

ISem 27

27th Internet Seminar
2023-2024

Harmonic Analysis

Techniques for
Elliptic Operators

LECTURE PHASE

Oct 2023 - Feb 2024

PROJECT PHASE

Mar 2024 - Jun 2024

FINAL WORKSHOP IN LUMINY

June 17-21, 2024

INFO & REGISTRATION



[mathematik.tu-darmstadt.de/
analysis/lehre_analysis/isem27](https://mathematik.tu-darmstadt.de/analysis/lehre_analysis/isem27)

ORGANIZERS

Moritz Egert (Darmstadt)
Robert Haller (Darmstadt)
Sylvie Monniaux (Marseille)
Patrick Tolksdorf (Karlsruhe)



TECHNISCHE
UNIVERSITÄT
DARMSTADT



Karlsruher Institut für Technologie



Aix-Marseille
université
Socialement engagée





Contents

1	Program	2
2	The Projects	4
3	Lectures	20
4	Participants	21

1 Program

Workshop “Harmonic analysis techniques for elliptic operators” – ISEM 27

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:30	Project Orange: (Wolfgang Arendt, Manuel Schlierf)	Project Gray (Amru Hussein)	9:30-10:30: Dorothee Frey I	Project Magenta (Andrea Carbonaro, Oliver Dragičević)	9:30-10:30: Pascal Auscher II
11:00-12:30	Project Red (Katharina Klioba, Christian Seifert, Sascha Trostorff)	Project Teal (Tim Böhnlein, Andrew Morris)	Project Lime (Peer Kunstmann)	Project Purple (Hendrik Vogt)	Project Green (René Hoffeld, Merlin Schmitz)
15:00-16:30	Project Blue (Julian Hölz, Jonathan Mui)		15:30-16:30: Pascal Auscher I	15:00-16:00 Dorothee Frey II	Project Olive (Jonas Sauer)
17:00-18:30	Project Violet (Simon Bortz)		Project Pink (Sebastian Bechtel)	16:00-17:15: Project Cyan (Emiel Lorist)	Project Yellow (Sahiba Arora, Jochen Glück, Felix Schwenninger)
				17:45-19:00: Project Brown (Marjeta Kramar- Fijavž, Ivica Nakić)	

Workshop “Harmonic analysis techniques for elliptic operators” – ISEM 27

Monday, 17th June 2024

Time	Speaker	Title of Talk
09:00-10:30	Project Orange	Parabolic maximal regularity and the Kato square root property
11:00-12:30	Project Red	\mathcal{R} -sectorial Operators and Maximal Regularity
15:00-16:30	Project Blue	Littlewood-Paley Theory
17:00-18:30	Project Violet	$T(1)$ and $T(b)$ theorems

Tuesday, 18th June 2024

Time	Speaker	Title of Talk
09:00-10:30	Project Gray	Functional calculus for tuples of sectorial and bi-sectorial operators
11:00-12:30	Project Teal	Quadratic estimates for perturbed Dirac operators and applications

Wednesday, 19th June 2024

Time	Speaker	Title of Talk
09:30-10:30	Dorothee Frey I	Paradifferential methods for elliptic operators I
11:00-12:30	Project Lime	Bounded imaginary powers and complex interpolation
15:30-16:30	Pascal Auscher I	Weak solutions of abstract parabolic problems I
17:00-18:30	Project Pink	The Kato square root problem on open sets

Thursday, 20th June 2024

Time	Speaker	Title of Talk
09:00-10:30	Project Magenta	p -ellipticity
11:00-12:30	Project Purple	L^p -extrapolation à la Blunck-Kunstmann and Shen
15:00-16:00	Dorothee Frey II	Paradifferential methods for elliptic operators II
16:00-17:15	Project Cyan	H^∞ -calculus and square functions on Banach spaces
17:45-19:00	Project Brown	Gaussian estimates of heat kernels on domains

Friday, 21st June 2024

Time	Speaker	Title of Talk
09:30-10:30	Pascal Auscher II	Weak solutions of abstract parabolic problems II
11:00-12:30	Project Green	Strichartz Estimates
15:00-16:30	Project Olive	Maximal L^p Regularity for the Dirichlet Laplacian on a Wedge
17:00-18:30	Project Yellow	Characterisation of sectorial operators on L^p -spaces possessing a bounded H^∞ -calculus

\mathcal{R} -sectorial Operators and Maximal Regularity

Project red

Project coordinators: Katharina Klioba (Hamburg), Christian Seifert (Hamburg), Sascha Trostorff (CAU Kiel)

Participants: Francisco Carvalho (Delft), Drech Vusconty Miassangana (Casablanca), Maximilian Ruff (Karlsruhe)

In the ISem, we have encountered sectorial operators A on a Hilbert space H . In Lecture 6 we have defined the exponential e^{-tA} for $t > 0$ if the sectoriality angle of A is smaller than $\frac{\pi}{2}$, the so-defined family $(e^{-tA})_{t>0}$ is called the semigroup associated with A . In Proposition 6.6 it was shown that the semigroup yields the solution to the abstract Cauchy problem

$$\begin{aligned}\partial_t u(t) + Au(t) &= 0, \quad (t > 0) \\ u(0+) &= u_0\end{aligned}$$

by setting $u(t) := e^{-tA}u_0$. In the same way, one can solve the equation

$$\begin{aligned}\partial_t u(t) + Au(t) &= f(t), \quad (t > 0), \\ u(0+) &= 0,\end{aligned}\tag{2.1}$$

by computing the convolution of e^{-tA} with f ; that is,

$$u(t) := \int_0^t e^{-(t-s)A} f(s) ds.$$

One can now show that sectoriality of A yields the maximal L_2 -regularity of (2.1); that is, if $f \in L_2(0, \infty; H)$ then the so-defined solution u satisfies $u \in H^1(0, \infty; H)$ or equivalently (due to (2.1)) $Au \in L_2(0, \infty; H)$. It is the main object of this project to generalise this result to operators on Banach spaces X .

As we will see, sectoriality is not enough to ensure maximal regularity of (2.1). In fact, some stronger property is needed, namely \mathcal{R} -sectoriality, which in the Hilbert space case is equivalent to sectoriality. Moreover, the goal to prove such a result for all Banach spaces turns out to be too ambitious, so we will restrict our attention to so-called UMD spaces (sometimes also called \mathcal{HT} -spaces to reflect their relation to the Hilbert transform). This class of Banach spaces turns out to be suited for the application of techniques from Fourier analysis, which will be one of the main tools to prove our goal, which can be formulated as:

Maximal regularity of (2.1) in a UMD space is equivalent to \mathcal{R} -sectoriality of A .

The main source for this project will be [1], where our main result can be found in Theorem 4.4. Moreover, we will have a look at elliptic operators in divergence form, now on $L_p(\mathbb{R}^n)$ and not on $L_2(\mathbb{R}^n)$, and study the \mathcal{R} -sectoriality of those operators. If time permits, we can continue the study of elliptic operators, now on half-spaces and on domains.

