Frontiers in Sub-Riemannian Geometry

Aux Frontières de la Géométrie Sous-Riemannienne

Centre International de Rencontres Mathématiques (CIRM)

Marseille, 25 – 29 November 2024

Presentation

Sub-Riemannian geometry has grown significantly since the early 1990's, in particular through its close links with several domains of mathematics such as geometric control theory, the theory of partial differential equations or geometric measure theory. Sub-Riemannian geometry also plays a major role in many mathematical applications like for example in robotics, quantum control or neuro-geometry. The aim of this conference is to bring together researchers working in different areas related to sub-Riemannian geometry, with different backgrounds, to share the most recent results with multiple points of view and so to foster interactions between research groups and to contribute to the training of young researchers.

La géométrie sous-riemannienne a connu un essor considérable depuis le début des années 90 nourri en particulier par des problématiques en lien avec de nombreux domaines mathématiques parmi lesquels on peut citer la théorie géométrique du contrôle, la théorie des équations aux dérivées partielles ou encore la théorie géométrique de la mesure. La géométrie sous-riemannienne joue par ailleurs un rôle important dans de nombreuses applications des mathématiques comme par exemple en robotique, en contrôle quantique ou en neuro-géométrie. Cette conférence a pour objectif de réunir des chercheurs et chercheuses travaillant dans des domaines liés à la géométrie sous-riemannienne issus d'horizons divers afin de partager les résultats les plus récents sous des points de vue multiples et ainsi favoriser un maximum d'interactions entre groupes de recherches et contribuer à la formation des plus jeunes.

Scientific Committee
Comité scientifique

Ugo Boscain
Ludovic Rifford
Séverine Rigot
Luca Rizzi
Aissa Wade

Organizing Committee
Comité d'organisation

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Speakers

Plenary speakers

Andrei Agrachev (SISSA, Italy)
André Belotto da Silva (Paris Cité University, France)
Véronique Fischer (University of Bath, UK)
Valentina Franceschi (University of Padua, Italy)
Erlend Grong (University of Bergen, Norway)
Karen Habermann (University of Warwick, UK)
Enrico Le Donne (University of Fribourg, Switzerland)
Andrea Merlo (University of Pais Vasco, Spain)
Omar Mohsen (Paris-Saclay University, France)
Roberto Monti (University of Padua, Italy)
Pierre Pansu (Paris-Saclay University, France)
Dario Prandi (Paris-Saclay University, France)
Anton Thalmaier (University of Luxembourg, Luxembourg)
Emmanuel Trélat (Sorbonne University, France)
Davide Vittone (University of Padua, Italy)

Short speakers

Shahriar Aslani (University of Toronto, Canada)
Eugenio Bellini (University of Milano–Bicocca, Italy)
Tania Bossio (University of Padua, Italy)
Tommaso Rossi (Sorbonne University, France)
Alessandro Socionovo (Sorbonne University, France)
Florin Suciu (Paris Dauphine University, France)
Lucia Tessarolo (Sorbonne University, France)
Daniele Tiberio (SISSA, Italy)

Poster speakers

Magalie Benefice (Institute of Mathematics of Bordeaux, France)
Gaia Bombardieri (University of Padua, Italy)
Alejandro Bravo-Doddoli (University of Michigan, US)
Nicklas Day (Texas A&M University, US)
Marco Di Marco (University of Padua, Italy)
Liangbing Luo (University of Connecticut, US)
Luca Nalon (University of Fribourg, Switzerland)
Nicola Paddeu (University of Fribourg, Switzerland)
Alessandro Scagliotti (Technical University of Munich, Germany)
Ye Zhang (Okinawa Institute of Science and Technology, Japan)

Schedule

	Mon 25th	**Tue 26th**	**Wed 27th**	**Thu 28th**	**Fri 29th**
9:00	Agrachev⋆	Habermann⋆	Pansu	Franceschi⋆	Fischer
10:00			**Coffee Break**		
10:30	Vittone⋆	Trélat⋆	Grong	Merlo	*Bellini*
11:30	*Tessarolo*	*Bossio*	*Suciu*	*Socionovo*	
12:00			**Lunch**		
14:00	Le Donne	Mohsen		Monti	
15:00	**Coffee Break**		**Poster Session**	**Coffee Break**	
15:30	Belotto da Silva	Thalmaier		Prandi	
16:30	*Tiberio*	*Rossi*		*Aslani*	

Plenary talks with ⋆ will be recorded and archived in the CIRM's audiovisual library.

