List of talks

1. Héritage speakers

Serge Cantat (CNRS & Université de Rennes). — Rigidity and examples.

I will describe problems concerning rigidity properties of automorphisms of complex projective varieties.

Antoine Chambert-Loir (Université Paris-Cité). — Intersection theory and heights : birational perspective.

The preceding États de la recherche on this topic happened in Rennes, 2006, and the organizers of the present edition asked me to make the bridge between these two sessions. My 2006 talks were devoted to the theory of heights and equidistribution theorems for algebraic dynamical systems. I will start from there by presenting the framework allowed by Arakelov geometry, and explaining the recent manuscript of X. Yuan and S.-W. Zhang who provide a birational perspective to these concepts. The theory is a bit complex and technical but I will try to emphasize the parallel between those ideas and the ones that lie at the ground of pluripotential theory in complex analysis, or in the theory of b-divisors in algebraic geometry.

2. Minicourses

Laura DeMarco (Harvard univ.)— Geometry and algebra of preperiodic points in $\mathbb{P}^N$.

In these lectures, we will examine a series of conjectures about the geometry of preperiodic points for endomorphisms of $\mathbb{P}^N$. Lecture 1 will focus on the Dynamical Manin-Mumford Conjecture (DMM), formulated by Shouwu Zhang in the 1990s as an extension of the well-known Manin-Mumford Conjecture (which investigated the geometry of torsion points in abelian varieties and was proved in the early 1980s by Raynaud). The DMM aims to classify the subvarieties of $\mathbb{P}^N$ containing a Zariski-dense set of preperiodic points. Lectures 2 and 3 will be devoted to conjectures that treat families of maps on $\mathbb{P}^N$. One conjecture in particular was inspired by the recently-proved "Relative Manin-Mumford" theorem of Gao-Habegger for abelian varieties, but the dynamical version turns out to be closely related to the study of dynamical stability and to contain many previously-existing questions/conjectures/results about moduli spaces of maps on $\mathbb{P}^N$. These lectures are based on joint work with Myrto Mavraki.

Stéphane Lamy (Université de Toulouse). — The group of tame automorphisms.

The group $\mathrm{Aut}(\mathbb{A}^n)$ of polynomial automorphisms of the affine space is an inter- esting huge group, and a slightly simpler group is its subgroup $\mathrm{Aut}(\mathbb{A}^n)$ of tame automorphisms. Natural questions about these groups include:

- does they admit normal subgroups beside the obvious subgroup of automorphisms with Jacobian 1?
- do they satisfy a Tits alternative?
- what are the possible dynamical degrees of their elements?

The method to investigate these questions is via some actions on some metric spaces, namely the coset complex and the valuation complex, that we plan to introduce in detail. The lectures will focus on the following three cases: the group $\mathrm{Aut}(\mathbb{A}^2) = \mathrm{Tame}(\mathbb{A}^2)$ following Chapter 7 of my book in preparation; then the group $\mathrm{Tame}(\mathbb{A}^3)$ (Lamy, Lamy-Przytycki, Blanc-Van Santen); and finally the group $\mathrm{Tame}(Q(\mathbb{A}^4))$ of tame automorphisms of $\mathbb{A}^4$ preserving a nondegenerate quadratic form (Bisi-Furter-Lamy, Martin, Dang).

Yusheng Luo (Cornell university). — Degeneration of rational maps and hyperbolic components.

In this mini course, I will discuss how degenerating sequences of rational maps can be studied using geometric and arithmetic tools. I will also discuss applications to study the boundary of hyperbolic components, length spectrum and rescaling limits, and some differences for sequences vs holomorphic families.

Junyi Xie (BICMRS, Beijing). — Complexity theory in arithmetic dynamical systems

It is a fundamental problem to measure the complexity of a dynamical system. In this lecture, we discuss this problem for arithmetic dynamics in terms of topology, algebra and arithmetic. In particular, the notion of dynamical degrees, which can be viewed as an algebraic analogy of "entropy", plays a key role. We will see how it applies to study the orbits, periodic points and action of cohomologies.

