NSA at CIRM

Marseille, June 3–7, 2024

Schedule and Abstracts

Monday, June 3

08.50 - 09.00	*Welcome & Opening*
09.00 - 09.40	**M. Hitrik:** *Magic angles and classically forbidden regions for twisted bilayer graphene*
09.45 - 10.10	**N. Dencker:** *The generic instability of differential operators*
10.15 - 10.40	**A. Luger:** *Quasi-Herglotz functions and non-selfadjoint operators*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**J.-C. Cuenin:** *Spectral inequalities for Schrödinger operators with complex potentials*
12.00 - 12.25	**N. M. Schiavone:** *Uniform resolvent estimates, smoothing effects and spectral stability for the Heisenberg sublaplacian*
12.30 - 14.30	*Lunch*
14.30 - 15.10	**C. Tretter:** *Challenges for non-selfadjoint spectral problems in analysis and computation*
15.15 - 15.40	**F. Štampach:** *A Borg-Marchenko type theorem for Schrödinger operators with complex potentials*
15.45 - 16.15	*Coffee break*
16.15 - 18.00	**Posters** **A. Balmaseda:** *A quantum particle in a box with moving walls: dynamics and controllability* **A. Chaigneau:** *Numerical insights on the generalized Steklov problem on planar domains* **C. Drysdale:** *Taming non-Hermitian Eigenvalue Problems: Verified spectra of the imaginary cubic oscillator* **L. Heriban:** *Non-self-adjoint relativistic point interactions* **D. Kramár:** *Spectral stability of quantum systems on the half-line* **J. Kříž:** *Bound states in soft quantum waveguides and layers* **O. Kubů:** *First-order superintegrability with complex magnetic fields* **R. Kvasničková:** *Absence of embedded boundstates in semi-Dirac semi-metals* **V. Laliena:** *Scattering of spin waves by one dimensional solitons* **E. J. Leguizamon Quinche:** *Limit point and limit circle case criteria for Sturm-Liouville problems with complex potentials on a ray* **A. Maichine:** *tba* **S. Petpradittha:** *Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials* **M. Tušek:** *Non-local relativistic δ-shell interactions* **N. Weber:** *Weakly coupled bound states of Schrödinger operators with complex potentials in one and two dimensions* **M. Winklmeier:** *Spectral inclusions for perturbations of normal operators and applications* **M. Zaccaron:** *Geometry deformations of the Maxwell operator*
19.30	*Dinner*

Tuesday, June 4

09.00 - 09.40	**G. Metafune:** *L^p estimates for degenerate problems in the half-space*
09.45 - 10.10	**L. Baldelli:** *Curved quantum nonlinear waveguides*
10.15 - 10.40	**Y. Pinchover:** *The space of Hardy-weights for quasilinear equations: Maz'ya-type characterization and sufficient conditions for existence of minimizers*
10.45 - 11.15	*Coffee break*
11.15 - 11.40	**A. Hussein:** *Non-self-adjoint scattering on graphs*
11.45 - 12.10	**A. Comech:** *Virtual levels of linear operators in Banach spaces. Application to Schrödinger operators in 2D*
12.30 - 14.30	*Lunch*
14.30 - 14.55	**C. Cazacu:** *Hardy inequalities for magnetic p-Laplacians*
15.00 - 15.25	**B. Gerhat:** *Powers of discrete Laplacians and Hardy-type inequalities*
15.30 - 16.00	*Coffee break*
16.00 - 17.00	**Open Problems**
19.30	*Dinner*

Wednesday, June 5

09.00 - 09.40	**K. Cherednichenko:** *Functional model for generalised resolvents and its application to time-dispersive media*
09.45 - 10.10	**P. Kleinhenz:** *Stability and optimal backward uniqueness of second order equations with unbounded damping*
10.15 - 10.40	**A. Arnal:** *Energy decay of solutions of the wave equation with unbounded damping at infinity*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**J. Faupin:** *Spectral decomposition of some non-self-adjoint operators*
12.00 - 12.25	**T. ter Elst:** *Dynamic boundary conditions for divergence form operators with Hölder coefficients*
12.30 - 14.30	*Lunch*
14.30 - 19.30	*Free afternoon*
19.30	*Dinner*

