Approximation by Diffusion and homogenization of nonlinear kinetic models

Nourelhouda Khedhiri

Abstract

Our motivation is the derivation of fluid models, (here a Homogenized Energy Transport Model) as an approximation by diffusion of non-linear transport equations, eventually coupled with the Poisson equation, and describing transport in the heterogeneous composite media (semiconductors. super or supra conductors, etc.). Here We are concerned with a Boltzmann equation associated with a nonlinear collision operator modeling electron-electron and electron-impurity collisions and a potential with macroscopic and microscopic variations, leading to a phenomenon of homogenization.

A convective bulk-surface Cahn-Hilliard model

Jonas Stange

Abstract

We consider a general class of convective bulk-surface Cahn–Hilliard systems with singular potentials. In contrast to classical Neumann boundary condi-tions, the dynamic boundary conditions of Cahn-Hilliard type allow for dynamic changes of the contact angle between the diffuse interface and the boundary, a convection-induced motion of the contact line as well as absorption of material by the boundary. The coupling conditions for bulk and surface quantities involve nonnegative parameters K, L, whose choice declares whether these conditions are of Dirichlet, Robin or Neumann type. I present some recent results on the well-posedness of this system for singular potentials. We make use of the Yosida approximation to regularise these potentials which allows us to apply the results for regular potentials and eventually pass to the limit in this approximation scheme to obtain a global-in-time weak solution. Afterwards, under additional assumptions on the mobility functions and the velocity fields, we use standard difference-quotient arguments and regularity theory for elliptic systems with bulk-surface coupling to prove the existence of a unique global-in-time strong solution. These higher regularity results can then be used to establish separation properties of the phase-fields. This is based on joint work with Patrik Knopf (University of Regensburg).

On global well-posedness of inhomogeneous Navier-Stokes equations with bounded density allowing vacuum

Feng Shao

Abstract

I will present the joint work with Tiantian Hao, Dongyi Wei and Zhifei Zhang on the global well-posedness of inhomogeneous Navier-Stokes equations (INS) with bounded density. We solve Lions' density patch problem (without compatibility conditions) about the preseving of boundary regularity of a density patch and Lions' open problem on the 2-D uniqueness of weak solutions. Moreover, we extend Leray's 2-D global well-posedness result of weak solutions in $L^2(\mathbb{R}^2)$ on the classical Navier-Stokes equations (NS) to (INS) (in which case our uniqueness requires the positive lower bound of the density), and we also extend Fujita-Kato's celebrated result on the global well-posednesss of (NS) in $\dot{H}^{1/2}(\mathbb{R}^3)$ to (INS). Particularly, our proof of uniqueness is based on a duality argument and a surprising finding that the estimate $t^{1/2}\nabla u \in L^2(0,T;L^\infty(\mathbb{R}^d))$ instead of $\nabla u \in L^1(0,T;L^\infty(\mathbb{R}^d))$ is enough to ensure the uniqueness of the solution.

Well-posedness and stability for the two-phase periodic quasistationary Stokes flow

Daniel Boehme

Abstract

The two-phase horizontally periodic quasistationary Stokes flow in $\mathbb{R}^2$, describing the motion of two immiscible fluids with equal viscosities that are separated by a sharp interface, parameterized as the graph of a function $f = f(t)$, is considered in the general case when both gravity and surface tension effects are included. Using potential theory, the moving boundary problem is formulated as a fully nonlinear and nonlocal parabolic problem for the function f. Based on abstract parabolic theory, it is shown that the problem is well-posed in all subcritical spaces $\mathrm{H}^r(\mathbb{S})$, $r \in (3/2, 2)$. Moreover, the stability properties of the flat equilibria are analyzed in dependence on the physical properties of the fluids. This is joint work with Bogdan-Vasile Matioc

Modeling Reactive Micropolar Fluid Flow and Thermal Explosion with Spherical Symmetry

