WINTER SCHOOL ON GALOIS REPRESENTATIONS AND AUTOMORPHIC FORMS: ABSTRACTS

1. Representation Theory of p-adic reductive groups (Anne-Marie Aubert)

We will start by introducing the p-adic reductive groups and some of their subgroups, including parabolic and Levi subgroups, maximal compact subgroups, parahoric subgroups and their reductive quotients. A p-adic reductive group is a topological group $G = \mathbb{G}(F)$, where F is a nonarchimedean local field whose residue field has characteristic p, and $\mathbb{G}$ is a connected reductive group over F. These groups G include groups like $\mathrm{GL}_n(F)$, $\mathrm{PGL}_n(F)$, $\mathrm{SL}_n(F)$, $\mathrm{Sp}_{2n}(F)$, $\mathrm{SO}_n(F)$, their inner forms $\mathrm{GL}_m(D)$, $\mathrm{PGL}_n(D)$, $\ldots$, with D/F a division algebra, and the groups of exceptional types G_2, F_4, E_6, E_7 and E_8.

We will next set out the basic properties of the representations of G, focusing on the different possible notions of inductions. Numerous examples of representations will be provided, e.g. supercuspidal representations, Iwahori-spherical representations and unipotent representations.

Then we will study the category $\mathbf{R}(G)$ of smooth complex representations of G: give its decomposition into a product of indecomposable full subcategories, the *Bernstein blocks*, introduce certain twisted affine Hecke algebras, and use them to analyze the structure of the blocks. In the last part of the lectures, we will introduce enhanced Langlands parameters and mirror the decomposition of $\mathbf{R}(G)$ on the Galois side of the local Langlands correspondence.

Bibliography:

[1] *A.-M. Aubert*, Representation Theory of p-Adic Reductive Groups, in "K-Theory and Representation Theory", LMS Textbook Series 494, Cambridge University Press, 2024.

[2] *A.-M. Aubert*, Correspondences between affine Hecke algebras and applications, arXiv:2311.03203, to appear in "Representations and characters: revisiting some aspects of the works of Harish-Chandra and Weil", Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore.

[3] *T. Kaletha and G. Prasad*, Bruhat-Tits Theory: a new approach, New Mathematical Monography 44, Cambridge University Press, 2023.

[4] *D. Renard*, Représentations des groupes réductifs p-adiques, Société Mathématique de France, https://perso.pages.math.cnrs.fr/users/david.renard/livre.pdf

2. Automorphic forms, automorphic periods (Pierre-Henri Chaudouard)

The course will be an introduction to some basic notions on automorphic forms. The focus will be on the construction of some L-functions and the links between certain integrals of automorphic forms, called periods, and certain special values of these L-functions.

Prerequisites: we will recall the basic facts we need. However, some basic knowledge of adèles will be helpful (see e.g. chapter 5 of [6] or section 3 of [5] or beginning of chapter 2 of Getz-Hahn)

Bibliography:

[1] *D. Bump*, Automorphic forms and representations, Cambridge Univ. Press, Cambridge, 1997;

[2] *Bump, D.; Cogdell, J. W.; de Shalit, E.; Gaitsgory, D.; Kowalski, E.; Kudla, S.* An introduction to the Langlands program. Lectures presented at the Hebrew University of Jerusalem, Jerusalem, March 1216, 2001. Edited by Joseph Bernstein and Stephen Gelbart. Birkh auser Boston)

[3] Codgell's lectures in *Cogdell, James W. ; Kim, Henry H. ; Murty, M. Ram* Lectures on automorphic L-functions. Fields Institute Monographs, 20. American Mathematical Society, Providence,

[4] *Getz, J. R. ; Hahn, H.* An introduction to automorphic representations with a view toward trace formulae. Graduate Texts in Mathematics, 300.

[5] *Knapp, A. W.* Introduction to the Langlands program. Representation theory and automorphic forms (Edinburgh, 1996), 245302, Proc. Sympos. Pure Math., 61, Amer. Math. Soc.,

[6] *Ramakrishnan, D. ; Valenza, R.* Fourier analysis on number fields. Graduate Texts in Mathematics, 186. Springer-Verlag

3. Geometric Sen theory, locally analytic representations and the Drinfeld tower (Gabriel Dospinescu)

This course will be an introduction to the methods introduced by Lue Pan and further developped by Rodriguez Camargo and Benchao Su, Tian Qiu, giving detailed information about the locally analytic vectors in the completed cohomology of Shimura varieties.