Thursday, June 6

09.00 - 09.40	**A.-S. Bonnet-Ben Dhia:** *Spectrum and pseudospectrum of reflectionless frequencies in waveguides*
09.45 - 10.10	**M. Ruiz:** *Spectral analysis of the material-independent modes for the Helmholtz equation*
10.15 - 10.40	**F. Ferraresso:** *Essential spectrum of dissipative Maxwell systems*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**C. Lancien:** *Spectrum of random quantum channels*
12.00 - 12.25	**A. Sukhtayev:** *Spectral decomposition and decay to grossly determined solutions for a simplified BGK model*
12.30 - 14.30	*Lunch*
14.30 - 14.55	**T. D. Nguyen:** *Dislocated Dirac operator*
15.00 - 15.25	**S. Tumanov:** *On Molchanov's criterion for the compactness of the resolvent for the non-selfadjoint Sturm–Liouville operator*
15.30 - 16.00	*Coffee break*
16.00 - 16.25	**C. Trunk:** *Essential spectra of Sturm–Liouville operators and their indefinite counterpart*
16.30 - 16.55	**I. Nakić:** *Spectrum of operators on equilateral metric graphs*
19.30	*Festive dinner*

Friday, June 7

09.00 - 09.40	**J. Behrndt:** *The generalized Birman-Schwinger principle*
09.45 - 10.10	**R. Yang:** *Pluri-harmonic solutions to the Maxwell's equations and Yang-Mills equations*
10.15 - 10.40	**J. M. Perez Pardo:** *On the stability of non-autonomous Schrödinger equations and applications to Quantum Control*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**M. Kachanovska:** *Limiting absorption principle for a degenerate problem modelling a hybrid resonance in plasma*
12.00 - 12.25	**L. Grubišić:** *Model order reduction of the resolvent using subspace embeddings with random Khatri–Rao products*
12.30 - 14.30	*Lunch*

Abstracts

Invited talks

1. **Jussi Behrndt**

   Title: *The generalized Birman-Schwinger principle*

   Abstract: In this talk we discuss a generalized Birman-Schwinger principle in the non-self-adjoint context. In particular, we provide a detailed discussion of geometric and algebraic multiplicities of eigenvalues of the basic operator of interest (e.g., a Schrödinger operator) and the associated Birman-Schwinger operator, and additionally offer a careful study of the associated Jordan chains of generalized eigenvectors of both operators.

   This talk is based on a joint paper with Tom ter Elst (Auckland, New Zealand) and Fritz Gesztesy (Baylor, US).

2. **Anne-Sophie Bonnet-Ben Dhia**

   Title: *Spectrum and pseudospectrum of reflectionless frequencies in waveguides*

   Abstract: This work is done in collaboration with Lucas Chesnel and Vincent Pagneux.

   Let us consider an infinite uniform acoustic waveguide with a compactly supported perturbation (obstacle, variation of the sound speed, deformation of the boundary...). A reflectionless frequency is such that there exists an incident wave (combination of propagating modes) which goes through the perturbation without producing any backward reflection. We proved in [1] that these reflectionless frequencies correspond to real eigenvalues of a non-selfadjoint operator. This operator is PT-symmetric if the perturbation has some spatial symmetry. The main idea is to use a complex scaling (or equivalently Perfectly Matched Layers) with conjugated parameters on each side of the perturbation. The present work concerns the approximation of the spectrum, when these PMLs are truncated.

   We focus for simplicity on the 1D case similarly to [2]. The spectrum is then included in an infinite cone, whose boundary is the essential spectrum (characterized by Weyl sequences). A strange phenomenon occurs for the interior of the cone: in presence of a perturbation, it contains only discrete eigenvalues but it is entirely contained in the spectrum in the unperturbed case. On the other hand, with truncated PMLs, the Fredholm theory ensures the discreteness of the spectrum for both the unperturbed and the perturbed cases.

   This leads us to analyze the evolution of the spectrum with truncated PMLs, when the length L of the PMLs tends to infinity while the size ε of the perturbation tends to 0. The results are completely different according to the order of the two limits. Taking the limit when $L \to +\infty$ of the spectrum for $\varepsilon = 0$ gives the positive real axis. Permuting the order of these two limits, we get the union of the boundary of the cone introduced above and of a sequence of real values tending to infinity. This can be explained by studying how the pseudospectrum of the unperturbed operator depends on L. This is very similar to the phenomenon described in [3] concerning the finite section approximation of Toeplitz operators.