Wed 27th	**Poster Session**	
14:00 – 17:00	*Benefice*	*Luo*
	Bombardieri	*Nalon*
	Bravo-Doddoli	*Paddeu*
	Day	*Scagliotti*
	Di Marco	*Zhang*

The posters will be displayed during the whole week in Room S2 at the library.

Plenary talks

Andrei Agrachev (SISSA, Italy)

Sub-Riemannian geometry of osculating curves

Simplest geometric example of a nonholonomic constraint is one for the movement of the tangent line along a smooth plane curve. We obtain a better contact with the curve and more interesting constraints if we substitute tangent lines with "osculating" algebraic curves of degree $n > 1$. My talk is devoted to the vector distributions and sub-Riemannian structures raised from these geometric models, starting from the osculating conics and cubics.

André Belotto da Silva (Paris Cité University, France)

On a geometrical approach to the Sard Conjecture

I will present a geometrical approach to study the Sard Conjecture in sub-Riemannian geometry (SR Geometry). I will start by explaining how the Conjecture can be interpreted as a geometrical problem concerning the behavior of a characteristic singular foliation in the cotangent bundle. I will then show how we can settle the Conjecture in two main cases: generic smooth co-rank 1 distributions, and analytic co-rank 1 distributions which generate non-spllitable characteristic foliations. These are recent results in collaboration with Parusinki and Rifford.

Véronique Fischer (University of Bath, UK)

TBA

TBA

Valentina Franceschi (University of Padua, Italy)

Mean convex mean curvature flow in the Heisenberg group
(is this related with the isoperimetric problem?)

The mean curvature flow (MCF) describes the evolution of a hypersurface in time, where the velocity at each point is given by its mean curvature vector (i.e., the unit normal vector multiplied by the mean curvature). When initiated with a sphere in $\mathbb{R}^n$, the MCF will shrink it homothetically to a point in finite time. In this talk, we introduce an adaptation of the mean convex MCF within the Heisenberg group setting. Our initial objective was to explore potential connections between this flow and the Heisenberg isoperimetric problem. We will discuss the existence and uniqueness of solutions and prove that the Pansu sphere does not evolve homothetically under the MCF. This work is based on joint research with Gaia Bombardieri and Mattia Fogagnolo.

Erlend Grong (University of Bergen, Norway)

Non-linear data and landmarks — Applications of sub-Riemannian geometry

Geometric statistics is the topic of analyzing data on a manifold where we cannot use addition to define standard concepts such as the mean and covariance. Our approach to this topic is to consider the diffusion mean, where mean and covariance is determined as the optimal initial condition for a certain diffusion process on the frame bundle. We will show that estimating this initial condition is related to the sub-Riemannian geometry of the frame bundle. We will furthermore show that sub-Riemannian geometry also determine the most likely path for how one shape can be transformed into another under a stochastic differential equation. More precisely, if we describe a shape through a collection of landmarks, then a most probable paths for these landmarks follow sub-Riemannian geodesics. The research is part of a joint work with Stefan Sommer (Copenhagen).

