Geometric Sciences in Action: from Geometric Statistics to Shape Analysis
Centre International de Rencontres Mathématiques, Lumini, Marseille France
27 – 31 May, 2024

Organizing Committee:
Martin Bauer (Florida State University)
Blanche Buet (Université Paris-Saclay)
Alice Le Brigant (Université Paris 1 Panthéon-Sorbonne)
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Darryl Holm (Imperial College London)
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Alain Trouvé (École Normale Supérieure)

Conference Schedule

Monday, May 27, 2024

- **08:30 – 09:00** The organizers
  Welcome address and information about the conference

Geometric Statistics and Stochastic Geometric Mechanics

Chair: Xavier Pennec

- **09:00 – 09:30** Erlend Grong, University of Bergen, Norway
  Score matching and sub-Riemannian bridges
- **09:45 – 10:15** Huiling Le, University of Nottingham, UK.
  Stein's method on stratified spaces?

- **10:30 – 11:00** Coffee Break

- **11:00 – 11:30** Hans Munthe-Kaas, UiT The Norwegian Arctic University, Norway
  Connection algebras: between algebra, geometry and computations.
- **11:45 – 12:15** Karen Habermann, University of Warwick, UK
  Long-time existence of Brownian motion on configurations of two landmarks

- **12:30 – 15:00** **Lunch Break**

Chair: Stefan Sommer

- **15:00 – 15:30** Victor Panaretos, EPFL Lausanne, CH
  Geometrical Statistics in the Bures-Wasserstein Space
- **15:45 – 16:15** Stephan Huckemann, Georg-August-Universität Göttingen, DE
  A Lower Bound for Estimating Fréchet Means

- **16:30 – 17:00** Coffee Break

Software Presentations I

Chair: Alice Le Brigant

- **17:00 – 17:30** Luís F. Pereira, Università degli Studi di Milano-Bicocca, IT & UC Santa Barbara, USA
  Geomstat, a Python package for Riemannian geometry in machine learning
- **17:30 – 18:00** Mathieu Carriere, Université Côte d'Azur and Inria, FR
  Topological Data Analysis with the Gudhi library

Tuesday, May 28, 2024

Diffeomorphic Methods and Applications to Medical Imaging and Biology

Chair: Stephen Preston

- **09:00 – 09:30** Laurent Younes, Johns Hopkins University, USA
  A scale space framework for diffeomorphic shape analysis.
- **09:45 – 10:15** Barbara Gris, Sorbonne Université, FR
  An implicit formulation for incorporating different priors into a deformation model.

- **10:30 – 11:00** Coffee Break

- **11:00 – 11:30** Nicolas Charon, University of Houston, USA
  Unbalanced optimal transport under path constraints in measure space
- **11:45 – 12:15** Francois-Xavier Vialard, Univ. Gustave Eiffel, FR
  Synthetic nonnegative cross-curvature lifts to the Wasserstein space.

- **12:30 – 15:00** **Lunch Break**

Chair: Klas Modin

- **15:00 – 15:30** Boris Khesin, University of Toronto, CA
  Fluids, diffeomorphisms, and shapes.
- **15:45 – 16:15** Alice Le Brigant, University Paris 1, FR
  The L^p-Fisher-Rao metric and α-connections.

- **16:30 – 17:00** Coffee Break

Poster session

- **17:00 – 19:00** Poster Competition
  Contributions from Shreya Arya, Benjamin Beaudett, Jonathan Cerqueira, Théo Dumont, Emmanuel Hartman, Florine Hartwig and Sascha Beutler, Sadashige Ishida, Erik Jansson, Lars Lammers, Levin Maier, Elodie Maignant and Anna Calissano, Mao Nishino, Guillaume Olikier, Morten Akhøj Pedersen, Lidiya Pryymak, Guillaume Sérieys, Tom Szwagier, Yann Thanwerdas.

Wednesday, May 29, 2024;
Chair: M. Bauer

Curves and Surfaces I

- **09:00 – 09:30** Peter Michor, University of Vienna, AT
  Symplectic Structures on the space of space curves.
- **09:45 – 10:15** Barbara Tumpach, University of Lille, FR
  Totally geodesic submanifolds in the manifold SPD of symmetric positive-definite real matrices.