Bibliography

- [1] R. Denk, M. Hieber, and J. Prüss. *\mathcal{R} -boundedness, Fourier multipliers and problems of elliptic and parabolic type*, volume 788 of *Mem. Am. Math. Soc.* Providence, RI: American Mathematical Society (AMS), 2003.

Strichartz Estimates

Project green

Project coordinators: René Hosfeld (Berlin), Merlin Schmitz (Wuppertal)

Participants: Chiara Alessi (Ferrara), Stefano Böhmer (Lund), Rafael Hirsch (Karlsruhe), Pablo Merino San José (University of the Basque Country)

In his seminal work from 1977, ROBERT S. STRICHARTZ [1] considered two problems concerning the Fourier transform, namely:

Given a subset $S \subset \mathbb{R}^n$ and a positive measure $d\mu \in \mathcal{S}'(\mathbb{R}^n)$ supported on S ;

- For which $1 \leq p < 2$ does $f \in L^p(\mathbb{R}^n)$ imply that \hat{f} has a well-defined restriction to S in $L^2(d\mu)$ with

$$\left(\int |\hat{f}|^2 d\mu \right)^{1/2} \leq c_p \|f\|_p?$$

- For which $2 < q \leq \infty$ does the tempered distribution $F d\mu$ for $F \in L^2(d\mu)$ have a Fourier transform in $L^q(\mathbb{R}^n)$ satisfying

$$\|\widehat{F d\mu}\|_q \leq c_q \left(\int |F|^2 d\mu \right)^{1/2} ?$$

While not being the first one to do so, he pioneered in applying such estimates to the analysis of dispersive equations such as the Schrödinger equation

$$i \frac{d}{dt} u(t) + \Delta u(t) = f(t), \quad u(0) = u_0$$

and the wave equation

$$\frac{d^2}{dt^2} u(t) - \Delta u(t) = f(t), \quad u(0) = u_0, \quad \frac{d}{dt} u(0) = u_1,$$

where the Fourier-representation of Δ is vital. This led to a series of similar results, known as *Strichartz estimates*, describing the space-time behavior of solutions. In the following years, many famous mathematicians extended this theory, e.g. to nonlinear equations, culminating in the proof of the “endpoint case” by MARKUS KEEL and TERENCE TAO [2].

Although the proofs of these results make use of many tools we learned throughout the lectures, e.g. interpolation and the Hardy-Littlewood-Sobolev inequality, their application is complementary to what we have seen so far.

In this project we want to study the development of Strichartz estimates and focus on their use for the (nonlinear) Schrödinger and wave equation. Depending on the students level, we will further investigate the endpoint case.

Bibliography

- [1] Robert S. Strichartz. Restrictions of Fourier Transforms to Quadratic Surfaces and Decay of Solutions of Wave Equations. *Duke Mathematical Journal*. Vol. 44, no. 3, pp. 705–714, 1977.
- [2] Markus Keel and Terence Tao. Endpoint Strichartz estimates. *American Journal of Mathematics*. Vol. 120, no. 5, pp. 955–980, 1998.
- [3] Jean Ginibre and Giorgio Velo. Smoothing Properties and Retarded Estimates for Some Dispersive Evolution Equations. *Communications in Mathematical Physics*. Vol. 144, no. 1, pp. 163–188, 1992.

Littlewood-Paley Theory

Project blue

Project coordinators: Julian Hölz (Wuppertal), Jonathan Mui (Wuppertal)

Participants: Paul Czerny (Kaiserslautern), Sefika Kuzgun (Rochester), Ivan Puljiz (Zagreb), Siliang Weng (Karlsruhe),

In ISem27, we have learned about quadratic estimates for sectorial operators of the form

$$\int_0^\infty \|f(tL)u\|^2 \frac{dt}{t} \lesssim \|u\|^2,$$

where f is a suitable holomorphic function defined on a sector. As hinted in Lecture 8 of the ISem, such estimates are related to a wider topic known as *Littlewood-Paley theory*. The goal of this project is to explore some aspects of this theory in the specific context of analytic semigroups $(T_t)_{t \geq 0}$, following Chapter 3 of the influential monograph of Stein [2]. These semigroups arise naturally, for example, in the study of diffusion processes and partial differential equations.

It will be of particular interest to extend the analysis from Hilbert spaces to L^p spaces with $1 < p < \infty$, and to obtain the *Littlewood-Paley inequality*

$$\left\| \left(\int_0^\infty \left| t \frac{d}{dt} T_t f \right|^2 \frac{dt}{t} \right)^{1/2} \right\|_p \leq C_p \|f\|_p \quad (2.2)$$

for all $f \in L^p(X, \mu)$, where $C_p > 0$ only depends on the semigroup and p . An important consequence of estimate (2.2) is the semigroup *maximal inequality*

$$\left\| \sup_{t>0} |T_t f| \right\|_p \leq M_p \|f\|_p,$$

which in turn has many applications in the study of the long-term behaviour of semigroups.

Depending on the interests of the students, we could investigate further connections to martingale theory (as in [2, Chapter 4]) or applications to the Ornstein-Uhlenbeck semigroup in finite or infinite dimensions (as in [1, Chapter 10]).

Bibliography

- [1] Giuseppe Da Prato and Jerzy Zabczyk. *Second order partial differential equations in Hilbert spaces*, volume 293, London Mathematical Society Lecture Note Series. Cambridge University Press, Cambridge, 2002.
- [2] Elias M. Stein. *Topics in harmonic analysis related to the Littlewood-Paley theory*, volume 63, Ann. Math. Stud. Princeton University Press, Princeton, NJ, 1970.

H^∞ -calculus and square functions on Banach spaces

Project cyan

Project coordinator: Emiel Lorist (Delft)

Participants: Andrew Pritchard (Newcastle), Himani Sharma (Karlsruhe), Johannes Stojanow (Hamburg)

To solve the Kato conjecture in the lectures, we first reformulated the Kato property as a square function estimate. One of the main characters in this reformulation was McIntosh's theorem, which states that a sectorial operator L on a Hilbert space H has a bounded H^∞ -calculus if and only if for some (equivalently all) nonzero $f \in H_0^\infty(S_\varphi)$ the quadratic estimate

$$\left(\int_0^\infty \|f(tL)u\|_H^2 \frac{dt}{t} \right)^{1/2} \approx \|u\|_H, \quad u \in H, \quad (2.3)$$

holds. Since neither the definition of the H^∞ -calculus, nor the statement of McIntosh's theorem explicitly use the Hilbert space structure of H , one may wonder if this theorem is also true for Banach spaces. This would, for example, be a useful tool in the study of the Kato property in $L^p(\Omega)$ with $p \neq 2$.

In [1], it was shown that for a sectorial operator L on $L^p(\Omega)$ the quadratic estimates need to be adapted, taking the form

$$\left\| \left(\int_0^\infty |f(tL)u|^2 \frac{dt}{t} \right)^{1/2} \right\|_{L^p(\Omega)} \approx \|u\|_{L^p(\Omega)}, \quad u \in L^p(\Omega). \quad (2.4)$$

Note that (2.3) and (2.4) coincide for $p = 2$ by Fubini's theorem.

