Diophantine Approximation and Transcendence

Cirm – November 10−14, 2025

Program

Shabnam Akhtari

Pennsylvania State University (USA)

Title : *Monogenic Orders in Quartic Number Fields*

Abstract : An order O in a number field is called monogenic if it can be generated by one element over the integers, that is $O = \mathbb{Z}[\alpha]$. In this case, we call α a monogenizer of O. Since $\mathbb{Z}[\alpha] = \mathbb{Z}[\pm\alpha + c]$, for any integer c, we call two algebraic integers α and α' equivalent if $\alpha + \alpha'$ or $\alpha - \alpha'$ is a rational integer. By a monogenization of O, we mean an equivalence class of monogenizers of O. Győry has shown that there are finitely many monogenizations for a given order. An interesting problem is to count the number of monogenizations of a given monogenic order. First, we will observe, for a given order O, that $O = \mathbb{Z}[\alpha]$ in α, is indeed a Diophantine equation, namely an index form equation. Then we will modify some algorithmic approaches, due to Gaál, Pethő and Pohst for finding solutions of index form equations in quartic number fields to obtain new and improved upper bounds for the number of monogenizations of a quartic order. This approach allows us to explore questions about algebraic relations between monogenizers of a given order, which are inspired by the work of Bérczes, Evertse, and Győry on multiply monogenic orders.

Demi Allen

University of Exeter (UK)

Title : *An inhomogeneous Khintchine–Groshev Theorem without monotonicity*

Abstract : The classical (inhomogeneous) Khintchine–Groshev Theorem tells us that for a monotonic approximating function $\psi : \mathbb{N} \to [0, \infty)$ the Lebesgue measure of the set of (inhomogeneously) ψ-well-approximable points in $\mathbb{R}^{nm}$ is zero or full depending on, respectively, the convergence or divergence of $\sum_{q=1}^{\infty} q^{n-1}\psi(q)^m$. In the homogeneous case, it is now known that the monotonicity condition on ψ can be removed whenever $nm > 1$, and cannot be removed when $nm = 1$. In this talk, I will discuss recent progress towards removing monotonicity from the inhomogeneous Khintchine–Groshev Theorem. This talk will mostly be based on some joint work with Felipe Ramírez (Wesleyan, US) as well as work in progress with Manuel Hauke (Graz, Austria) and Felipe Ramírez (Wesleyan, US).

Dmitry Badziahin

University of Sydney (Australia)

Title : *On the t-adic Littlewood "conjecture" over finite fields*

Abstract : In the last years there was a lot of interest in various analogues of the famous Littlewood conjecture in Diophantine approximation. One of them is the so-called t-adic Littlewood conjecture (t-LC) that states as follows. Let $\mathbb{F}$ be a finite field and $\mathbb{F}((t^{-1}))$ be the space of Laurent series over $\mathbb{F}$ with the induced t-adic norm. For the series $f(t)$ from this space, let $||f(t)||$ be the distance from it to the nearest polynomial. Then for all series $f(t)$, the infimum of the expression $|N| \cdot ||N \cdot t^k \cdot f(t)||$ where N runs through all polynomials in $\mathbb{F}[t]$ and k runs through all nonnegative integers, equals zero.

It was widely believed that the Littlewood conjecture together with all its analogues are true unless the recent work of Adiceam, Lunnon and Nesharim who constructed a counterexample for the t-adic Littlewood conjecture when $char(\mathbb{F}) = 3$. Later, Garrett and Robertson constructed counterexamples for fields of characteristics between 3 and 11. Their method is heavily computational and does not allow to disprove the t-LC for any infinite series of characteristics.

In this talk I will speak about our recent work with Adiceam where we manage to provide a purely mathematical way that provides counterexamples to the t-LC for all characteristics that are congruent to 3 modulo 4.