- **10:30 – 11:00** Coffee Break

- **11:00 – 11:30** Eric Klassen, Florida State University, USA
  Elastic Metrics on Spaces of Euclidean Curves: Theory and Algorithms.
- **11:45 – 12:15** Stephen C. Preston, Brooklyn College, USA
  Isometric immersions and the waving of flags

- **12:30 – 15:00** **Lunch Break**

- **15:00 – 19:00** Social Afternoon: Hiking in Luminy

Thursday, May 30, 2024

Graphs, Networks and Stratified Spaces

- **09:00 – 09:30** Facundo Memoli, Ohio State University, USA
  The exact determination of Gromov-type distances between spheres.
- **09:45 – 10:15** Tom Needham, Florida State University, USA
  Geometry and Topology of Spaces of Structured Matrices.

- **10:30 – 11:00** Coffee Break

- **11:00 – 11:30** Blanche Buet, Université Paris Saclay, FR
  Flagfolds: multi-dimensional varifolds to handle discrete surfaces
- **11:45 – 12:15** Karl-Theodor Sturm, University of Bonn, DE.
  Metric Spaces and Synthetic Curvature Bounds

- **12:30 – 15:00** **Lunch Break**

- **15:00 – 15:30** Ezra Miller, Duke, USA.
  What is a Gaussian on a singular space?
- **15:45 – 16:15** Victor-Emmanuel Brunel, Inria, FR
  Estimation of generalized barycenters in metric spaces.

- **16:30 – 17:00** Coffee Break

Curves & Surfaces II

- **17:00 – 17:30** Kathrin Welker, University of Freiberg, DE
  On the optimization of piecewise-smooth shapes.
- **17:45 – 18:15** Anuj Srivastava, Florida State University, USA
  On Statistical Inferences Involving Shape Data.

Friday, May 31, 2024

Software Presentations II

- **09:00 – 09:30** Emmanuel Hartman, Florida State University, USA
  Elastic shape analysis of surfaces with second-order Sobolev metrics.
- **09:30 – 10:00** Jean Feydy, Inria, FR
  The geometric software stack: past, present, future.
- **10:00 – 10:30** Gefan Yang
  Jax Geometry and Hyperiax

- **10:30 – 11:00** Coffee Break

Probabilities & Shapes

- **11:00 – 11:30** Irene Kaltenmark, Université Paris Cité, FR
  I. IA vs. humans : what shape space for the mind ? II. Partial matchings of curves and surfaces.
- **11:45 – 12:15** James Benn, Inria, FR
  The Geometry of Right-Invariant Metrics on Lie Groups.

- **12:30 – 14:00** **Lunch Break**

- **14:00 – 14:20** Josua Sassen, ENS Paris-Saclay, FR
  Low-dimensional Product Submanifolds of the Space of Discrete Shells.
- **14:30 – 14:50** Anna Calissano, Imperial College, UK
  Analysis of Unlabelled Graphs: Graph Space Geometry and Generalized Geodesic Modelling.
- **15:00 – 15:20** Libby Baker, University of Copenhagen, DK
  Conditioning infinite-dimensional stochastic processes with applications to shapes
- **15:30** End of the conference

Book of Abstracts

Libby Baker
Title: Conditioning infinite-dimensional stochastic processes with applications to shapes
Abstract: In this talk we present the theory of stochastic shape matching. That is, we consider a stochastic process defined in the diffeomorphism group and then develop an infinite dimensional analogue of Doob's h-transform, used to condition this process to hit a certain end point. To do so, we employ an infinite-dimensional version of Girsanov's theorem. This results in a stochastic process on shapes, that can be conditioned to hit a specific shape at a specific time. We apply this to butterfly outlines, to model how the change in shape of butterflies evolves over time.

James Benn
Title: The Geometry of Right-Invariant Metrics on Lie Groups
Abstract: This talk reviews the geometry of a right invariant metric on a (possibly infinite dimensional) Lie Group. New results on the geodesic equations and Riemannian exponential map will be presented, along with some applications to conjugate points and geodesic stability. It is known that the classical BCH formula for diffeomorphism groups does not hold due to the local surjectivity failure of the intrinsic group exponential map; this talk will close with a discussion how our results on Riemannian exponential maps of right-invariant Sobolev type metrics could be used to obtain a Riemannian BCH-type formula for the diffeomorphism group.

Victor-Emmanuel Brunel
Title: Estimation of barycenters in convex spaces
Abstract: In metric spaces that lack a linear structure, barycenters provide a canonical extension of linear averaging. In this talk, we are interested in the problem of estimating the barycenter of a distribution, given iid data. We work under a geometric assumption on the underlying space, ensuring that barycenters are defined as solutions to (geodesically) convex optimization problems and we present statistical guarantees for several estimators, some of which that can be computed efficiently from streamed data.