Angela Bašić-Šiško

Abstract

In this work, a model for the flow and thermal explosion of a reactive micropolar gas governed by a generalized equation of state is investigated. This model is particularly relevant for the description of gas behavior in microtubes during combustion. By using a generalized equation of state instead of the ideal gas law, the model reproduces the gas dynamics more accurately. Furthermore, the inclusion of micropolar fluid mechanics, taking into ac- count microscale effects, provides a more suitable framework than classical flow models to describe flows of fluids with a distinct particle structure on small scales. In this study, we focus on the spherically symmetric solution for a fluid confined between two concentric spheres. We derive the associated boundary- initial value problem with homogeneous boundary conditions, introduce the concept of a generalized solution and analyze its existence. In particular, we prove that a generalized solution to the problem exists locally in time.

Virial Theorem and Instability of Water-Waves

Haocheng Yang

Abstract

We analyze the dynamics of two layers of immiscible, inviscid, incompressible, and irrotational fluids through a full nonlinear system. Our goal is to establish a virial theorem and prove the polynomial growth of slope and curvature of the interface over time when the fluid below is no denser than the one above. These phenomena, known as Rayleigh-Taylor instability and Kelvin-Helmholtz instability, will be proved for a broad class of regular initial data, including the case of 2D overlapping interface.

Well-posedness of the Euler equations for a stratified ocean in isopycnal coordinates

Théo Fradin

Abstract

The dynamics of the ocean can be described by the incompressible Euler equations, completed with a transport equation for the density. For large scale dynamics, the ocean is assumed stratified, meaning that it is layered by sheets of constant density, namely isopycnals. This work is devoted to the study of perturbations around a shear flow equilibrium. A fruitful viewpoint is the one of isopycnal coordinates, which uses the density as the vertical coordinate, flattening the isopycnals. The system becomes quasi-2D and thus is sometimes used in numerical studies. While the well-posedness in Sobolev spaces has been treated in Eulerian coordinates (see [2]), a loss of derivatives appears in the reformulation in isopycnal coordinates, treated in [1] by the addition of a diffusion term. This work focuses on how to keep track of the symmetry of the system in Eulerian coordinates, while using the advantageous quasi-2D structure of the reformulation in isopycnal coordinates. More precisely, the main result is a well-posedness theorem in Sobolev spaces on the system in isopycnal coordinates without diffusion. The time of existence is uniform with respect to the size of the perturbation, and with the additional assumptions of medium amplitude regime and small shear velocity, it is also uniform in the shallow-water parameter. This work is based on [3] and is part of my PhD thesis under the supervision of Vincent Duchêne and David Lannes.

References

[1] V. Duchêne and R. Bianchini. On the hydrostatic limit of stably stratified fluids with isopycnal diffusivity. June 2022.

[2] B. Desjardins, D. Lannes, and Jean-Claude Saut. Normal mode decomposition and dispersive and nonlinear mixing in stratified fluids. Water Waves, 3(1):153–192, July 2020.

[3] Well-posedness of the Euler equations in a stably stratified ocean in isopycnal coordinates.

On steady solutions to the MHD equations with inhomogeneous boundary conditions for the magnetic field

Jiri Neustupa

Abstract

We consider the steady MHD equations for a viscous incompressible fluid in a 3D bounded and generally multiply connected domain with the no-slip boundary condition for the velocity and the inhomogeneous generalized impermeability boundary conditions for the magnetic field. We provide an appropriate definition of the weak solution and present results on its existence, continuous dependence on the data when the data tend to zero and uniqueness in a “small” neighborhood of the zero solution.

On the motion of fluid in a moving domain and applications to fluid structure interaction

Sarka Necasova

Abstract

We analyze a system of partial differential equations governing the interaction between two compressible mutually noninteracting fluids and a shell of Koiter type encompassing a time dependent domain filled by the fluids or the case of one compressible fluid and a shell of a Koiter type with Navier boundary condition. It is a joint work with M. Kalousek, Y. Liu, S. Mitra.