The connection between H^∞ -calculus and quadratic estimates in [1] is not yet as clean as the statement we know in the Hilbert space setting. Only after introducing randomness, through a notion called \mathcal{R} -sectoriality, we arrive at a formulation in $L^p(\Omega)$ fully analogous to McIntosh's theorem [3]. In this project, we will explore the intricacies of McIntosh theorem in $L^p(\Omega)$. Moreover, we will discuss what happens in a general Banach space X [2]. Note that (2.4) does not have an obvious interpretation in this case, as $|x|^2$ has no meaning for $x \in X$!

Bibliography

- [1] M. Cowling, I. Doust, A. McIntosh, and A. Yagi. Banach space operators with a bounded H^∞ functional calculus. *J. Austral. Math. Soc. Ser. A*, 60(1):51–89, 1996.
- [2] N.J. Kalton and L. Weis. The H^∞ -functional calculus and square function estimates. In *Selecta. Volume 1.*, pages 716–764. Basel: Birkhäuser/Springer, 2016.
- [3] C. Le Merdy. On square functions associated to sectorial operators. *Bull. Soc. Math. France*, 132(1):137–156, 2004.

p-ellipticity

Project magenta

Project coordinators: Andrea Carbonaro (Genova) and Oliver Dragičević (Ljubljana)

Participants: Nikita Cernomazov (Frankfurt), Arnaud Dumont (Birmingham), Bruno Predojević (Zagreb), Floris Roodenburg (Delft)

The notion of an *elliptic* partial differential equation goes back to 1889, when it was introduced by Du Bois-Reymond [5, p. 265]. Eventually, the study of such equations and associated operators evolved into one of major subareas of mathematical analysis. The essence of ellipticity was described by L. Evans in his classic textbook [6, p.327] as follows: “[...] *calculations are often technically difficult but eventually yield extremely powerful and useful assertions concerning the smoothness of weak solutions. As always, the heart of each computation is the invocation of ellipticity: the point is to derive analytic estimates from the structural, algebraic assumption of ellipticity.*” [emphasis by L.E.]

Starting from around 1950s, a great deal of attention has been devoted to operators with *complex* coefficients. It has emerged that many of their properties significantly differ from those of their real-valued counterparts. In 2016, Carbonaro and Dragičević [1] introduced the concept of *p*-ellipticity. It involves an algebraic condition, not more elaborate than the classical ellipticity itself, which relates the exponent $p \in (1, \infty)$ to the complex matrix function A featuring in the divergence-form operator $Lu = -\operatorname{div}(A\nabla u)$. The case $p = 2$ corresponds to the classical ellipticity. Another condition, yet equivalent to *p*-ellipticity, was formulated independently of Carbonaro and Dragičević by Dindoš and Pipher [4]. Remarkably, they did so while focusing on problems much unrelated to those studied by Carbonaro and Dragičević.

Numerous results that followed since then indicate that *p*-ellipticity is the key condition for the L^p -theory of elliptic partial differential operators with complex coefficients. Moreover, *a posteriori* it turned out to be crucial even outside that realm, more precisely, in the holomorphic functional calculus [3]. The goal of the project is to study some of the above phenomena, with emphasis on applications to the theory of operator semigroups.

Bibliography

- [1] A. CARBONARO, O. DRAGIČEVIĆ: *Convexity of power functions and bilinear embedding for divergence-form operators with complex coefficients*, J. Eur. Math. Soc. (JEMS) 22 (2020), no. 10, pp. 3175–3221.
- [2] A. CARBONARO, O. DRAGIČEVIĆ: *Bilinear embedding for divergence-form operators with complex coefficients on irregular domains*, Calc. Var. Partial Differential Equations 59 (2020), no. 3, paper no. 104, 36 pp.
- [3] A. CARBONARO, O. DRAGIČEVIĆ: *Functional calculus for generators of symmetric contraction semigroups*, Duke Math. J. 166 (2017), 937–974.
- [4] M. DINDOŠ, J. PIPHER: *Regularity theory for solutions to second order elliptic operators with complex coefficients and the L^p Dirichlet problem*, Adv. Math. 341 (2019), 255–298.
- [5] P. DU BOIS-REYMOND: *Ueber lineare partielle Differentialgleichungen zweiter Ordnung*, J. reine angew. Math. 104 (1889), no.1, 241–301.
- [6] L. C. EVANS: *Partial differential equations*, 2nd Edition, AMS, Providence, RI, 2010.

Characterisation of sectorial operators on L^p -spaces possessing a bounded H^∞ -calculus

Project yellow

Project coordinators: Sahiba Arora (Twente), Jochen Glück (Wuppertal), Felix Schwenninger (Twente)

Participants: Salah-Eddine Chorfi (Mararrakech), Guillaume Neuntens (Jena), Christoph Schwerdt (Rostock), Ilka Wöckel (Kaiserslautern)

We have seen in [1, Chapter 6] that on a Hilbert space (the negative of) a sectorial operator L with sectoriality angle $\varphi_L < \pi/2$ generates a holomorphic semigroup $(e^{-tL})_{t \geq 0}$. Moreover, due to McIntosh's theorem [1, Theorem 8.4], we have characterised which operators have a bounded H^∞ -calculus.

Leaving the Hilbert space setting, we characterise sectorial operators on reflexive L^p -spaces that have a bounded H^∞ -calculus of angle $< \pi/2$. In particular, we prove the following:

Theorem. *Let (Ω, μ) be a σ -finite measure space and let $p \in (1, \infty)$. For a sectorial operator L on a subspace-quotient of $L^p(\Omega)$, the following are equivalent.*

- (i) L has a bounded H^∞ -calculus of angle $\varphi_L < \pi/2$.
- (ii) $(e^{-tL})_{t \geq 0}$ is similar to a positive, contractive, and analytic semigroup on some $L^p(\tilde{\Omega})$.

The implication (ii) \Rightarrow (i) is due to Weis [2] while the other implication was shown by Fackler [3].

Bibliography

- [1] ISem 27, Lecture Notes, 2024. <https://moodle-guest.hrz.tu-darmstadt.de/mod/resource/view.php?id=677>
- [2] L. Weis. Operator-valued Fourier multiplier theorems and maximal L_p -regularity. *Mathematische Annalen*, vol. 319, no. 4, 735–758 (2001).
- [3] S. Fackler. On the structure of semigroups on L_p with a bounded H^∞ -calculus. *Bull. Lond. Math. Soc.*, vol. 46, no. 5 pp. 1063–1076 (2014).

Functional calculus for tuples of sectorial and bi-sectorial operators

Project gray

Project coordinator: Amru Hussein (Kaiserslautern)

Participants: Bernhard Aigner (Freiberg), Simon Bau (Konstanz), Simon Murmann (Jena), El Assad Ouro-Koura (Darmstadt)

In the ISem lectures [1] we have learned about the notions of sectorial operators and the bounded H^∞ -calculus. The prime example for operators with these properties is the Laplace operator in \mathbb{R}^d .