Bibliography:

[1] *Lue Pan* On locally analytic vectors of the completed cohomology of modular curves II. To appear in Annals of Mathematics. arXiv:2209.06366

[2] *Lue Pan* On locally analytic vectors of the completed cohomology of modular curves. Forum of Mathematics, Pi , Volume 10 , 2022 , e7

[3] *J. E. Rodríguez Camargo* Locally analytic completed cohomology arXiv:2209.01057

[4] *J. E. RodrÃguez Camargo* Geometric Sen theory over rigid analytic spaces , To appear in J.E.M.S, arXiv:2205.02016

[3] *B. Su and T. Qiu* Locally analytic vectors in the completed cohomology of unitary Shimura curves

4. Geometrization of the Langlands correspondence (Linus Hamann)

Let $G/\mathbf{Q}_p$ be a connected reductive group over the p-adic numbers. The local Langlands correspondence is a bridge between the analytic world (smooth irreducible representations of $G(\mathbf{Q}_p)$) and the arithmetic world (representations of the Weil group of $\mathbf{Q}_p$). Recently, Fargues and Scholze recast this correspondence in terms of what has become known as the categorical local Langlands conjecture, replacing smooth representations with certain ℓ-adic sheaves on Bun_G, the moduli stack of G-bundles on the Fargues-Fontaine curve, and Weil group representations with coherent sheaves on the moduli stack of such representations. On the one hand, this encodes the usual local Langlands correspondence and its various refinements in a very beautiful way. On the other hand, it also gives insight into other important phenomena in arithmetic geometry. For example, both sides of this correspondence are equipped with a Hecke action that encodes information about how local Langlands is realized in the cohomology of local and global Shimura varieties. The goal of the course will be to give a survey of these recent developments, highlighting its connection to classical local Langlands and the cohomology of Shimura varieties.

Bibliography:

Categorical Local Langlands

[1] Geometrization of the local Langlands correspondence, L. Fargues and P. Scholze

[2] Eilenberg/Hausdorff Lectures on the geometrization of the local Langlands correspondence, L. Fargues

[3] Beijing notes on the categorical local Langlands conjecture, D. Hansen

[4] Tame Categorical Langlands Correspondence, X. Zhu

Smooth Representations and Classical Local Langlands:

[5] The B(G)-parametrization of the local Langlands Correspondence, A. Bertolini-Meli and M. Oi

[6] The local Langlands conjecture, T. Kaletha and O. TaÃ¯bi

[7] The local Langlands conjecture for non quasi-split groups, T. Kaletha

[8] Representations of p-adic groups (1992 Lecture notes), J. Bernstein

p-adic geometry and Shimura Varieties:

[9] Towards a theory of local Shimura varieties, M. Rapoport and E. Viehmann

[10] Berkeley Lectures on p-adic geometry, J. Weinstein and P.Scholze

[11] Etale Cohomology of Diamonds, P. Scholze

[12] A PEL Type Igusa stack and the p-adic geometry of Shimura varieties, M. Zhang

5. A geometric realization of affine Hecke algebras (Peng Shan)

The goal of these lectures is to explain a geometric realization of affine Hecke algebras using equivariant K-theory of Steinberg variety discovered by Kazhdan-Lusztig and Ginzburg. We will start by basic properties of the Springer resolution and Steinberg varieties as well as a brief introduction of equivariant K-theory. Then we will explain the geometric realisation of affine Hecke algebras. Finally, we will explain how this was used to obtain the Deligne-Langlands classification of irreducible modules for affine Hecke algebras.

Bibliography:

[1] *Kazhdan, D., and Lusztig, G.* Equivariant K-Theory and Representations of Hecke Algebras. II. Inventiones Mathematicae 80, no. 2 (1985): 209–31. https://doi.org/10.1007/BF01388604.

[2] *Kazhdan, D. and Lusztig, G.* Proof of the Deligne-Langlands Conjecture for Hecke Algebras. Inventiones Mathematicae 87, no. 1 (1987): 153–215. https://doi.org/10.1007/BF01389157.

[3] *Chriss, N. and Ginzburg, V.* Representation Theory and Complex Geometry. BirkhÃ¤user Boston, 2010. http://link.springer.com/10.1007/978-0-8176-4938-8. Chapter 3, and chapters 5-8.