   References

   [1] A.-S. Bonnet-Ben Dhia, L. Chesnel and V. Pagneux, Trapped modes and reflectionless modes as eigen- functions of the same spectral problem, Proc. R. Soc. A., 2018.

   [2] H. Hernandez-Coronado, D. Krejcirik and P. Siegl, Perfect transmission scattering as a PT - symmetric spectral problem, Physics Letters A, 375(22), 2011.

   [3] L. N. Trefethen and M. Embree, Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators, Princeton University Press, 2005.

3. **Kirill Cherednichenko**

   Title: *Functional model for generalised resolvents and its application to time-dispersive media*

   Abstract: Motivated by recent results concerning the asymptotic behaviour of differential operators with highly contrasting coefficients, which have involved effective descriptions involving generalised resolvents, in this talk I will present the functional model for a typical example of the latter. This provides a spectral representation for the generalised resolvent, which can be utilised for further analysis, in particular the construction of the scattering operator in related wave propagation setups. This is joint work with Yulia Ershova and Sergey Naboko.

4. **Jean-Claude Cuenin**

   Title: *Spectral inequalities for Schrödinger operators with complex potentials*

   Abstract: I will first review the state of the art concerning Schrödinger operators on Euclidean space with complex L^p potentials. Then I will discuss some recent results in the case when the potential has some structure or is random. Finally, I will present new sharp estimates on compact manifolds.

5. **Jérémy Faupin**

   Title: *Spectral decomposition of some non-self-adjoint operators*

   Abstract: We will consider in this talk non-self-adjoint operators in Hilbert spaces given as relatively compact perturbations of a self-adjoint operator. Typical examples are Schrödinger operators with bounded, complex potentials vanishing at infinity. We will describe abstract conditions insuring that the Hilbert space admits a direct sum decomposition into H-invariant subspaces, generalizing the well-known spectral decomposition of self-adjoint operators in terms of their spectral measures. A central role in the talk will be played by spectral singularities, an abstract notion corresponding to that of real resonances for Schrödinger operators. We will also present a useful regularized functional calculus for non-self-adjoint operators.

   This is joint work with Nicolas Frantz.

6. **Michael Hitrik**

   Title: *Magic angles and classically forbidden regions for twisted bilayer graphene*

   Abstract: Magic angles are a topic of current interest in condensed matter physics and refer to a remarkable theoretical (Bistritzer–MacDonald, 2011) and experimental (Jarillo-Herrero et al, 2018) discovery: two sheets of graphene twisted by a certain (magic) angle display unusual electronic properties, such as superconductivity. In this talk, we shall discuss a simple periodic Hamiltonian describing the chiral limit of twisted bilayer graphene (Tarnopolsky-Kruchkov-Vishwanath, 2019), whose spectral properties are thought to determine which angles are magical. We show that the corresponding eigenfunctions decay exponentially in suitable geometrically determined regions as the angle of twisting decreases, which can be viewed as a form of semiclassical analytic hypoellipticity. This is joint work with Maciej Zworski.

7. **Maryna Kachanovska**

   Title: *Limiting absorption principle for a degenerate problem modelling a hybrid resonance in plasma*

   Abstract: We address a problem that arises in the modeling of resonances in cold plasma. Mathematically, this phenomenon can be described by a boundary-value problem for a second-order partial differential equation (PDE). The difficulty arises from the fact that the principal part of the differential operator contains a smooth coefficient which changes its sign inside the domain. While the underlying operator admits a self-adjoint extension (see [Baouendi, Goulaouic 1969]), it seems that the corresponding functional setting does not accurately represent the physical situation of hybrid resonance. This stems from the fact that the domain of such an operator does not contain singular functions which are at the origin of plasma heating, see [Nicolopoulos, Campos-Pinto, Despres, Ciarlet Jr. 2020].