Karen Habermann (University of Warwick, UK)

Score matching for simulating sub-Riemannian diffusion bridge processes

Simulation of conditioned diffusion processes is an essential tool in inference for stochastic processes, data imputation, generative modelling, and geometric statistics. Whilst simulating diffusion bridge processes is already difficult on Euclidean spaces, when considering diffusion processes on Riemannian manifolds the geometry brings in further complications. In even higher generality, advancing from Riemannian to sub-Riemannian geometries introduces hypoellipticity, and the possibility of finding appropriate explicit approximations for the score, the logarithmic gradient of the density, of the diffusion process is removed. We handle these challenges and construct a method for bridge simulation on sub-Riemannian manifolds by demonstrating how recent progress in machine learning can be modified to allow for training of score approximators on sub-Riemannian manifolds. Since gradients dependent on the horizontal distribution, we generalise the usual notion of denoising loss to work with non-holonomic frames using a stochastic Taylor expansion, and we demonstrate the resulting scheme both explicitly on the Heisenberg group and more generally using adapted coordinates. Joint work with Erlend Grong (Bergen) and Stefan Sommer (Copenhagen).

Enrico Le Donne (University of Fribourg, Switzerland)

Asymptotic geometry of Riemannian nilpotent groups

Asymptotic cones of Riemannian nilpotent Lie groups are Carnot groups. The volume of balls in Carnot groups grows exactly as a power of the radius. Heuristically, the better the asymptotic cone approximates a Riemannian group, the closer to a polynomial the volume growth becomes. I will discuss several results obtained over the last few years in collaboration with Breuillard, Nalon, Nicolussi Golo, Ryoo, Tettamanti, and Tyson.

Omar Mohsen (Paris-Saclay University, France)

On maximally hypoelliptic differential operators

The class of maximally hypoelliptic differential operators is a large class of differential operators which contains elliptic operators as well as Hörmander's sum of squares. I will present our work where we define a principal symbol generalising the classical principal symbol for elliptic operators which should be thought of as the analogue of the principal symbol in sub-Riemannian geometry. Our main theorem is that maximal hypoellipticity is equivalent to invertibility of our principal symbol, thus generalising the main regularity theorem for elliptic operators and confirming a conjecture of Helffer and Nourrigat. While defining our principal symbol, we will answer the question: What is the tangent space in sub-Riemman geometry in the sense of Gromov? If time permits, I will also talk about the heat kernel of maximally hypoelliptic differential operators. This is partly joint work with Androulidakis and Yuncken.

Roberto Monti (University of Padua, Italy)

TBA

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Pierre Pansu (Paris-Saclay University, France)

Beckmann's variational problem in higher dimensions

In 1952, M. Beckmann related the optimal transport problem to a variational problem on currents of dimensions 0 and 1. We propose a possible extension of this correspondence to higher dimensions. The sub-Riemannian version is a work in progress with A. Baldi and B. Franchi.

Dario Prandi (Paris-Saclay University, France)

Magnetic Hardy inequalities in the Heisenberg group

We introduce a notion of magnetic field in the Heisenberg group and we study its influence on spectral properties of the corresponding magnetic (sub-elliptic) Laplacian. We show that uniform magnetic fields uplift the bottom of the spectrum. For magnetic fields vanishing at infinity, including Aharonov–Bohm potentials, we derive magnetic improvements to a variety of Hardy-type inequalities for the Heisenberg sub-Laplacian, relying on a novel sharp Hardy inequality for the Folland–Stein operator. In particular, we establish a sub-Riemannian analogue of Laptev and Weidl sub-criticality result for magnetic Laplacians in the plane. This is joint work with Biagio Cassano, Valentina Franceschi and David Krejcirik.

Anton Thalmaier (University of Luxembourg, Luxembourg)

The differentiation of sub-Riemannian heat flows

The effect of curvature on the behavior of the heat flow on a Riemannian manifold is a classical problem. A quantitative measurement of this behavior is encoded most directly in terms of gradient estimates and Harnack inequalities, involving constants depending only on a lower Ricci curvature bound, the dimension of the manifold, etc. While it is well-understood how to derive such results for Laplacians on Riemannian manifolds, the case of sub-Laplacians is afflicted with the difficulties inherent to sub-Riemannian geometry: the lack of a canonical substitute of the Levi-Civita connection, relevant connections naturally have torsion, etc. In this talk, we give an account on these issues from a probabilistic point of view, including a discussion of crucial integration by parts formulae.