Blanche Buet
Title: Flagfolds: multi-dimensional varifolds to handle discrete surfaces
Abstract: We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well. While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We performed numerical computations of mean curvature and Gaussian curvature on point clouds in $\mathbb{R}^3$ to illustrate this approach. Though flexible, varifolds require the knowledge of the dimension of the shape to be considered. By interpreting the product of the Principal Component Analysis, that is the covariance matrix, as a sequence of nested subspaces naturally coming with weights according to the level of approximation they provide, we are able to embed all d-dimensional Grassmannians into a stratified space of covariance matrices. Building upon the proposed embedding of Grassmannians into the space of covariance matrices, we generalize the concept of varifolds to what we call flagfolds in order to model multi-dimensional shapes.

Anna Calissano
Title: Analysis of Unlabelled Graphs: Graph Space Geometry and Generalized Geodesic Modelling
Abstract: Sets of graphs (or networks) arise in many different fields, from medicine to finance, from sport to the social sciences. The analysis of unlabelled graphs or networks is far from trivial due to the highly non-Euclidean nature of such data. We describe Graph Space as a possible geometric embedding for a set of unlabelled graphs, i.e. graphs with no node correspondence across observations. Graph Space is a quotient space, but it is not a manifold, requiring the definition of statistical methods beyond the tangent space approach. We introduce the Align All and Compute algorithm and use it for both estimating generalized geodesic principal components

and generalized geodesic regression models, showing how to interpolate between unlabelled graphs. We demonstrate the flexibility of the framework on both simulated data, public transport system data, and Fifa 2018 player passing network data.

Mathieu Carrière
Title: Topological Data Analysis with the Gudhi library
Abstract: Topological Data Analysis (TDA) is a field of data science that is gathering increasing attention in the recent years, due to its ability to produce descriptors of topological flavor that (a) can be computed on a wide range of data sets, and (b) encode a unique type of information that is very useful yet missed by other standard descriptors. However, integration of such descriptors in standard machine learning pipelines is not straightforward, be it Mapper complexes or persistent homology. In this talk, I will present the main difficulties associated to the creation of topological machine learning pipelines, and then I will go over the various methods and solutions proposed over the recent years to handle these issues and implemented in the Gudhi library, including, e.g., the statistical treatment of Mapper, the representation of (multi-parameter) persistent homology, and the differentiability of topological descriptors.

Nicolas Charon
Title: Unbalanced optimal transport under path constraints in measure space
Abstract: We will present a variation of the unbalanced optimal transport model and Wasserstein Fisher-Rao metric on positive measures, in which one imposes additional affine integral equality constraints. This is motivated by multiple examples from mathematics and applied mathematics that naturally involve comparing and interpolating between two measures in particular subspaces or in which one enforces some constraints on the interpolating path itself. Building from the dynamic formulation of the Wasserstein Fisher-Rao metric, we introduce a class of constrained problems where the interpolating measure at each time must satisfy a given stationary or time-dependent constraint in measure space. We then specifically derive general conditions under which the existence of minimizing paths can be guaranteed, and then examine some of the properties of the resulting models and the metrics that are induced on measures. We will further hint at the potential of this approach in various specific situations such as the comparison of measures with prescribed moments, the unbalanced optimal transport under global mass evolution or obstacle constraints, and emphasize some connections with the construction of Riemannian metrics on the space of all convex shapes in an Euclidean space. We shall conclude with a few remaining unsolved/open questions.

Jean Feydy
Title: The geometric software stack: past, present, future
Abstract: The software ecosystem for geometric machine learning is evolving quickly. Modern libraries such as PyTorch, PyVista or Taichi open exciting research directions, but also challenge the long-term viability of our research projects: as major libraries get deprecated from one year to the next, how can we build software that is both cutting edge and future-proof? In this presentation, I will discuss the main tradeoffs involved in those decisions, and share some practical tips learnt through the development of the KeOps, GeomLoss and scikit-shapes libraries.