3. Plenary talks

Valentin Huguin (University of Toronto). — The moduli spaces of polynomial maps and multipliers at small cycles.

I will show that the multipliers at the cycles with periods 1 and 2 provide a good description of the space P_d of polynomial maps of degree d modulo conjugation by affine transformations. More precisely, the elementary symmetric functions of the multipliers at the cycles with periods 1 and 2 induce a finite birational morphism from P_d onto its image. This result arises as a direct consequence of the following two facts:

1. For each integer $p > 1$, any sequence of complex polynomials of degree d with bounded multipliers at its cycles with period p is necessarily bounded in $\mathrm{P}_d(\mathbb{C})$.
2. A generic conjugacy class of complex polynomials of degree d is uniquely determined by its multipliers at its cycles with periods 1 and 2.

I will present a quantitative version of the first statement, which also holds over various valued fields of characteristic 0. The second statement proves a conjecture by Hutz and Tepper and strengthens a recent results by Ji and Xie in the particular case of polynomial maps.

Rohini Ramadas (University of Warwick). — The tropical moduli space correspondence.

The tropical moduli space correspondence is a piecewise-linear dynamical system on a polyhedral complex. It is related to the dynamics "near infinity" of Thurston's pullback map on Teichmüller space. I will give a preliminary report, including some work in progress with C. Favre.

Sabya Mukherjee (HBCSE, Tata institute of fundamental research). — The role of algebraic functions in combination theorems.

The main objects of this talk are algebraic correspondences on compact Riemann surfaces that combine actions of rational maps and Kleinian groups. We will discuss a general mechanism for constructing such correspondences. This involves manufacturing appropriate meromorphic maps on subsets of the sphere via analytic (surgery) techniques and characterizing the resulting meromorphic maps as algebraic functions. We will illustrate this recipe with concrete examples, thus producing algebraic correspondences on surfaces of arbitrary genus. We will also address questions regarding degenerations of the above correspondences as the ‘group part’ or the ‘rational map part’ (or both) degenerates.

Julia Schneider (University of Sheffield). — Generating the plane Cremona group by involutions.

The plane Cremona group $\mathrm{Cr}(2, K)$ is the group of birational transformations of the projective plane that are defined over the field K. In this talk, I will present the following result, obtained in a joint work with S. Lamy: The plane Cremona group over any perfect field is generated by involutions. While it is classically known that this is true for algebraically closed fields, I will explain how to use Sarkisov links (elementary birational maps) to obtain a set of generators and how to decompose them into involutions. If time permits, I will also report on what is known about generation by involutions of Cremona groups $\mathrm{Cr}(n, K)$ in higher dimensions.

Gabriel Vigny (Université d’Amiens). — Julia, Mandelbrot and Hénon.

Quite recently, it was shown that the Mandelbrot set is not the Julia set of a rational map (Ghioca, Krieger and Nguyen for polynomials and Luo for rational maps), a long-standing question in complex dynamics. Generalizing this result, I will explain how several dynamically defined fractals in $\mathbb{C}$ and $\mathbb{C}^2$ which arise from different type of polynomial dynamical systems cannot be the same objects. One of our main results, whose proof I will explain, is that the closure of Misiurewicz PCF cubic polynomials (the strong bifurcation locus) cannot be the Julia set of a regular polynomial endomorphism of $\mathbb{C}^2$. I will also give corollaries in terms of the number of points in $\mathbb{C}^2$ which are simultaneously periodic points of a polynomial endomorphism and PCF parameters (j.w. Thomas Gauthier).

4. Short addresses

Marc Abboud. (EPFL) *Intersection of orbits of automorphisms of affine surfaces.*

We prove the following result. If S is a normal complex affine surface and f, g are automorphisms with first dynamical degree > 1 having an orbit intersecting infinitely many times then f and g share a common iterate. In particular, this holds for Henon maps over the affine plane. This result was previously shown by Ghioca, Tucker and Zieve for polynomial maps of the complex line. The problem of intersection of orbits is related to the dynamical Mordell-Lang conjecture and has sparked a lot of work in the recent years.