François Ballaÿ

University of Caen Normandie (France)

Title : *A generalization of Yuan's equidistribution theorem*

Abstract : Yuan's equidistribution theorem (2008) describes the asymptotic distribution of the Galois orbits of generic sequences of algebraic points in a projective variety over a number field, under the assumption that the heights of the points converge to the normalized height of the variety. This theorem applies for heights arising from polarized dynamical systems, which include the case of Néron-Tate heights on abelian varieties previously treated by Szpiro, Ullmo and Zhang. I will present a generalization of Yuan's theorem that allows more flexibility in the choice of the height function. This result can be applied to some non-polarized dynamical systems, and recovers Kühne's equidistribution theorem on semi-abelian varieties. This is joint work with Martín Sombra (ICREA & Universitat de Barcelona).

Fabrizio Barroero

University of Roma Tre (Italy)

Title : *Distinguished categories, Σ-orbits and Mordell-Lang*

Abstract : In this talk, I will report on a joint work with Gabriel Dill. A few years ago we introduced distinguished categories with the aim of providing a general formal framework in which certain statements and facts about unlikely intersections can be formulated and proved at the same time in several different settings, e.g., for semi-abelian varieties and connected mixed Shimura varieties. More recently, inspired by work of Aslanyan and Daw, we introduced, in the same general framework, the notion of Σ-orbits which generalizes both finite rank subgroups of abelian varieties as well as Hecke orbits in Shimura varieties. We showed that our Σ-orbits have

useful functorial properties and we used them to formulate two general statements of Mordell-Lang type. We obtained some conditional as well as some unconditional results.

Laura Capuano

University of Roma Tre (Italy)

Title : *Singular intersections of curves and subgroups in tori and in abelian varieties*

Abstract : It can be proved that, if X is an irreducible curve in a torus defined over a number field, then the set of all the points of X lying in a proper algebraic group is always infinite, even under the hypothesis that X is not contained in a proper algebraic group. In a joint work with F. Ballini and N. Ottolini we prove that, if one looks at the multiplicities of intersections, the set of points where this intersection is singular is a finite set. This statement, which fits in the general framework of problems of Unlikely Intersections, generalizes a previous result of Marché and Maurin for curves in a torus of dimension 2.

Sara Checcoli

Institut Fourier - University of Grenoble-Alpes (France)

Title : *If a machine did it, it is probably transcendental (even p-adically)*

Abstract : Continued fraction expansions form a well-established bridge between algebraic properties of numbers and combinatorics on words.

Beyond the classical real case, one may also consider p-adic continued fractions, which have received increasing attention in recent years.

In this talk, we study the algebraicity of numbers whose p-adic continued fractions are generated by classes of words that generalize the classical automatic, periodic, and palindromic ones. Our main theorem shows that, under mild assumptions, such numbers are necessarily either algebraic of degree at most two or transcendental.

This result provides a p-adic counterpart to the theorems of Bugeaud and Adamczewski–Bugeaud in the real setting, and at the same time extends previous works that were confined to specific choices of p-adic floor functions and to more restrictive classes of words.

This is joint work with Laura Capuano, Marzio Mula and Lea Terracini.

Christopher Daw

University of Reading (UK)

Title : *Some new cases of Zilber-Pink in $Y(1)^3$*

Abstract : I will discuss joint work with M. Orr (Manchester) and G. Papas (IAS) on the Zilber-Pink conjecture for $Y(1)^3$. This is known for so-called asymmetric curves by the 2012 work of Habegger-Pila. More recently, an approach known as the G-function method, has yielded further cases, namely, curves intersecting (∞, ∞, ∞) (D-Orr) and curves intersecting a

special point in the boundary (Papas). In this work, we extend the method to deal with curves intersecting a boundary modular curve, and to give an unconditional result for points with few places of supersingular reduction.