In this project we will see that these notions can be extended in several directions. Instead of the elliptic Laplace operator $-\Delta$ on \mathbb{R}^d a possible prototype could be the parabolic operator $\partial_t - \Delta$ on $\mathbb{R} \times \mathbb{R}^d$. The first observation using Fourier transforms is that $-\Delta$ defines a sectorial operator of angle smaller than $\pi/2$ while ∂_t does not. However, its spectral properties can be captured accurately by the notion of bi-sectoriality. This means that the spectrum is contained in a so-called *bi-sector* with resolvent estimates outside, compare Figure 2.1.

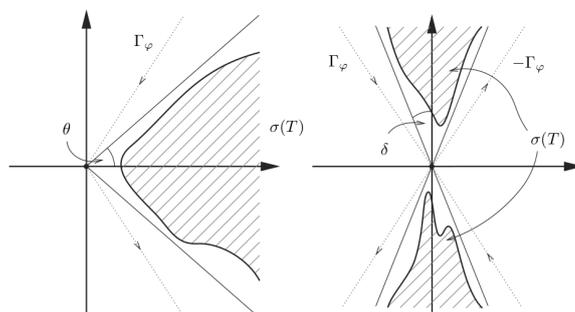


Figure 2.1: Sectorial and bi-sectorial operators, cf. [3, Fig. 1.1 and 1.2]

Second, to carry over the strategy of Fourier transforms – transforming derivatives to multiplications – one needs to transform simultaneously with respect to the time and space variables. This leads – instead of the bounded H^∞ -calculus for a single operator – to a bounded H^∞ -calculus for tuples, that is one, two or more operators. So, instead of only giving meaning to $f(-\Delta)$, one considers also objects of the form $f(\partial_t, -\Delta)$.

Third, the functions used in the functional calculus have been \mathbb{C} -valued holomorphic functions. This can be extended to include holomorphic functions taking values in a complex Banach space.

These three extensions have been addressed by DORE and VENNI in their article [2] amongst others. One elaboration can be found also in the book [3] by DENK and KAIP where general mixed order systems are treated as an application. Starting with these references we will examine how the above mentioned extensions of the theory can be accomplished and why some applications require exactly these methods.

Bibliography

- [1] ISem 27, Lecture Notes, 2023/24.
- [2] G. Dore and A. Venni. H^∞ functional calculus for sectorial and bi-sectorial operators. *Studia Math.*, 166(3):221–241, 2005. <https://doi.org/10.4064/sm166-3-2>
- [3] R. Denk and M. Kaip. *General parabolic mixed order systems in L_p and applications*. Birkhäuser/Springer, Cham, 2013, <https://doi.org/10.1007/978-3-319-02000-6>

Gaussian estimates of heat kernels on domains

Project brown

Project coordinators: Marjeta Kramar Fijavž (Ljubljana), Ivica Nakić (Zagreb)

Participants: Thomas Adrian (Darmstadt), Giulio Pecorella (Modena), Jakob Paulo Witt (Kaiserslautern)

This project can be seen as a complement to the off-diagonal estimates studied in Lecture 11 of [1], where the so-called L^2 -off-diagonal estimates for resolvents and semigroups generated by elliptic operators are obtained. The estimates were based on the representation of the corresponding resolvent as a convolution operator with the Bessel kernel. In this project we aim to establish Gaussian estimates for the (heat) kernel of the semigroup generated by a second order operator A in divergence form with real, not necessarily symmetric, second order coefficients on an open subset Ω of \mathbb{R}^d satisfying various boundary conditions. Here, Gaussian estimates are estimates of the heat kernel $K_t(x; y)$ of the corresponding semigroup of the form

$$|K_t(x; y)| \leq ct^{-d/2} e^{-b|x-y|^2 t^{-1}} e^{\omega t},$$

valid for almost all $(x, y) \in \Omega^2$ and uniformly for all $t > 0$, where $b, c > 0$ and $w \in \mathbb{R}$ are some constants.

We will start with Gaussian estimates for heat kernels of strictly elliptic operators satisfying Dirichlet boundary conditions given in [2] and introduce main tools, including the important concept of ultracontractivity. In the next step, we will extend the estimates to a more general setting using [3] and [4]. Finally, if time permits, we shall see how these estimates can be translated to the setting of heat equation on a network by studying [5].

Bibliography

- [1] ISem 27, Lecture Notes, 2024. <https://moodle-guest.hrz.tu-darmstadt.de/mod/resource/view.php?id=677>
- [2] E. B. Davies. *Heat kernels and spectral theory*. Cambridge University Press, 1989.
- [3] W. Arendt and A. F. M. ter Elst. Gaussian estimates for second order elliptic operators with boundary conditions. *Journal of Operator Theory* 38 (1):87-130, 1997.
- [4] E. M. Ouhabaz. *Analysis of Heat Equations on Domains*. LMS Monograph Series 30, Princeton University Press, Princeton, 2004.
- [5] D. Mugnolo. Gaussian estimates for a heat equation on a network. *Networks and Heterogeneous Media*, 2 (1):55–79, 2006.

Bounded imaginary powers and complex interpolation

Project lime

Project coordinator: Peer Christian Kunstmann (Karlsruhe)

Participants: Khalid Baadi (Paris), Paul Beckermann (Kaiserslautern), Abdelhakim Dahmani (Aalen), Jens de Vries (Twente)

Let A be an injective sectorial operator in a Hilbert space X . Such an operator is densely defined and has dense range. We know from this year's ISEM lectures that A has fractional powers A^α for any $\alpha \in \mathbb{C}$, given by the holomorphic functional calculus. In this project we shall study the property that the purely imaginary powers A^{it} , $t \in \mathbb{R}$, are bounded linear operators on X , i.e., that there exist $M \geq 1$ and $\omega \geq 0$ such that

$$\|A^{it}\| \leq M e^{\omega|t|}, \quad t \in \mathbb{R}. \quad (\text{BIP})$$

Clearly, if A has a bounded H^∞ -functional calculus then (BIP) holds, and the ω is related to the optimal angle of the H^∞ -calculus, which is – as we know – the same as the optimal sectoriality angle. Actually, also the converse holds, namely that (BIP) implies boundedness of the H^∞ -calculus. Another equivalent condition is the property $D(A^\theta) = [X, D(A)]_\theta$ for $\theta \in (0, 1)$ and the same for the adjoint operator A^* . Here, $[X, D(A)]_\theta$ denotes the *complex interpolation space* between X and the domain $D(A)$ of A equipped with the graph norm. This is important for applications as it allows to calculate domains of fractional powers A^α of A using just the information of its domain $D(A)$ (some examples are, e.g., in [10]).

