically interpolating classes in Betti cohomology, first introduced by Hida and then given a more representation-theoretic interpretation by Emerton. The other is by p-adically interpolating classes in coherent cohomology, once again pioneered by Hida and generalised in recent years by Boxer and Pilloni. I will explain these two constructions and then discuss joint work in progress with James Newton and Juan Esteban Rodríguez Camargo that aims to compare them.

- Yiwen Ding

Title: $\mathrm{Fil}^{\max}$ and $\mathrm{Fil}^{\text{2nd-max}}$

Abstract: Let ρ be an n-dimensional de Rham representation of $\mathbb{Q}_p$ with regular Hodge–Tate weights. A central problem in the p-adic Langlands program is to recover the Hodge filtration on $D_{\mathrm{dR}}(\rho)$ on the automorphic side. We first recall Breuil's conjecture on how to see $\mathrm{Fil}^{\max}$ of $\wedge^i D_{\mathrm{dR}}(\rho)$, for $i = 1, \ldots, n-1$—the one-dimensional non-zero Hodge filtration of maximal index—via certain extensions of locally analytic $\mathrm{GL}_n(\mathbb{Q}_p)$-representations. We then discuss the existence of extra locally algebraic constituents in the crystalline case, and show

that the corresponding extensions allow us to see various $\mathrm{Fil}^{\mathrm{2nd}-\max}$, which are enough to determine the full Hodge filtration. This is a joint work with Christophe Breuil.

- Gabriel Dospinescu

  Title: $\mathfrak{n}$-homology and the p-adic Jacquet–Langlands correspondence

  Abstract: I will try to explain the role of $\mathfrak{n}$-homology in the study of Scholze's functor between locally analytic representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ and those of its inner forms, as well as in the study of the coherent cohomology of the Lubin–Tate tower and of Hecke eigenspaces in overconvergent modular forms. This is joint work, very much in progress, with Benchao Su and Juan Esteban Rodriguez-Camargo.

- Toby Gee

  Title: The reduction mod p of crystalline Galois representations

  Abstract: I will report on joint work with Bhargav Bhatt and Mark Kisin on the inertial weights of the reduction mod p of local p-adic Galois representations.

- Florian Herzig

  Title: Global candidates and the locality question for GL_2

  Abstract: Given a 2-dimensional mod p Galois representation ρ of Gal_K, where K is a non-trivial finite unramified extension of $\mathbb{Q}_p$, we hope to associate a smooth mod p representation $\pi(\rho)$ of $\mathrm{GL}_2(K)$. If ρ arises globally from automorphic forms there is a natural (non-canonical) global candidate for $\pi(\rho)$. We discuss some of what is known and hoped for regarding these global candidates. This is joint work with C. Breuil, Y. Hu, K. Kozioł, S. Morra, B. Schraen and S.W. Shin.

- Yuanyang Jiang

  Title: Locally analytic completed cohomology of Hilbert modular varieties

  Abstract: We generalize a result of Lue Pan on locally analytic completed cohomology of modular curves to the case of Hilbert modular varieties. As an application, we prove that for parallel weight Hecke classes appearing in the completed cohomology of Hilbert modular varieties, de Rhamness of the associated Galois representation will imply classicality. One central problem is to understand certain "partial de Rham cohomology", which we will prove to be classical by developing a locally analytic version of Jacquet–Langlands transfer of D-modules using theory of analytic de Rham stacks.

- Kalyani Kansal

  Title: Aspects of the geometry of the Emerton–Gee stack and applications to local–global compatibility

  Abstract: Let F be a totally real number field, unramified at p. In this talk, we will discuss some aspects of a proof of local-global compatibility at p for mod p representations arising from degree zero cohomology of partial weight one Hilbert modular forms. We will assume the other weights are sufficiently small, i.e. between 1 and $p+1$. This proof comes from work in progress, joint with Levin and Savitt. The discussion will focus primarily on accessing the geometry of the Emerton—Gee stack parametrizing two dimensional mod p and p-adic Galois representations of a finite unramified extension of $\mathbb{Q}_p$, and in particular how loci parameterizing mod p representations admitting crystalline lifts with specific (small) Hodge—Tate weights sit inside

the stack. This will also include ideas from the work of Le Hung, Mezard and Morra, as well as my previous joint works with various collaborators.