Barbara Gris
*Title:*An implicit formulation for incorporating different priors into a deformation model
Abstract: One of the goals of shape analysis is to model and characterise shape evolution. We focus on methods where this evolution is modeled by the action of a time-dependent diffeomorphism, which is characterised by its time-derivatives: vector fields. Reconstructing the evolution of a shape from observations then amounts to determining an optimal path of vector fields whose flow of diffeomorphisms deforms the initial shape in accordance with the observations. However, if the space of considered vector fields is not constrained, optimal paths may be inaccurate from a modeling point of view. To overcome this problem, the notion of deformation module allows to incorporate prior information from the data into the set of considered deformations and the associated metric. I will present this generic framework as well as the Python library IMODAL, which allows to perform registration using such structured deformations. More specifically, I will focus on a recent implicit formulation where the prior can be expressed as a property that the generated vector field should satisfy. This imposed property can be of different categories that can be adapted to many use cases, such as constraining a growth pattern or imposing divergence-free fields.

Erlend Grong
Title: Score matching and sub-Riemannian bridges
Abstract: We discuss how to simulate bridge processes by conditioning a stochastic process on a manifold whose generator is a hypo-elliptic operator. This operator is, up to a drift-term, the sub-Laplacian of a bracket-generating sub-Riemannian structure, meaning in particular that it has positive smooth density everywhere. The logarithmic gradient of this density is called the score, and we show that it is needed to describe the generator of the bridge process. We therefore discuss several methods for how we can estimate the score using a neural network, with examples.
The results are from a joint work with Stefan Sommer (Copenhagen) and Karen Habermann (Warwick).

Karen Habermann
Title: Long-time existence of Brownian motion on configurations of two landmarks
Abstract: In computational anatomy and, more generally, shape analysis, the Large Deformation Diffeomorphic Metric Mapping framework models shape variations as diffeomorphic deformations. An important shape space within this framework is the space consisting of shapes characterised by $n \geq 2$ distinct landmark points in $\mathbb{R}^d$. In diffeomorphic landmark matching, two landmark configurations are compared by solving an optimization problem which minimizes a suitable energy functional associated with flows of compactly supported diffeomorphisms transforming one landmark configuration into the other one. The landmark manifold Q of n distinct landmark points in $\mathbb{R}^d$ can be endowed with a Riemannian metric g such that the above optimization problem is equivalent to the geodesic boundary value problem for g on Q. Despite its importance for modeling stochastic shape evolutions, no general result concerning long-time existence of Brownian motion on the Riemannian manifold (Q, g) is known. I will present joint work with Philipp Harms and Stefan Sommer on first progress in this direction which provides a full characterization of long-time existence of Brownian motion for configurations of exactly two landmarks, governed by a radial kernel.

Emmanuel Hartmann
Title: Elastic shape analysis of surfaces with second-order Sobolev metrics
Abstract: We present a set of numerical methods for Riemannian shape analysis of 3D surfaces within the setting of invariant (elastic) second-order Sobolev metrics. More specifically, we address the computation of geodesics and geodesic distances between unregistered surfaces represented as 3D meshes with potentially varying sampling or mesh structures. Building on this, we present tools for the statistical shape analysis of sets of surfaces, including methods for estimating Karcher means and performing tangent PCA on shape populations, and for computing parallel transport along paths of surfaces. The numerical framework is open access available at `https://github.com/emmanuel-hartman/H2_SurfaceMatch`.

Irene Kaltenmark
Title: I. IA vs. humans : what shape space for the mind ? II. Partial matchings of curves and surfaces.
Abstract: Part 1. A short historical and philosophical excursion will allow me to introduce the concept of individuation developed by the philosopher Bernard Stiegler.
Part 2. The matching of analogous shapes is a central problem in computational anatomy. However, inter-individual variability, pathological anomalies or acquisition methods sometimes challenge the assumption of global homology between shapes. In this talk, I will present an asymmetric data attachment term characterizing the inclusion of one shape in another. This term is based on projection on the nearest neighbor with respect to the metrics of varifold spaces. Varifolds are representations of geometric objects, including curves and surfaces. Their specificity is to take into account the tangent spaces of these objects and to be robust to the choice of parametrization. This new data attachment term extends the scope of application of the pre-existing methods of matching by large diffeomorphic deformations (LDDMM). The partial registration is indeed induced by a diffeomorphic deformation of the source shape. The anatomical (topological) characteristics of this shape are thus preserved. This is a joint work with Pierre-Louis Antonsanti and Joan Glaunès.