The project shall give an introduction to complex interpolation present these results. The complex interpolation method had been introduced in [2]. A short introduction can be found in [5, Chapter IV], nice further literature are [1, 7, 9]. The mentioned equivalences are in [10] and [8] (note that [10] preceded the invention of H^∞ -functional calculus in [8]).

Remarks: (a) The important property that (BIP) allows to calculate domains of fractional powers of sectorial operators persists to Banach spaces X (see [9, 1.15.3]), but the other implications break down. However, there are suitable “reformulations” of corresponding properties (see [4]), but this will not be part of the project.

(b) There is another equivalence to (BIP) in Hilbert space, see [6], which is interesting in the context of the ISEM but can probably not be part of the project.

(c) The other important application of (BIP) is the Dore-Venni Theorem (see [3]) on the closedness of the sum of two closed operators in a UMD-Banach space. This result and its later variants have found countless applications to elliptic and parabolic problems. However, also these topics will not be part of the project.

Bibliography

- [1] Jöran Bergh, Jörgen Löfström, Interpolation spaces. An introduction. Grundlehren der Mathematischen Wissenschaften, No. 223. Springer-Verlag, Berlin-New York, 1976.
- [2] A.-P. Calderón, Intermediate spaces and interpolation, the complex method. *Studia Math.* 24 (1964), 113–190.
- [3] Giovanni Dore, Alberto Venni, On the closedness of the sum of two closed operators. *Math. Z.* 196 (1987), no. 2, 189–201.
- [4] Nigel Kalton, Peer Kunstmann, Lutz Weis, Perturbation and interpolation theorems for the H^∞ -calculus with applications to differential operators. *Math. Ann.* 336 (2006), no. 4, 747–801.
- [5] Yitzhak Katznelson, An introduction to harmonic analysis. Third edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2004.
- [6] Christian Le Merdy, The similarity problem for bounded analytic semigroups on Hilbert space. *Semigroup Forum* 56 (1998), no. 2, 205–224.
- [7] Alessandra Lunardi, Interpolation theory. Third edition. Appunti. Scuola Normale Superiore di Pisa (Nuova Serie), 16. Edizioni della Normale, Pisa, 2018.
- [8] Alan McIntosh, Operators which have an H^∞ functional calculus. Miniconference on operator theory and partial differential equations (North Ryde, 1986), 210–231, *Proc. Centre Math. Anal. Austral. Nat. Univ.*, 14, Austral. Nat. Univ., Canberra, 1986.
- [9] Hans Triebel, Interpolation theory, function spaces, differential operators. North-Holland Mathematical Library, 18. North-Holland Publishing Co., Amsterdam-New York, 1978.
- [10] Atsushi Yagi, Coïncidence entre des espaces d'interpolation et des domaines de puissances fractionnaires d'opérateurs. *C. R. Acad. Sci. Paris Sér. I Math.* 299 (1984), no. 6, 173–176.

Maximal L^p Regularity for the Dirichlet Laplacian on a Wedge

Project olive

Project coordinator: Jonas Sauer (Jena)

Participants: Leon Berghoff-Flüel (Darmstadt), Sara Mountassir (Marrakech), Philip Preußler (Twente), Paul Stephan (Konstanz)

In this project, we want to explore the notion of maximal L^p regularity and apply our newly gained knowledge in order to understand how to solve the equation

$$\begin{cases} \partial_t u - \Delta u = f & \text{in } J \times \Omega, \\ u = 0 & \text{on } J \times \partial\Omega, \\ u = 0 & \text{on } \{0\} \times \Omega \end{cases}$$

in suitable L^p -based spaces, where

$$\Omega = \{x \in \mathbb{R}^2 \mid x_1 > 0, 0 < x_2 < x_1\}$$

is an infinite wedge. As a guideline, we will work with the paper [2] by Prüss and Simonett.

The notion of maximal L^p regularity for a sectorial operator L has been mentioned in Chapter 6.2 of the lecture notes of the ISem27, but we will shift our perspective slightly: Instead of fixing a non-trivial initial configuration and solving a homogeneous differential equation, we start with zero initial data and make the problem non-trivial by introducing a general inhomogeneity f on the right-hand side of the differential equation.

So what is maximal L^p -regularity for a sectorial operator L on a Banach space X for us? For $p \in (1, \infty)$, $f \in L^p(J; X)$ and $T \in (0, \infty]$ and $J = (0, T)$, there is always a concept of a unique mild solution $u \in C(J; X)$ to the problem

$$\begin{cases} u' + Lu = f & \text{in } J, \\ u(0) = 0, \end{cases}$$

which is given by Duhamel's formula $\int_0^t e^{-(t-s)L} f(s) ds$. We say that L admits maximal L^p regularity on J , if for all $f \in L^p(J; X)$ this mild solution takes its values in $D(L)$ almost everywhere and satisfies $Lu \in L^p(J; X)$ (and thus also $u' \in L^p(J; X)$). In other words, not only is the sum $u' + Lu$ in the same space as f (which is trivial, because this sum is f), but both summands u' and Lu are so individually, too. This is vital information when dealing with nonlinear (e.g. quasilinear) problems, because it opens the door for a proof of existence and uniqueness of solutions via a Picard iteration. In this project, however, we will be more modest and stick to the linear theory: Our goal will be to show maximal L^p regularity of the Dirichlet Laplacian on the wedge Ω . Our starting point will be a corollary of a theorem by Kalton and Weis [1], which states that for two operators A and B with a bounded H^∞ calculus, the operator $A + B$ admits a bounded H^∞ calculus as well, as long as the underlying Banach space is fulfilling a mild geometric property. In the whole space, choosing $A := \partial_t$ and $B := -\Delta$ on the Banach space $L^p(J \times \mathbb{R}^n)$ almost immediately shows that $-\Delta$ admits maximal L^p regularity on $X = L^p(\mathbb{R}^n)$. But on the wedge one has to be more careful, and one of the main goals of this project will be to detect and properly understand the pitfalls that arise from the special geometry of the domain Ω with its non-smooth tip at the origin. In particular, the underlying Banach space X cannot necessarily be taken to be $L^p(\Omega)$ anymore. So which Banach space X would be a good substitute? Let's find out... by joining the project!

Bibliography

[1] N. J. Kalton and L. Weis. The H^∞ -Calculus and Sums of Closed Operators. *Math. Ann.*, 321(2):319–345, 2001.

[2] J. Prüss and G. Simonett. H^∞ -Calculus for the Sum of Non-Commuting Operators. *Trans. Amer. Math. Soc.*, 359(8):3549–3565, 2007.