Emmanuel Trélat (Sorbonne University, France)

From gas giant planets to the spectral theory of subelliptic Laplacians

This is a work with Yves Colin de Verdière, Charlotte Dietze and Maarten De Hoop, motivated by recent works by M. De Hoop on inverse problems for sound wave propagation in gas giant planets. On such planets, the speed of sound is isotropic and tends to zero at the surface. Geometrically, this corresponds to a Riemannian manifold with a boundary whose metric blows up near the boundary. With appropriate variable changes, we can reduce the study of the Laplacian–Beltrami to that of a kind of sub-Riemannian Laplacian. In this talk, I will explain how to approach the spectral analysis of such operators, and in particular how to calculate Weyl's law.

Davide Vittone (University of Padua, Italy)

Stokes' theorem in Heisenberg groups

We introduce the notion of submanifolds with boundary with intrinsic C^1 regularity in the setting of sub-Riemannian Heisenberg groups. We present a Stokes' Theorem for such submanifolds involving the integration of Heisenberg differential foms introduced by Rumin. This is a joint work with M. Di Marco, A. Julia and S. Nicolussi Golo.

Short talks

Shahriar Aslani (University of Toronto, Canada)

Bumpy metric theorem for co-rank 1 sub-Riemannian metrics

We use methods of Hamiltonian dynamics to show that for a generic sub-Riemannian metric defined on a fixed co-rank 1 distribution (for a generic metric while we preserve the distribution), all strictly normal periodic orbits are non-degenerate. A periodic orbit is called non-degenerate if its associated linearized Poincare map does not admit roots of unity as an eigenvalue. The same result also holds for reversible sub-Finsler metrics. This is a joint work with Ke Zhang.

Eugenio Bellini (Universita di Milano-Bicocca, Italy)

Quantitative tightness for three dimensional contact manifolds: a sub-Riemannian approach

A contact structure on a three dimensional manifold is a completely non-integrable plane field. The topological properties of such structures are often subtle and difficult to detect. Indeed, even the harmless-looking statement that there are (at least) two different contact structures on $\mathbb{R}^3$ is highly non-trivial to prove. In this talk I will describe some recent results concerning the relations between contact topology and sub-Riemannian geometry. The focus will be on tightness questions, both semi-local and global, and on geometric detection of overtwisted disks. In particular I will present a K-contac version of Cartan–Hadamard Theorem: the universal cover of any negatively curved K-contact manifold is the Heisenberg group. This is a joint work with A. Agrachev, S. Baranzini and L. Rizzi.

Tania Bossio (University of Padua, Italy)

Tubes in sub-Riemannian geometry: Steiner's and Weyl's tube formulae

The tube of radius r over a submanifold S of $\mathbb{R}^n$ is, roughly speaking, the set of points at distance within r from S. Steiner and Weyl proved that the volume of this set is a polynomial in r of degree n. The coefficients of such a polynomial carry information about the curvature of the submanifold. In this talk, we investigate the validity of a Steiner and Weyl-like formula where the ambient space is a sub-Riemannian manifold, extending previous results obtained in the Heisenberg group. In the specific case of a 3D contact sub-Riemannian manifold, we provide a geometric interpretation of the coefficients of the Taylor expansion of the volume as the size of the tube tends to zero in terms of sub-Riemannian curvature objects. The obtained results are in collaboration with Davide Barilari, Luca Rizzi and Tommaso Rossi.