Boris Khesin
Title: Fluids, diffeomorphisms, and shapes
Abstract: We discuss ramifications of Arnold's group-theoretic approach to ideal hydrodynamics as the geodesic flow for a right-invariant metric on the group of volume-preserving diffeomorphisms. It turns out that many

equations of mathematical physics, such as the motion of vortex sheets or fluids with moving boundary, have Lie groupoid, rather than Lie group, symmetries. We present their geodesic setting, which also allows one to describe multiphase fluids and Brenier's generalized flows. This also leads to a natural nonlocal $H^{-1/2}$-type "vorticity metricön the shape space of hypersurfaces, as well as to many open problems related to its properties. This is a joint work with Anton Izosimov.

Eric Klassen
Title: Elastic Metrics on Spaces of Euclidean Curves: Theory and Algorithms
Abstract: This talk is concerned with shape analysis of curves in Euclidean space. A one-parameter family of first-order Sobolev metrics on the shape space of immersed Euclidean curves was introduced in 2007 by Mio et al. Intuitively, the parameter gives the relative penalty on bending as opposed to stretching while deforming curves. We refer to this family of metrics as " elastic metrics". The elastic metrics are defined first on the space of parameterized curves; they are then shown to be parameterization invariant so that they descend to metrics on the shape space. In order to apply these metrics to data analysis, it is necessary to compute geodesics, first in the space of parameterized curves and then (by optimally registering two curves) in the shape space. For one member of this family (in the case of planar curves), Younes et al., in 2008, introduced a simplifying transformation that locally flattened the space of parameterized curves and thereby made it easy to compute geodesics. In 2010, Srivastava et al. introduced a similar simplifying transformation for a different member of the family of elastic metrics. This metric has come to be called the SRVF (square root velocity function) metric. It extends to all absolutely continuous curves in R^n for all $n > 0$. This transformation completely flattens the space of parameterized curves, turning it into a Hilbert space, in which the geodesics are straight lines and, hence, trivial to compute. In order to compute geodesics in shape space, the optimal registration of two curves (with respect to the SRVF) was usually approximated using dynamic programming. In 2015, Lahiri et al. developed an algorithm that yielded the precise optimal registration between PL curves. This is useful, since absolutely continuous curves can be approximated by PL curves.
In this talk, we show that all of the algorithms described above for the SRVF metric can be extended to every member of the family of elastic metrics. To be precise, every elastic metric can be extended to the space of all absolutely continuous curves in R^n, geodesics between parameterized curves can be computed quickly and precisely, optimal registration can be approximated by dynamic programming, and the precise algorithm of Lahiri et al. can be extended to all elastic metrics. When analyzing data consisting of curves, this provides the freedom to choose whichever elastic metric is the most useful for the given data, instead of restricting oneself to the SRVF.

Stephan Huckeman
Title: A Lower Bound for Estimating Fréchet Means
Abstract: Fréchet means are generalizations of the Euclidean expected value and are hence among the most popular nonparametric statistics for non-Euclidean data. In fact the umbrella of generalized Fréchet means nicely encompasses various other non-Euclidean statistics, for instance principal components geodesic, principle nested spheres, principle flows, barycentric subspaces and entire flags composed of these. They can suffer, however, from unprecedented non-Euclidean behavior, like smeariness and stickiness, which may manifest even for considerable high sample sizes. Here we focus on the effect of smeariness, namely how closeness of distributions with unique Fréchet means near distributions with nonunique means affects the estimation of the former. It turns out that, independent of sample size, it is not possible to uniformly estimate Fréchet means below a precision determined by the diameter of the set of nonunique Fréchet means nearby". Illustrating the relevance of our lower bound, examples of extrinsic, intrinsic, Procrustes, diffusion and Wasserstein means showcase either deteriorating constants or slow convergence rates of empirical Fréchet means for samples near the regime of nonunique means. This is joint work with Shayan Hundrieser and Benjamin Eltzer

Huiling Le
Title: Stein's method on stratified spaces?
Abstract: Using stochastic analysis, Stein's method has recently been successfully generalised to Riemannian manifolds. Moving further forward, we aim to develop the method to stratified spaces. In this talk, we discuss progress made and highlight some remaining issues.

Alice Le Brigant

Title: The L^p-Fisher-Rao metric and α-connections.

Abstract: Information geometry is a differential geometric framework to study spaces of probability distributions. Central tools of this framework are the Fisher-Rao metric, a Riemannian metric induced by the Fisher information on parametric statistical models, and the family of dual α-connections. Both the Fisher-Rao metric and the α-connections have non-parametric counterparts, which we discuss in this talk. We introduce the L^p-Fisher-Rao metrics, a family of Finsler metrics that generalize the Fisher-Rao metric, and show that their geodesics coincide with that of the alpha-connection, for $p = 2/(1-\alpha)$, on the space of smooth densities. This result no longer holds on the space of probability densities. This gives a new variational interpretation of α-geodesics as being energy minimizing curves.