Parabolic maximal regularity and the Kato square root property

Project orange

Project coordinators: Wolfgang Arendt (Ulm), Manuel Schlierf (Ulm)

Participants: Sofian Abahmami (Agadir), Henning Heister (Karlsruhe), Azam Jahandideh (Poznan), Vincenzo Leone (Salerno)

1 Introduction

Given $T > 0$ and Hilbert spaces V and H where $V \hookrightarrow H$, a mapping $a : [0, T] \times V \times V \rightarrow \mathbb{K}$ is called *non-autonomous form* if $a(\cdot, v, w) : [0, T] \rightarrow \mathbb{K}$ is measurable for all $v, w \in V$ and

$$|a(t, v, w)| \leq M \|v\|_V \|w\|_V \quad \text{for all } t \in [0, T] \text{ and } v, w \in V$$

for some $M \geq 0$. The form is said to be *coercive* if there exists $\alpha > 0$ with

$$\operatorname{Re} a(t, v, v) \geq \alpha \|v\|_V^2 \quad \text{for all } t \in [0, T] \text{ and } v \in V.$$

An elegant result of Lions shows well-posedness of the problem

$$\begin{cases} u'(t) + \mathcal{A}(t)u(t) &= f(t) \\ u(0) &= u_0 \end{cases} \quad (\text{P})$$

where $f \in L^2(0, T; V')$ and $u_0 \in H$. Here, we consider the usual embedding $H \hookrightarrow V'$ and the family of operators $\mathcal{A}(t) \in \mathcal{L}(V, V')$ given by

$$\langle \mathcal{A}(t)u, v \rangle = a(t, u, v) \quad \text{for all } t \in [0, T] \text{ and } v, w \in V.$$

In fact, one has *maximal regularity* in V' , i.e.

$$u \in H^1(0, T; V') \cap L^2(0, T; V).$$

Particularly, all the terms u' , $\mathcal{A}(\cdot)u(\cdot)$ and f belong to $L^2(0, T; V')$. Frequently however, the part $A(t)$ of $\mathcal{A}(t)$ in H given by

$$D(A(t)) = \{v \in V : \mathcal{A}(t)v \in H\}, \quad A(t)v = \mathcal{A}(t)v$$

is more important because this operator incorporates the boundary conditions. Thus, an important problem is the following

Lions' Problem (1961). *If $f \in L^2(0, T; H)$ and $u_0 \in V$, does this imply $u \in H^1(0, T; H)$?*

The answer is “No”, even if $u_0 = 0$. A first counterexample has been given by Dominik Dier (2014). It is based on the counterexample of McIntosh showing that $V \neq D(A^{\frac{1}{2}})$ is possible.

On the other hand, if the form a is sufficiently regular in time, then positive results hold by results of D. Dier, S. Fackler, E.M. Ouhabaz, C. Spina and others.

2 Organization of the project

The project is organized in the following parts.

1. Consider the Gelfand triple $V \hookrightarrow H \hookrightarrow V'$ and let $\mathcal{A} : V \rightarrow V'$ be the operator associated to an autonomous, coercive form a on V and let A be the part of \mathcal{A} in H . Moreover, denote by $(T(t))_t$ the contractive, holomorphic C_0 -semigroup on H generated by $-A$, cf. [AVV19, Theorem 5.8]. The goal is then to prove that

$$T(\cdot)x \in H^1(0, T; H) \quad \text{if and only if} \quad x \in D(A^{\frac{1}{2}}). \quad (2.5)$$

The main steps in the proof are outlined in [ADF17, Section 4]. One of the main ingredients and the main focus of this talk lies in understanding that $D(A^{\frac{1}{2}}) = [H, D(A)]_{\frac{1}{2}}$. That is, the domain of the square root is an interpolation space! The proof of this fact is a special case of [Haa06, Theorem 6.6.9].

2. Lions' theorem on maximal regularity in V' , cf. [AVV19, Theorem 17.15] and the above introduction. A key argument in the proof involves Lions' representation theorem, cf. [AVV19, Theorem 17.11].

3. Dier's counterexample, cf. [ADF17, Example 5.1] and [Die14].

4. A positive result: maximal regularity in H for Lipschitz continuous forms. More precisely, we suppose that the non-autonomous form $a: [0, \tau] \times V \times V \rightarrow \mathbb{K}$ can be written as $a = a_1 + b$ where a_1 and b are bounded non-autonomous forms on V with the following requirements:

- (i) a_1 is symmetric, i.e. $a_1(t, x, y) = \overline{a_1(t, y, x)}$ for $x, y \in V$ and $0 \leq t \leq \tau$;
- (ii) a_1 is coercive, i.e. there exists $\alpha > 0$ with $a_1(t, x, x) \geq \alpha \|x\|_V^2$ for all $x \in V$, $0 \leq t \leq \tau$;
- (iii) a_1 is Lipschitz continuous, i.e. there exist $M'_1 \geq 0$ with

$$|a_1(t, x, y) - a_1(s, x, y)| \leq M'_1 |t - s| \|x\|_V \|y\|_V$$

for all $0 \leq t \leq \tau$ and all $x, y \in V$;

- (iv) There exists $M_b \geq 0$ with $|b(t, x, y)| \leq M_b \|x\|_V \|y\|_H$ for all $0 \leq t \leq \tau$ and $x, y \in V$.

Then the statement of Lions' Problem as above holds true. For reference, cf. [ADLO14] and [AVV19, Theorem 18.2].

This talk's goal is giving a proof, possibly under somewhat stronger regularity assumptions on the form. For instance, if one even assumes C^1 regularity instead of Lipschitz regularity in time, then many technicalities become easier to handle.

Bibliography

- [ADF17] Wolfgang Arendt, Dominik Dier, and Stephan Fackler. J. L. Lions' problem on maximal regularity. *Arch. Math. (Basel)*, 109(1):59–72, 2017.
- [ADLO14] Wolfgang Arendt, Dominik Dier, Hafida Laasri, and El Maati Ouhabaz. Maximal regularity for evolution equations governed by non-autonomous forms. *Adv. Differential Equations*. 19(11-12):1043–1066. 2014.
- [AVV19] Wolfgang Arendt, Hendrik Vogt, and Jürgen Voigt. Form methods for evolution equations. In *Lecture Notes of the 18th International Internet seminar*, volume 6, 2019.
- [Die14] Dominik Dier. *Non-autonomous Cauchy problems governed by forms: maximal regularity and invariance*. PhD thesis, Universität Ulm, 2014.
- [Haa06] Markus Haase. *The functional calculus for sectorial operators*, volume 169 of *Operator Theory: Advances and Applications*. Birkhäuser Verlag, Basel, 2006.

The Kato square root problem on open sets

Project pink

Project coordinator: Sebastian Bechtel (Delft)

Participants: Cody Michael Hutcheson (Tuscaloosa), Timotheus Schmatzler (Stockholm), Tolgahan Tasci (Wuppertal), Mattes Wittig (Hamburg)

Even though we have introduced elliptic operators $-\operatorname{div}A\nabla$ on an open set $\Omega \subseteq \mathbb{R}^n$ and subject to Dirichlet boundary conditions during this ISEM, we could only solve the Kato square root problem on the full Euclidean space $\Omega = \mathbb{R}^n$. This was not due to laziness but there are serious complications when treating non-trivial open sets, for instance:

- What is the right substitute for dyadic cubes, which played an eminent role in the final lectures?
- Unperturbed differential operators like ∇ , $-\Delta$ and $-\operatorname{div}$ cannot be manipulated on the Fourier side anymore!
- Geometric complications, for instance in the usage of Sobolev–Poincaré inequalities.