Julian Demeio

University of Hannover (Germany)

Title : *(Absence of) Bounded generation and semi-simplicity*

Abstract : Using methods from Diophantine approximation, namely a theorem of Evertse based on the Schlickewei-Schmidt Subspace theorem, we prove that linear groups boundedly generated by semi-simple elements are necessarily virtually abelian. To do so, we frame bounded generation in the more general context of (Purely) Exponential Parametrizations (PEP) for subsets of affine spaces, a concept which unifies different issues. Our study shows in particular that for a (PEP) set over a number field, the asymptotic distribution of its points of Height at most T is always $\sim c(\log T)^r$, with certain constants $c > 0$ and $r \in \mathbb{Z}_{\geq 0}$. (This shape fits with a well-known viewpoint first put forward by Manin.). This is joint work with Pietro Corvaja, Andrei Rapinchuk, Jinbo Ren and Umberto Zannier

Gabriel Dill

University of Neuchâtel (Switzerland)

Title : *Points of small height and where to find them – a group-theoretic criterion*

Abstract : The height of an algebraic number is a measure for its arithmetic complexity. While numbers of height zero are classified by Kronecker's theorem (they are precisely 0 and the roots of unity), many questions remain open about numbers of small but non-zero height. The famous conjecture of Lehmer predicts an essentially best possible lower bound for the height of such numbers. A related question is whether, given an algebraic extension of the rationals, it contains numbers of arbitrarily small height.

Rémond formulated a generalization of Lehmer's conjecture, which yields a characterization of points of small height on abelian varieties or algebraic tori that are defined over some infinite algebraic extensions of the rational numbers, generated by the division points of certain finitely generated subgroups. For instance, the field generated over the rationals by all roots of 2 contains some obvious points of very small height (0, small fractional powers of 2 multiplied by roots of unity) and the conjecture implies that it contains no others.

If the finitely generated subgroup is of positive rank, then even weaker versions of Rémond's conjecture (analogues of Dobrowolski's theorem) are wide open. Recently, Pottmeyer identified a necessary group-theoretic condition for Rémond's conjecture to hold and showed that it is satisfied for the multiplicative group. In joint work with Sara Checcoli, we show that Pottmeyer's condition is also satisfied for arbitrary almost split semiabelian varieties, using results from Kummer theory.

Vesselin Dimitrov

Caltech (USA)

Title : *The arithmetic of power series, II*

Abstract : I will explain how the classical Thue hypergeometric Pade method gets encoded into the formal framework of the previous talk, and how it continues with the more recent methodology of multivalent holonomy bounds. My focus here will be on the algebraic Apery limits, and therefore on effectivity. I will outline a self-contained proof of a basic, but completely explicit holonomy bound that includes a Diophantine approximation term. The hypergeometric reduction is itself a simple consequence of the observation that, with the correct normalization, the generating functions of the usual Hermite-Pade binomial forms turn out to be dihedral algebraic functions. In this way, we readily effectivize the Thue-Siegel square root exponent for Thue's original special case of high order roots from a fixed rational number. A well-known geometry of numbers argument of Bombieri's allows to thereafter recover an effective height bound on the solutions to the general S-unit equation in two variables. These are not new results, but their proofs are simple enough to explain in full at the 30 minutes talk. This is joint work with Frank Calegari and Yunqing Tang. Ongoing efforts (which we are happy to discuss) are under way to extend this method over to less smooth-going cases where Baker's linear forms in logarithms, or the Bombieri-Cohen equivariant Thue-Siegel method, no longer help.

Colin Faverjon

University Picardie Jules Verne (France)

Title : *Mahler's method*

Abstract : In 2007, Adamczewski and Bugeaud proved that irrational automatic real numbers are transcendental, using the p-adic subspace theorem. This result had previously been wrongly announced as proved by Mahler's method. Only recently has a correct proof following Mahler's approach been given. This approach has the advantage of dealing with algebraic independence, whereas the subspace theorem mainly yields transcendence results. As a consequence, we obtain statements of the following type: irrational real numbers that are automatic in multiplicatively independent bases are algebraically independent — in particular, they are distinct. Furthermore, we are able to characterize the algebraic relations that might exist between real numbers that are automatic in the same base.