Facundo Mémoli

Title: The exact determination of Gromov-type distances between spheres.

Abstract: Distances such as the Gromov-Hausdorff distance and its Optimal Transport variants (Gromov-Wasserstein distance and relatives) are nowadays often invoked in applications related to data classification. Interestingly, the precise value of these distances on pairs of canonical shapes is known only in very limited cases. In this talk, I will describe lower bounds for the Gromov-Hausdorff distance between spheres (endowed with their geodesic distances) which we prove to be tight in some cases via the construction of optimal correspondences. These lower bounds arise from a certain version of the Borsuk-Ulam theorem for discontinuous functions. Time permitting, I will cover recent work on the precise determination of the Gromov-Wassertein distance between spheres as well.

Peter Michor

Title: Symplectic Structures on the space of space curves

Abstract: For $c \in \mathrm{Imm}(S^1, \mathbb{R}^3)$ the 2-form

$$\Omega_c^{MW}(h,k) = \int_{S^1} \det(D_s c, h, k) ds,$$

where $ds = |c'(\theta)| d\theta$ and $D_s = \frac{1}{|c'(\theta)|}\partial_\theta$, induces the Marsden-Weinstein symplectic structure[1] on the shape space $\mathrm{Imm}(S^1, \mathbb{R}^3)/\operatorname{Diff}(S^1)$, corresponding to a Kähler structure. The Hamiltonian flow for the length functional is the binormal flow. In this talk, I will present other natural symplectic structures related to this one. Based on collaboration with Martin Bauer and Sadashige Ishida.

Ezra Miller

Title: What is a Gaussian on a singular space?

Abstract: In the classical central limit theorem (CLT), distributions of sample means are asymptotically Gaussian. On manifolds, the same is true for Fréchet means of finite samples, where now the limiting Gaussian distributions naturally live in tangent spaces, which are linear. What happens when the sample space is singular? Limiting distributions still live in tangent spaces, but now those spaces are singular cones. What should be meant by a Gaussian, for singular CLT purposes? How should it be characterized? Should it relate to linear Gaussians? And how should it behave under duality? These are the goals in joint work with Jonathan Mattingly and Do Tran.

Hans Munthe-Kaas

Title: Connection algebras: between algebra, geometry and computations.

Abstract: Butcher's B-series have been a central tool in numerical analysis for over 50 years. More recently they have been generalised to Lie-Butcher series and related series for analysing flows on Lie groups, homogeneous manifolds, symmetric spaces and other geometries. Such series are becoming increasingly important in many application areas. Their algebraic properties are intimately related to the geometry of the domain, through the algebras of the connection in the particular geometry. The term "Connection algebras" cover the special cases of pre-Lie algebras (B-series) for Euclidean geometry, post-Lie algebras for Lie groups, Lie-admissible triple

[1]Marsden, J., and Weinstein, A. Coadjoint orbits, vortices, and Clebsch variables for incompressible fluids. Physica D: Nonlinear Phenomena 7, 1 (1983), 305-323.

algebras for symmetric spaces, as well as the case of general (non-invariant connections), relating to the work of A.V. Gavrilov.

Tom Needham
Title: Geometry and Topology of Spaces of Structured Matrices
Abstract: A finite unit norm tight frame (FUNTF) is a spanning set of unit vectors in a finite-dimensional Hilbert space such that the spectrum of singular values of an associated operator is constant. In signal processing applications, it is desirable to use FUNTFs to encode signals, as such representations are proven to be optimally robust to noise. This naturally gives rise to questions about the geometry and topology of the space of FUNTFs. For example, the conjecture that every space of FUNTFs is connected was open for 15 years, and slight variants of this problem still remain open. I will discuss recent work with Clayton Shonkwiler, where we answer several questions about random matrix theory and optimization in spaces of structured matrices, using tools from symplectic geometry and geometric invariant theory.