- Hiroki Kato

  Title: An arithmetic approach to an ordinarity conjecture in commutative algebra

  Abstract: I will discuss an ordinarity conjecture concerning invariants appearing in commutative algebra/singularity theory, especially focusing on its relationship to the ordinarity conjecture on the Newton and Hodge polygons of motives defined over number fields. The bridge between these settings is provided by a reinterpretation of Bernstein–Sato theory via nearby cycles. As a consequence, we will see that existing (very rare) known instances of the conjecture for motives already yield some new cases in commutative algebra. Joint work with Eamon Quinlan-Gallego and Daichi Takeuchi.

- Vaughan McDonald

  Title: Eigenvarieties over CM fields and Galois representations

  Abstract: Eigenvarieties are parameter spaces for finite slope p-adic automorphic forms of varying weight. These objects have become increasingly popular for studying the Fontaine—Mazur conjecture, which leads us to ask what kinds of Galois representations appear on eigenvarieties. Our main result shows that for eigenvarieties for the group GL_n over a CM field, the associated Galois representations are trianguline at all p-adic places, resolving a conjecture of Hansen (following Kisin, Colmez, Bellaiche—Chenevier). The strategy of proof (which could be of independent interest) is to embed eigenvarieties for GL_n into an eigenvariety for a $2n$-variable unitary group.

- Vytautas Paškūnas

  Title: On mod p Jacquet-Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$

  Abstract: We show that the restriction of Scholze's functor to blocks in the category of locally admissible mod p representations of GL2(Qp), corresponding to very generic rhobar, is fully faithful. This is joint work in progress with Yongquan Hu.

- Juan Esteban Rodríguez Camargo

  Title: Cartier duality in p-adic Hodge theory

  Abstract: Cartier duality is a fascinating phenomenon across different incarnations of algebraic geometry. However, a proper foundation of the theory has been missing for years, and it was only recently found by Scholze and Stefanich in their theory of Gestalten. In this talk, I will discuss some applications of their general formalism in p-adic Hodge theory, more precisely, in the Cartier duality between the Simpson gerbe and the Hodge-Tate stack, and the Cartier duality for locally analytic Banach-Colmez spaces.

- Benjamin Schraen

  Title: Multiplicities in the socle of locally analytic representations and functors on the category O

  Abstract: A conjecture of Breuil describes the isomorphism classes of finite slope, locally analytic representations that appear in the socle of the locally analytic representation $\Pi(\rho)$ of $\mathrm{GL}_n(\mathbb{Q}_p)$ (conjecturally) associated to a potentially semistable n-dimensional representation rho of the absolute Galois group of Qp. When Pi(rho) comes from the global framework of

p-adic automorphic forms on definite unitary groups, this conjecture has been established in the crystalline case under some mild hypotheses. However, the problem of determining the multiplicities of these finite slope representations in the socle remained unsolved. In joint work with Hellmann and Hernandez, we proposed a conjectural description of these multiplicities in terms of an exact functor on the category O constructed by Bezrukavnikov. In this talk, I will discuss the proof new cases of this conjecture. This is a joint work with Eugen Hellmann and Valentin Hernandez.

- Benchao Su

  Title: Locally analytic vectors in the completed cohomology of quaternionic Shimura curves

  Abstract: We use the methods introduced by Lue Pan to study the locally analytic vectors of the completed cohomology of Shimura curves associated to an indefinite quaternion algebra D which is ramified at a prime number p. Let $D_p^\times$ be the group of units of D at p. Using p-adic uniformization of the quaternionic Shimura curves, we compute the Hecke eigenspace of the completed cohomology with the Hecke eigenvalues associated to a classical automorphic form on another quaternion algebra $\bar{D}$ (switching invariants of D at p, ∞). We present this locally analytic $D_p^\times$-representation using the de Rham complex of the Lubin–Tate tower of dimension 1. This is analogous to the Breuil–Strauch conjecture for the group $\mathrm{GL}_2(\mathbb{Q}_p)$. We will also give some applications to the p-adic Langlands program. This is a joint work with Zhenghui Li and Zhixiang Wu.