Ziyang Gao

University of California, Los Angeles (USA)

Title : *Generic positivity of the heights of Gross-Schoen and Ceresa cycles*

Abstract : In this talk, I will explain a generic positivity result of the Beilinson-Bloch height of Gross-Schoen and Ceresa cycles. We prove that given any number field K, the Gross-Schoen/Ceresa cycle associated with a generic curve has non-negative Beilinson-Bloch height. We obtain this result as a consequence of a Northcott property. This is joint work with Shouwu Zhang.

Thomas Gauthier

University Paris-Saclay (France)

Title : *Uniformity in the dynamical Bogomolov problem*

Abstract : The dynamical Bogomolov conjecture is a dynamical counterpart of the classical Bogomolov conjecture. Roughly speaking, it states that given a polarized endomorphism $f : X \to X$ of a projective variety and a subvariety $Z \subset X$, all defined over a number field, the subvariety contains a Zariski dense and small sequence for an appropriate canonical height function if and only if it is preperiodic - except for obvious counter-examples. In joint works with Johan Taflin, and with Johan Taflin and Gabriel Vigny, we study uniform versions of this conjecture. We prove several results. As a particular case, we provide a dynamical proof of a uniform version of a Bogomolov type statement for algebraic tori.

Richard Griffon

University Clermont Auvergne (France)

Title : *A parallelogram inequality for abelian varieties over function fields*

Abstract : Given an abelian variety A over a number field, and two finite subgroups G,H of A, Rémond has recently proved a so-called "parallelogram inequality" which relates the Faltings heights of the quotients of A by G, H, $G \cap H$, and $G + H$. I will talk about a very recent work — joint with Le Fourn and Pazuki — in which we prove a perfect analogue of Rémond's inequality in the context of abelian varieties over function fields, where the rôle of the Faltings height is played by the differential height. This adds to our understanding of how isogenies interact with the height. I will introduce the relevant notions beforehand, and discuss an application of the parallelogram inequality in the function field setting to a bound on the height of an abelian subvariety of a given abelian variety. Time permitting, I will sketch the proof of our result.

Manuel Hauke

Graz University of Technology (Austria)

Title : *On twin primes and twisted diophantine approximation*

Abstract : In this talk, I will speak about dynamics of $(a_n\alpha)_n \mod 1$ for integer sequences $(a_n)_n$ and fixed irrational rotations α. The focus will be on the sequence of primes and other multiplicatively defined sequences, where gap statistics as well as twisted diophantine approximation will be considered. If time permits, I will outline the proof that includes a sieve coming from the twin prime counting problem, and establishing via random walks on Ostrowski digits an equidistribution result on diophantine Bohr sets mod d. This talk partially based on `https://arxiv.org/abs/2506.01736` and joint work with E. Kowalski `https://arxiv.org/abs/2502.08335`.

Yining Hu

Harbin Institute of Technology (China)

Title : *Automata, Algebra, and Continued Fractions*

Abstract : I will give an introduction to automatic sequences, starting with a classical example. Then I will talk about its link with algebra and number theory. Finally I will talk about Diophantine analysis in positive characteristics and recent results on automatic continued fractions.

Antoine Marnat

University Paris Est Créteil - Val-de-Marne (France)

Title : *About bad approximability and ratios of successive best approximations*

Abstract : For a real $m \times n$ matrix $\boldsymbol{\xi}$, we consider its sequence of best Diophantine approximation vectors $\boldsymbol{x}_i \in \mathbb{Z}^n$, $i = 1, 2, 3, ...$, the sequences of its norms $X_i = \|\boldsymbol{x}_i\|$ and the norms of remainders $L_i = \|\boldsymbol{\xi}\boldsymbol{x}_i\|$. It is known that, in the cases $m = 1$, bad approximability of $\boldsymbol{\xi}$ is equivalent to the boundedness of ratios $\frac{X_{i+1}}{X_i}$, while for $n = 1$ bad approximability of $\boldsymbol{\xi}$ is equivalent to the boundedness of ratios $\frac{L_i}{L_{i+1}}$. Moreover, carefully constructed example show that in the cases $m = 1$ and $n = 1$ boundedness of ratios $\frac{L_i}{L_{i+1}}$ and $\frac{X_{i+1}}{X_i}$ respectively (the order of ratios changed), does not imply bad approximability of $\boldsymbol{\xi}$. In the present talk, we study the impact of the boundedness of ratios on Diophantine properties of $\boldsymbol{\xi}$, in particular, what restrictions it gives for Diophantine exponents $\omega(\boldsymbol{\xi})$ and $\hat{\omega}(\boldsymbol{\xi})$.