Victor Panaretos
Title: Geometrical Statistics in the Bures-Wasserstein Space
Abstract: Covariance operators are fundamental in functional data analysis, providing the canonical means to analyse functional variation via the celebrated Karhunen–Loève expansion. These operators may themselves be subject to variation, for instance in contexts where multiple functional populations are to be compared. Statistical techniques to analyse such variation are intimately linked with the choice of metric on covariance operators, and the intrinsic infinite-dimensionality and of these operators. I will describe how the geometry and tools of optimal transportation can be leveraged to construct natural and effective statistical summaries and inference tools for covariance operators, taking full advantage of the nature of their ambient space. Based on joint work with Valentina Masarotto (Leiden), Leonardo Santoro (EPFL) and Yoav Zemel (EPFL).

Luís F. Pereira
Title: Geomstats: a Python package for Riemannian Geometry and Geometric Statistics
Abstract: Geomstats is an open-source Python package for computations, and statistics on nonlinear manifolds. We provide object-oriented and extensively unit-tested implementations. Manifolds can be equipped with Riemannian metrics with associated exponential and logarithmic maps, geodesics, and parallel transport. Some manifolds can also be endowed with additional mathematical structures, such as Lie group, or fiber bundle structures. Statistics and learning algorithms provide methods for estimation, clustering, and dimensionality reduction on manifolds. All associated operations are supported in different backends, namely NumPy, Autograd, and PyTorch. In this talk, we briefly introduce the main concepts in Riemannian geometry and discuss the package design. We show that Geomstats provides reliable building blocks to both foster research in differential geometry and statistics and democratize the use of Riemannian geometry in statistics and machine learning. The source code is freely available under the MIT license at `https://github.com/geomstats/geomstats`.

Stephen C. Preston
Title: Isometric immersions and the waving of flags
Abstract: A physical flag can be modeled geometrically as an isometric immersion of a rectangle into space, with one edge fixed along the flagpole. Its motion, in the absence of gravity and wind, can be modeled as a geodesic in the space of all isometric immersions, where the Riemannian metric is inherited from the kinetic energy on the much larger space of all immersions. In this talk I will show how to derive the geodesic equation, which turns out to be a highly nonlinear, nonlocal coupled system of two wave equations in one space variable, with tension determined by solving an ODE system.
This is joint work with Martin Bauer and Jakob Moeller-Andersen.

Josua Sassen
Title: Low-dimensional Product Submanifolds of the Space of Discrete Shells
Abstract: We introduce the construction of a low-dimensional nonlinear space capturing the variability of a non-rigid shape from a data set of example poses and efficient ways to parametrize it. The core of the approach is a Sparse Principal Geodesic Analysis (SPGA) on the Riemannian manifold of discrete shells. On the one hand, the SPGA is invariant to rigid body motions of the poses and supports large deformations. On the other hand,

directly parametrizing the constructed submanifolds with the (time-discrete) Riemannian exponential map is computationally demanding. Hence, we exploit a particular product structure of the submanifolds, namely that they are smoothly approximable by a direct sum of low-dimensional manifolds. To this end, we show how this structure can be used for simple grid-based and neural network-based approaches, where we separately learn approximations for low-dimensional factors and a subsequent combination. Finally, we demonstrate the effectiveness of our proposed approach with a series of numerical experiments. This is joint work with Klaus Hildebrandt (Delft), Martin Rumpf (Bonn), and Benedikt Wirth (Münster).

Karl-Theodor Sturm
Title: Metric Spaces and Synthetic Curvature Bounds”
Abstract: The talk offers a panoramic tour of metric geometry with selected highlights on synthetic curvature bounds. For metric spaces with nonpositive synthetic sectional curvature, we present the Law of Large Numbers and its recent application to the geometric mean of positive self-adjoint operators. We address the (open) question of whether the space of n-point spaces embeds isometrically into the space of spaces, both of which are metric spaces with nonnegative synthetic sectional curvature. Furthermore, we provide a brief survey on metric measure spaces with synthetic lower bounds on the Ricci curvature, including a fundamental compactness results for subsets of such spaces, and plenty of geometric and functional-analytic estimates on such spaces. Of particular interest are the transformation rules for the synthetic Ricci bounds under conformal changes and time changes. The latter will be exploited for the convexification of non-convex subsets of a metric space.

Anuj Srivastava
Title: On Statistical Inferences Involving Shape Data
Abstract: In this talk, I will cover problems of statistical modeling or inferences involving the shapes of some functional objects. These problems of interest include: (1) shape-constrained density or curve estimation, (2) shape regression models, (3) statistical models for shape sequences, (4) statistical modeling of shape graphs, and (5) learning pose manifolds of 3D objects.