Tommaso Rossi (Sorbonne University, France)

Tubes in sub-Riemannian geometry: a Weyl's invariance result

In 1939, Weyl derived a formula for the volume of a tube of small radius r around a submanifold S embedded in $\mathbb{R}^n$, showing that it is a polynomial in r whose coefficients are invariant, in the sense that they do not depend on the way S is isometrically embedded. In this talk, we discuss a Weyl's invariance result for curves in the Heisenberg groups: we prove that the volume of small tubes around non-characteristic curves does not depend on the way the curve is isometrically embedded, but only on its Reeb angle. Remarkably, the proof does not need the computation of the actual volume of the tube. This is a joint work with T. Bossio and L. Rizzi.

Alessandro Socionovo (Sorbonne University, France)

TBA

TBA

Florin Suciu (Paris Dauphine University, France)

Gradient flow on control space with rough initial condition

We consider the (sub-Riemannian type) control problem of finding a path going from an initial point x to a target point y, by only moving in certain admissible directions. We assume that the corresponding vector fields satisfy the Hörmander condition, so that the classical Chow–Rashevskii theorem guarantees the existence of such a path. One natural way to try to solve this problem is via a gradient flow on control space. However, since the corresponding dynamics may have saddle points, any convergence result must rely on suitable (e.g., random) initialization. We consider the case when this initialization is irregular, which is conveniently formulated via rough path theory. In some simple cases, we manage to prove that the gradient flow converges to a solution, if the initial condition is the path of a Brownian motion (or rougher). The proof is based on combining ideas from Malliavin calculus with Łojasiewicz inequalities. A possible motivation for our study comes from the training of deep Residual Neural Nets, in the regime when the number of trainable parameters per layer is smaller than the dimension of the data vector. Joint work with Paul Gassiat (Paris Dauphine-PSL).

Lucia Tessarolo (Sorbonne University, France)

On the Schrödinger Evolution on the Characteristic Foliation

Let M be a 3D contact sub-Riemannian manifold and S a surface embedded in M. We study the Schrödinger evolution of a particle constrained on the characteristic foliation $\mathcal{F}$ of S. Specifically, we define the Schrödinger operator on each leaf ℓ as $\Delta_\ell u = \mathrm{div}_\mu \nabla_\ell u$, where ∇_ℓ is the Euclidean gradient along the leaf and μ is the surface measure on S inherited from the Popp volume, using the sub-Riemannian normal to the surface. We then study the self-adjointness of the operator Δ_ℓ on each leaf by defining a notion of "essential self-adjointness at a point", in such a way that Δ_ℓ will be essentially self-adjoint on the whole leaf if and only is it is essentially self-adjoint at both its endpoints. We see how this local property at a characteristic point depends on a curvature-like invariant at that point. Additionally, we study self-adjoint extensions and we discuss the possibility of constructing extensions that allow communication of different leaves.

Daniele Tiberio (SISSA, Italy)

Sard properties for polynomial maps in infinite dimension and applications to the Sard conjecture

I will consider "polynomial maps" from an infinite-dimensional Hilbert space to a finite-dimensional Euclidean space (for instance, the endpoint maps of some control systems). It is well-known that the classical Sard theorem is false in this setting, and a general theory is still lacking. I will present sharp criteria for the validity of Sard-type theorems for these maps. As an application, I will present new results on the sub-Riemannian Sard conjecture in Carnot groups. This talk is based on a work in collaboration with Antonio Lerario and Luca Rizzi.

Poster Session

Magalie Benefice (Institute of Mathematics of Bordeaux, France)

TBA

Gaia Bombardieri (University of Padua, Italy)

TBA

Alejandro Bravo-Doddoli (University of Michigan, US)

TBA

Nicklas Day (Texas A&M University, US)

TBA

Marco Di Marco (University of Padua, Italy)

TBA

Liangbing Luo (University of Connecticut, US)

TBA

Luca Nalon (University of Fribourg, Switzerland)

TBA

Nicola Paddeu (University of Fribourg, Switzerland)

TBA

Alessandro Scagliotti (Technical University of Munich, Germany)

TBA

Ye Zhang (Okinawa Institute of Science and Technology, Japan)

TBA