Myrto Mawraki

University of Toronto (Canada)

Title : *Towards quantitative versions in the dynamical Bogomolov conjecture*

Abstract : Inspired by the analogy between torsion points on abelian varieties and preperiodic points of rational maps, Shou-Wu Zhang formulated a dynamical generalization of the Manin–Mumford and Bogomolov conjectures. These conjectures aim to classify subvarieties of $\mathbb{P}^n$ that contain a Zariski-dense or "generic" sequence of preperiodic points for an endomorphism f of $\mathbb{P}^n$ – or more generally, points of small canonical height with respect to f. Despite significant progress in special cases, the conjectures remain largely open. In this talk, we will discuss recent progress towards uniform and quantitative versions of the dynamical Bogomolov conjecture. This is joint work in progress with Jit Wu Yap.

Ricardo Menares

Pontificia Universidad Católica (Chile)

Title : *On the essential minimum of height functions*

Abstract : This is joint work with José Burgos Gil and Martín Sombra.

We present novel upper bounds on the essential minimum of a height function defined on a curve. Our results improve on previous estimates on the essential minimum of some remarkable heights, such as the Zhang-Zagier height. Our methods combine capacity theory (Rumely's

extensions of the Fekete-Sezego theorem) as well as Yuan's theorem on the equidistribution of small points.

Martin Orr

University of Manchester (UK)

Title : *Effective height bounds for very unlikely intersections in abelian varieties*

Abstract : Following an idea of André, Bombieri's Hasse principle for relations between values of G-functions can be used to obtain effective height bounds for certain very unlikely intersections in an abelian variety A (i.e. intersections between a curve C and algebraic subgroups of A of large codimension). This in turn implies effective height bounds for rational points in some curves. In this talk, I will discuss explicit calculations of these bounds.

Arnaud Plessis

Beijing Institute of Mathematical Sciences and Applications (China)

Title : *Bilu's equidistribution theorem for non-Galois invariant sets and application to a conjecture of Rémond*

Abstract : Denote by h the (absolute, logarithmic) Weil height. The celebrated Bilu's equidistribution theorem claims that for any sequence $(\alpha_n)_n$ of algebraic numbers satisfying $[\mathbb{Q}(\alpha_n) : \mathbb{Q}] \to +\infty$ and $h(\alpha_n) \to 0$, the sequence of Galois orbits $(\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}).\alpha_n)_n$ is equidistributed around the unit circle. Following a method due to Mignotte, I will explain in the first part of this talk why every sequence $(S_n)_n$ of finite subsets of $\overline{\mathbb{Q}}^*$ satisfying $|S_n| \to +\infty$ and $|S_n|^{-1} \sum_{\alpha \in S_n} h(\alpha) \to 0$ is equidistributed "in average" around the unit circle, that is,

$$\frac{1}{[\mathbb{Q}(S_n) : \mathbb{Q}]} \sum_{\sigma : \mathbb{Q}(S_n) \hookrightarrow \mathbb{C}} \left| \frac{1}{|S_n|} \sum_{\beta \in \sigma S_n} f(\beta) - \int_0^1 f(e^{2i\pi t}) dt \right| \to 0$$

for all bounded continuous functions $f : \mathbb{C}^* \to \mathbb{C}$. This is joint work with Amoroso.

In the second part of this presentation, which is a work in progress, I will explain how this equidistribution in average might help us to solve the degree one form of Rémond's conjecture, a deep conjecture related to the Zilber-Pink conjecture.