Chen Gong. (Ecole polytechnique) *Non-Archimedean techniques in the study of degenerating sequences.*

We employ Berkovich theory to associate a degenerating sequence with a rational map over a non-Archimedean field. This approach enables the transfer of dynamical information between complex and non-Archimedean rational maps. Using this framework, we analyze and control the multiplier of the sequence.

Alex Kapiamba (Harvard univ.) *Singular Parabolic Implosion*

Parabolic implosion is a powerful tool in complex dynamics which describes the perturbation of parabolic fixed points. In this setting, a multiple fixed point splits into several fixed points. In this talk, we will consider some singular perturbations of parabolic fixed points, where the multiple fixed point may split into fixed points and poles and critical points, and see that parabolic implosion can still be recovered. Based on joint work with Xavier Buff and Caroline Davis.

Irène Meunier (Univ. of Toulouse) *Dynamical degree of polynomial automorphisms of some affine* 3*-folds*

In this short talk, we will explain the computation of the first dynamical degree of some polynomial endomorphisms by minimizing a degree function over the valuation space. If time allows, we will discuss how this can be helpful to prove that dynamical degrees of polynomial automorphisms of $\mathbb{C}^3$ are algebraic numbers of degree at most 2 over $\mathbb{Q}$.

Jiarui Song. (Beijing univ.) *A high-codimensional Yuan's inequality and its application to higher arithmetic degrees.*

In this talk, we consider a dominant rational self-map of a normal projective variety defined over a number field. We discuss the arithmetic dynamical degree and the arithmetic degree of a subvariety, which extend the classical notions of dynamical degree and arithmetic degree of points. Extending Yuan's inequality to higher codimensions, we establish the existence of the arithmetic dynamical degrees and prove a relative degree formula relating arithmetic dynamical degrees to dynamical degrees. Furthermore, we derive an upper bound for the arithmetic degree of subvarieties, providing a conceptual proof of the classical result for points.

Virgile Tapiero (Univ. d'Orléans) *Algebraic braids and hyperbolic components of polynomial skew-products over the circle*

It is well known that for holomorphic families of rational maps, stability preserves hyperbolicity, and one can speak of hyperbolic components for these families. Examples are provided for each degree $d > 1$ by the shift locus S_d of the family of polynomial mappings of degree d. S_d is the set of polynomial mappings whose critical points all escape to infinity under iteration. Astorg-Bianchi recently proved that stability preserves hyperbolicity for holomorphic families of regular polynomial skew-products of $\mathbb{C}^2$, thus giving meaning to the notion of hyperbolic components in this context. Moreover, Astorg-Bianchi classified the hyperbolic components of the family of quadratic skew-products (with a fixed base) which are analogous to S_2.

In this talk, I will discuss some properties of the bifurcation and stability loci of the family of regular polynomial skew-products of degree $d > 1$ with a fixed base. When the base is z^d, I will present a new result which states that in this family, one can associate to each hyperbolic component analogous to S_d a topological invariant represented by an algebraic braid. In the quadratic case, these algebraic braids provide sufficient information to recover the classification of Astorg-Bianchi. The proof uses techniques of iterated monodromy.

Marco Vergamini (Pisa univ.) *Mixing and central limit theorems for Hénon maps*

Let f be a complex Hénon map and μ its unique measure of maximal entropy. Recently, Bianchi-Dinh proved that μ is exponentially mixing of all orders for all Hölder observables, and that all such observables satisfy the central limit theorem with respect to μ. De Thélin-Vigny generalized these results for a certain class of bounded plurisubharmonic observables. We prove that these properties hold for all, not necessarily bounded, plurisubharmonic observables. This is a joint work with Hao Wu.

Jit Wu Yap. (Harvard univ). *Quantitative Equidistribution of Small Points in Higher Dimensions.*

Let f be an endomorphism of $\mathbb{P}^n$ of degree $d \geq 2$. Yuan's theorem tells us that the Galois orbits of a generic sequence of preperiodic points equidistribute to the equilibrium measure. In dimension one, a quantitative version was proven by Favre and Rivera-Letelier. I will state a quantitative version in higher dimensions and sketch some consequences of it.