Usually, the Kato square root problem on open sets is studied in the scope of perturbed Dirac operators. Perturbed Dirac operators on \mathbb{R}^n will also be studied by another team. One direction for this project could be to investigate how the proof of square function estimates for perturbed Dirac operators on \mathbb{R}^n have to be modified if working on an open set Ω , but using “abstract” hypothesis on the geometry as ingredient [3]. To the converse, we could also take square function estimates for granted and show how the abstract geometric ingredients can be checked in concrete settings [1, 2], also for other boundary conditions than Dirichlet. To this end, questions in areas like interpolation theory, function spaces and so on could enter the scene.

That being said, the concrete content of this project will be adapted to the background and interests of the participants.

Bibliography

- [1] S. BECHTEL, M. EGERT, and R. HALLER-DINTELMANN. *The Kato square root problem on locally uniform domains*. Adv. Math. **375** (2020)
- [2] M. EGERT, R. HALLER-DINTELMANN, and P. TOLKSDORF. *The Kato Square Root Problem for mixed boundary conditions*. J. Funct. Anal. **267** (2014), no. 5, 1419–1461.
- [3] M. EGERT, R. HALLER-DINTELMANN, and P. TOLKSDORF. *The Kato Square Root Problem follows from an extrapolation property of the Laplacian*. Publ. Math. **61** (2016), no. 2, 451–483.

L^p -extrapolation à la Blunck-Kunstmann and Shen

Project purple

Project coordinator: Hendrik Vogt (Bremen)

Participants: Erik Heidrich (Kaiserslautern), Jonas Lenz (Mainz), Siguang Qi (Paris), Charlotte Söder (Darmstadt)

The aim of this project is a deeper investigation of off-diagonal estimates. In the ISem lectures, in Theorem 11.16, it has already been shown that off-diagonal estimates in combination with Sobolev embeddings lead to L^p -extrapolation for the resolvents of an elliptic operator L in divergence form on \mathbb{R}^n . More precisely, if $|\frac{1}{p} - \frac{1}{2}| < \frac{1}{n}$, then there exists $C > 0$ such that $\|(1 + tL)^{-1}u\|_p \leq C\|u\|_p$ for all $t > 0$, $u \in L^p \cap L^2(\mathbb{R}^n)$.

A related (more difficult!) question is for what range of $p \in (1, \infty)$ the norm equivalence $\|\sqrt{L}u\|_2 \simeq \|\nabla u\|_2$ from Theorem 12.1 (the Kato square root property for L !) extrapolates to $L^p(\mathbb{R}^n)$. It turns out that there are different ranges of p for the two estimates $\|\sqrt{L}u\|_p \lesssim \|\nabla u\|_p$ and $\|\nabla u\|_p \lesssim \|\sqrt{L}u\|_p$. The latter estimate is generally known as L^p -boundedness of the Riesz transform, and this is what shall be the core of the project.

Starting point of the project is the AMS memoir [1], which starts with an excellent introduction into the topic; you can find a preprint version of the memoir on the arXiv (with different numbering of theorems than in the published version, unfortunately). An important abstract L^p -extrapolation result is Theorem 1.1 in [1], the application Riesz transforms on L^p can be found in Section 4.1. This approach is due to Blunck and Kunstmann [2, 3]. If time permits, we can also study the approach of Shen [4] to Riesz transforms. The precise selection of topics will be decided among the participants of the project.

Bibliography

- [1] P. AUSCHER, *On necessary and sufficient conditions for L^p -estimates of Riesz transforms associated to elliptic operators on \mathbb{R}^n and related estimates*. Mem. Amer. Math. Soc. 186 (2007), no. 871.
- [2] S. BLUNCK, P. KUNSTMANN, *Calderón-Zygmund theory for nonintegral operators and the H^∞ functional calculus*. Rev. Mat. Iberoamericana 19, no. 3 (2003), 919–942.
- [3] S. BLUNCK, P. KUNSTMANN, *Weak-type (p, p) estimates for Riesz transforms*. Math. Z. 247, no. 1 (2004), 137–148.
- [4] Z. SHEN, *Bounds of Riesz transforms on L^p -spaces for second order elliptic operators*. Ann. Inst. Fourier 55 (2005), 173–197.

Quadratic Estimates for Perturbed Dirac Operators & Applications

Project teal

Project coordinators: Tim Böhnlein (Darmstadt), Andrew Morris (Birmingham)

Participants: Luca Haardt (Karlsruhe), Pablo Hidalgo-Palencia (Madrid), Wilhelm Treschow (Lund), Tobias Wang (Atlanta)

The solution of the Kato square-root problem shows us that the square-root $\sqrt{L_A}$ of a second-order divergence-form operator L_A with elliptic coefficients A has the domain and estimates comparable to that of the first-order gradient system $A\nabla$. There are advantages to working directly with similar first-order systems BD , where D is a self-adjoint first-order differential operator, and B is a bounded accretive multiplication operator encoding the elliptic coefficients A . For example, the square root $\sqrt{L_A}$, semigroup e^{-tL_A} and other important functions of L_A can be recovered as components using the holomorphic functional calculus for BD .

This project investigates just some of the benefits provided by the first-order perspective. The framework developed for perturbed Dirac operators by Axelsson–Keith–McIntosh in [4] allows us to obtain a unified proof of many results in the Calderón programme, such as the Kato square-root problem and boundedness of the Cauchy singular integral on Lipschitz curves. We have already encountered the beginning of this approach in the ISem Lecture Notes (see Exercises 2.4 and 2.5).

The results also allow us to deduce the quadratic estimates for BD -type systems obtained by Auscher–Axelsson–McIntosh in [2]. A further application, following the approach developed in [1, 3, 5], establishes L^2 -solvability of Dirichlet and Neumann boundary value problems on the upper half-space for elliptic equations $L_A u = 0$ with coefficients A in a certain block or Hermitian structure.

Bibliography

- [1] AUSCHER, P., AND AXELSSON, A. Weighted maximal regularity estimates and solvability of non-smooth elliptic systems I. *Invent. Math.* 184, 1 (2011), 47–115.
- [2] AUSCHER, P., AXELSSON, A., AND MCINTOSH, A. On a quadratic estimate related to the Kato conjecture and boundary value problems. In *Harmonic analysis and partial differential equations*, vol. 505 of *Contemp. Math.* Amer. Math. Soc., Providence, RI, 2010, pp. 105–129.
- [3] AUSCHER, P., AXELSSON, A., AND MCINTOSH, A. Solvability of elliptic systems with square integrable boundary data. *Ark. Mat.* 48, 2 (2010), 253–287.
- [4] AXELSSON, A., KEITH, S., AND MCINTOSH, A. Quadratic estimates and functional calculi of perturbed Dirac operators. *Invent. Math.* 163, 3 (2006), 455–497.
- [5] MORRIS, A.J., AND TURNER, A.J. Solvability for non-smooth Schrödinger equations with singular potentials and square integrable data. arXiv:2001.11901.