Anthony Poëls

University Claude Bernard Lyon 1 (France)

Title : *On approximation to a real number by algebraic numbers of bounded degree*

Abstract : In his seminal 1961 paper, Wirsing studied how well a given transcendental real number ξ can be approximated by algebraic numbers α of degree at most n for a given positive integer n, in terms of the so-called naive height $H(\alpha)$ of α. He showed that the exponent $\omega_n^*(\xi)$ which measures this quality of approximation is at least $(n+1)/2$. He also asked if we could even have $\omega_n^*(\xi) \geq n$ as it is generally expected. Since then, all improvements on Wirsing's lower bound were of the form $n/2 + O(1)$ until Badziahin and Schleischitz showed in 2021 that

$\omega_n^*(\xi) \geq an$ for each $n \geq 4$ with $a = 1/\sqrt{3} \approx 0.577$. In my talk, I will first present the background and ideas behind the proof of Wirsing's lower bound. Secondly, using a new approach that is partly inspired by parametric geometry of numbers, I will explain how we can obtain $\omega_n^*(\xi) \geq an$ for each $n \geq 2$, with $a = 1/(2 - \log 2) \approx 0.765$.

Yunqing Tang

University of California (USA)

Title : *The arithmetic of power series, I*

Abstract : We will first briefly discuss our approach to prove irrationality of certain periods. Our method uses rational approximations from the literature and we develop a new framework to make use of these approximations. The key ingredient is an arithmetic holonomy theorem built upon earlier work by André, Bost, Charles (and others) on arithmetic algebraization theorems via Arakelov theory. We will then discuss our recent result on irrationality measures. This is joint work with Frank Calegari and Vesselin Dimitrov.

Francesco Veneziano

University of Genova (Italy)

Title : *The field of definition of the iterates of a rational function*

Abstract : In this joint work with Solomon Vishkautsan we study how the field of definition of a rational function changes under iteration. We provide a complete classification of polynomials with algebraic coefficients with the property that one of their iterates is defined over a smaller field than the polynomial itself. We show with families of examples originating from algebraic groups that this characterization does not hold for rational functions. Finally, we also classify fractional linear transformations with this property.

Ingrid Vukusic

University of York (UK)

Title : *Balanced Fibonacci word rectangles*

Abstract : The infinite Fibonacci word 0100101001 . . . is perhaps the most famous Sturmian word. One of its basic properties is that it is balanced. That is, any two blocks of the same length have either the same sum or their sums are off by 1.

In this talk we will consider "Fibonacci word rectangles", i.e., blocks in the corresponding Hankel matrix. Using the software Walnut, we will fully characterise the balanced rectangles. I will also describe the result in terms of distribution modulo 1.

Joint work with Jeffrey Shallit. `https://www.arxiv.org/abs/2509.25994`

Robert Wilms

University of Caen Normandie (France)

Title : *Arakelov theory on Noetherian schemes*

Abstract : Arakelov theory serves as a useful tool for generating height functions in Diophantine geometry and Diophantine approximation. In this talk, I will provide a generalization of Arakelov theory to arbitrary Noetherian schemes. More precisely, I will construct a multi-linear pairing on the set of certain norm-like functions on a fixed Noetherian ring. A typical example of such a norm-like function is induced by the linear series of global sections of an ample line bundle on a projective variety and the multi-linear pairing coincides with the classical intersection pairing in this case. The same holds true for arithmetically ample hermitian line bundles on projective arithmetic varieties. If time allows, I will also discuss applications to other areas of mathematics.

Jit Wu Yap

Massachusetts Institute of Technology (USA)

Title : *Degeneration of Abelian Varieties by Ultrafilters*

Abstract : Degenerations have been very useful in proving uniformity results in diophantine geometry. Given a sequence (A_n/K_n) of abelian varieties over a valued field, I will explain how to use a recent construction by Luo and Favre-Gong to construct limits of our sequence. This will allow us to deduce uniformity results regarding torsion points on abelian varieties. This is joint work with Nicole Looper.