Barbara Tumpach
Title: Totally geodesic submanifolds in the manifold SPD of symmetric positive-definite real matrices
Abstract: We present necessary and sufficient conditions for a submanifold exp(E) of the manifold of symmetric positive-definite nxn real matrices SPD(n) to be totally geodesic for the affine Riemannian metric. A non-linear projection on a totally geodesic submanifold exp(E) is defined. Fiber bundle decompositions of SDP(n) follow, as will as corresponding decompositions of the general linear group GL(n). If time permits, extensions of this work to other homogeneous spaces will be addressed. This is joint work with. G. Larotonda.

François-Xavier Vialard
Title: Synthetic nonnegative cross-curvature lifts to the Wasserstein space
Abstract: In this talk, we insist on the concept of nonnegative cross-curvature and its synthetic definition for a general cost on a product space. Then, by using a formal argument we show why one can expect that such a property should be also true for the Wasserstein space. Then, we give examples of cost satisfying this synthetic nonnegative cross-curvature, in particular a new one with the Bures-Wasserstein case. We extend the result to the case of unbalanced optimal transport and show some potential applications.

Kathrin Welker
Title: On the optimization of piecewise-smooth shapes
Abstract: Shape optimization is concerned with identifying shapes (or subsets of $\mathbb{R}$) behaving in an optimal way with respect to a given physical system. It has been an active field of research for the past decades and is used for example in engineering. Many relevant problems in the area of shape optimization involve a constraint in the form of a partial differential equation (PDE). Theory and algorithms in shape optimization can be based on techniques from differential geometry, e.g., a Riemannian manifold structure can be used to define the distances of two shapes. Thus, shape spaces are of particular interest in shape optimization.
In this talk, we apply the differential-geometric structure of Riemannian shape spaces to the theory of classical PDE constrained shape optimization problems. We propose a space containing shapes in $\mathbb{R}^2$ that can be identified with a Riemannian product manifold but at the same time admits piecewise-smooth curves as elements. We

present algorithms to solve PDE constrained (multi-)shape optimization problems and give numerical results of these algorithms.
Joint work with: L. Pryymak (TU Bergakademie Freiberg, Germany) and T. Suchan (Helmut-Schmidt-University, Hamburg, Germany)

Laurent Younes
Title: A scale space framework for diffeomorphic shape analysis.
Abstract: We study a model, inspired by recent works of Miller, Trouvé and Tward, that develops shape comparisons in a continuous Riemannian scale space of diffeomorphisms. We provide basic inclusion properties of this scale space, geodesic equations, and propose, in particular, computational approaches to compute the reproducing kernel intervening in this equation. Preliminary experiments will be provided. This is joint work with Oscar Liu (JHU).

Benedikt Wirth
Title: Fourier discretizations of LDDMM and their convergence
Abstract: We will reconsider bandlimited LDDMM discretizations and discuss their behavior and convergence.

Posters

Shreya Arya
Title: Diffusion on Subanalytic Sets

Benjamin Beaudett
Title: Characterizing Pose Image Manifolds Using Geometry-Preserving GANs and Elasticae

Jonathan Cerqueira
Title: Discretized Sobolev metrics on curves

Théo Dumont
Title: Existence of Monge maps for the Gromov-Wasserstein problem

Emmanuel Hartman
Title: Geometric Deep Learning for Unregistered Shape Data

Florine Hartwig and Sascha Beutler
Title: Discrete geodesic calculus in the space of Sobolev curves

Sadashige Ishida
Title: A symplectic structure on implicit curves, and a symplectic version of optimal transport

Erik Jansson
Title: Shape analysis on matrix groups with applications to Cryo-EM microscopy

Lars Lammers
Title: Stickiness: a Blessing or a Curse?

David Loiseaux
Title: Towards Multiparameter Persistence for Machine Learning

Levin Maier
Title: On the interaction of Hamiltonian dynamics, infinite dimensional geometry and nonlinear PDEs

Elodie Maignant and Anna Calissano
Title: Barycentric subspace analysis of a set of graphs

Mao Nishino
Title: Path constrained unbalanced optimal transport

Guillaume Olikier
Title: Projected gradient descent accumulates at Bouligand stationary points

Morten Akhøj Pedersen
Title: Swallowtail shape dynamics

Lidiya Pryymak
Title: Riemannian geometry in shape optimization

Guillaume Sérieys
Title: Metamorphoses of manifold-valued images

Tom Szwagier
Title: Principal component analysis with flag manifolds

Yann Thanwerdas
Title: Invariant metrics on covariance and correlation matrices