$T(1)$ and $T(b)$ theorems

Project violet

Project coordinator: Simon Bortz (Tuscaloosa Alabama)

Participants: Daan van Dijk (Jena), Sven Lauterbach (Darmstadt), Alessandra Migliaccio (Ferrara)

The $T(1)$ theorem of David and Journé is one of the most remarkable theorems in harmonic analysis. The theorem reduces the study of L^p boundedness of a singular integral operator, T to testing a ‘testing condition’, that is, verifying $T(1)$ is in the space BMO . A simplistic view of these theorems is that they shift the task of verifying boundedness for all functions (globally) to that of verifying a condition on all cubes. More general testing conditions, e.g. ‘local $T(b)$ ’ conditions, allow one to adapt the testing function to the cube and/or weaken conditions on the operator. These ‘local $T(b)$ theorems’ are an important ingredient to the initial solution to the Kato problem.

The project will introduce the concepts of $T(1)/T(b)$ theory for singular integrals, Littlewood-Paley theory, Carleson measures and stopping time arguments. The goal is to present the ‘original’ proof of the Kato problem and, possibly, look at more recent developments.

Bibliography

[1] $T(1)$ and $T(b)$ Theorems and applications, In *Harmonic analysis and applications*, 155–197. IAS/Park City Math. Ser., 27.

Weak solutions of abstract parabolic problems

Pascal Auscher (Université Paris-Saclay)

Abstract:

We discuss a universal construction of weak solutions for parabolic problems on a Hilbert space when the time interval is allowed to be infinite. It leads to a construction of fundamental solution operators with the right homogeneous estimates. This is used to represent weak solutions of the Cauchy problem. The methods use functional calculus and rely on a “homogeneous” extension of Lions’ embedding. This extends in abstract context a method devised previously in joint work with M. Egert. This is joint work with my student K. Baadi.

Paradifferential methods for elliptic operators

Dorothee Frey (Karlsruhe Institute of Technology)

Abstract:

When dealing with nonlinear differential equations, it is often crucial to have sharp regularity bounds on the nonlinearities available. Classically, such results are obtained in the framework of the paradifferential calculus, which was developed by Bony, Coifman and Meyer, among others. It is based on an in-depth analysis of the interaction of frequencies of different sizes occurring in products of functions and a good understanding of resonances, and relies heavily on Littlewood-Paley theory. In this talk, we show how the standard paradifferential methods can be adapted to elliptic operators using holomorphic functional calculus. This includes Leibniz rules for operator-adapted Sobolev spaces and an endpoint Kato-Ponce commutator inequality.

4 Participants

Workshop “Harmonic analysis techniques for elliptic operators” – ISEM 27

Abahmami Sofian, Ibn Zohr University in Agadir

Adrian Thomas, TU Darmstadt

Aigner Bernhard, TU Bergakademie Freiberg

Alessi Chiara, University of Ferrara

Arendt Wolfgang, Ulm University

Arora Sahiba, University of Twente

Auscher Pascal, Université Paris-Saclay

Baadi Khalid, Université Paris-Saclay

Bau Simon, University of Konstanz

Bechtel Sebastian, TU Delft

Beckermann Paul, RPTU Kaiserslautern

Berghoff-Flüel Leon, TU Darmstadt

Böhmer Stefano, Lund University

Böhnlein Tim, TU Darmstadt

Bortz Simon, University of Alabama

Carbonaro Andrea, University of Genova

Carvalho Francisco, TU Delft

Cernomazov Nikita, Goethe University Frankfurt

Chorfi Salah-Eddine, Cadi Ayyad University

Czerny Paul, RPTU Kaiserslautern

Dahmani Abdelhakim, Hochschule of Aalen

van Dijk Daan, University Jena

Dragičević Oliver, University of Ljubljana

Dumont Arnaud, University of Birmingham

Egert Moritz, TU Darmstadt

Frey Dorothee, Karlsruhe Institute of Technology

Glück Jochen, University of Wuppertal

Haardt Luca, Karlsruhe Institute of Technology

Haase Markus, Kiel University

Haller Robert, TU Darmstadt

Heidrich Erik, RPTU Kaiserslautern

Heister Henning, Karlsruhe Institute of Technology

Hidalgo-Palencia Pablo, ICMAT, Madrid

Hirsch Rafael, Karlsruhe Institute of Technology

Hölz Julian, University of Wuppertal

Hosfeld Rene, TU Berlin

Hussein Amru, RPTU Kaiserslautern

Hutcheson Cody Michael, University of Alabama

Jahandideh Azam, Poznan University
Klioba Katharina, Hamburg University of Technology
Kramar-Fijavž Marjeta, University of Ljubljana
Kunstmann Peer, Karlsruhe Institute of Technology
Kuzgun Sefika, University of Rochester
Lauterbach Sven, TU Darmstadt
Lenz Jonas, Johannes Gutenberg-University Mainz
Leone Vincenzo, University of Salerno
Lorist Emiel, TU Delft
Merino San José Pablo, University of the Basque Country
Miassangana Drech Vusconty, Hassan II University
Migliaccio Alessandra, University of Ferrara
Monniaux Sylvie, Université Aix-Marseille
Morris Andrew, University of Birmingham
Mountassir Sara, University Cadi Ayyad
Mui Jonathan, University of Wuppertal
Murmman Simon, University Jena
Nakić Ivica, University of Zagreb
Neuttiens Guillaume, University Jena
Ouro-Koura El Assad, TU Darmstadt
Pecorella Giulio, University of Modena and Reggio Emilia
Predojević Bruno, University of Zagreb
Preußler Philip, University of Twente
Pritchard Andrew, Newcastle University
Puljiz Ivan, University of Zagreb
Qi Siguang, Sorbonne University
Roodenburg Floris, TU Delft
Ruff Maximilian, Karlsruhe Institute of Technology
Sauer Jonas, University Jena
Schlierf Manuel, Ulm University
Schmatzler Timotheus, Stockholm University
Schmitz Merlin, University of Wuppertal
Schnwenninger Felix, University of Twente
Schwerdt Christoph, University of Rostock
Seifert Christian, Hamburg University of Technology
Sharma Himani, Karlsruhe Institute of Technology
Söder Charlotte, TU Darmstadt
Stephan Paul, University of Konstanz
Stojanow Johannes, Hamburg University of Technology
Tasci Tolgahan, University of Wuppertal
Tolksdorf Patrick, Karlsruhe Institute of Technology
Treschow Wilhelm, Lund University
Trostorff Sascha, CAU Kiel

Vogt Hendrik, University of Bremen
de Vries Jens, University of Twente
Wang Tobias, TU Darmstadt/Georgia Tech
Weng Siliang, Karlsruhe Institute of Technology
Witt Jakob Paulo, RPTU Kaiserslautern
Wittig Mattes, Hamburg University of Technology
Wöckel Ilka, RPTU Kaiserslautern