   Following the idea of this latter work, we investigate the limiting absorption principle for this problem, for the case when the absorption is added into the principal symbol. One of the primary challenges lies in the definition of the domain of the limiting operator. This latter operator, in turn, appears to be non-self-adjoint. We illustrate our findings with some numerical experiments. This talk is based on a joint work with Patrick Ciarlet, Jr. and Etienne Peillon (ENSTA, France).

8. **Cécilia Lancien**

   Title: *Spectrum of random quantum channels*

Abstract: The main question that we will investigate in this talk is: what does the spectrum of a quantum channel typically looks like? We will see that a wide class of random quantum channels generically exhibit a large spectral gap between their first and second largest eigenvalues. This is in close analogy with what is observed classically, i.e. for the spectral gap of transition matrices associated to random graphs. In both the classical and quantum settings, results of this kind are interesting because they provide examples of so-called expanders, i.e. dynamics that are converging fast to equilibrium despite their low connectivity. We will also present implications in terms of typical decay of correlations in 1D many-body quantum systems. If time allows, we will say a few words about ongoing investigations of the full spectral distribution of random quantum channels. This talk will be based on: arXiv:1906.11682 (with D. Perez-Garcia), arXiv:2302.07772 (with P. Youssef) and arXiv:2311.12368 (with P. Oliveira Santos and P. Youssef).

9. **Giorgio Metafune**

Title: *L^p estimates for degenerate problems in the half-space*

Abstract: In this talk I will present recent results on elliptic and parabolic problems governed by the singular elliptic operators

$$L = y^{\alpha_1}\Delta_x + y^{\alpha_2}(D_{yy} + \frac{c}{y}D_y - \frac{b}{y^2}), \qquad \alpha_1, \alpha_2 \in \mathbb{R}$$

in the half-space $\mathbb{R}^{N+1}_+ = \{(x,y) : x \in \mathbb{R}^N, y > 0\}$.

More general operators containing mixed derivatives or variable coefficients are allowed, but I will focus on the model case to explain methods and results. The aim is not only to prove generation of a semigroup but also to describe the domain as a Sobolev space. We work in L^p_m which is the L^p space with respect to the measure $y^m\, dx\, dy$ for a range of admissible m. The operator is symmetric only for $m = c - \alpha_2$ but our study covers all cases, in particular that of the Lebesgue measure (corresponding to $m = 0$).

This talk is based on a series of papers in collaboration with L. Negro and C. Spina, University of Salento and uses methods of semigroups and vector-valued harmonic analysis. Similar results, with different methods have been obtained by H. Dong, Brown University.

10. **Christiane Tretter**

Title: *Challenges for non-selfadjoint spectral problems in analysis and computation*

Abstract: Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods. As an application, Maxwell's equations with conductivity will be considered.

(joint work with S. Boegli, M. Marletta, and F. Ferraresso)

Contributed talks

1. **Antonio Arnal**

Title: *Energy decay of solutions of the wave equation with unbounded damping at infinity*

Abstract: We study the long-time behaviour of the solutions of the wave equation with damping a unbounded at infinity and defined in an open (possibly unbounded) set $\Omega \subset \mathbb{R}^n$. Using multiplier methods, Ikehata and Takeda (2017) proved polynomial decay rates for unbounded continuous damping when $n \geq 3$. Our results significantly extend their findings by assuming minimal regularity conditions on a and placing no restriction on the space dimension n. We apply different methods that rely upon recent advances in the theory of operator semigroups on Hilbert spaces and the spectral analysis of the resolvent of the generator G of the equation.

This presentation is based on joint work with B. Gerhat, J. Royer and P. Siegl.

2. **Laura Baldelli**

Title: *Curved quantum nonlinear waveguides*

Abstract: In the last decade, there has been an increasing interest in the p-Laplacian, which plays an important role in geometry and partial differential equations. The p-Laplacian is a natural generalization of the Laplacian. Although the Laplacian has been much studied, not much is known about the nonlinear case $p > 1$.

Motivated by these facts, the purpose of the present paper is to review recent developments in the spectral theory of a specific class of quantum waveguides modeled by the Dirichlet Laplacian, i.e. $p = 2$, in unbounded tubes of uniform cross-section rotating w.r.t. the Tang frame along infinite curves in Euclidean spaces of arbitrary dimension. We discuss how the spectrum depends upon three geometric deformations: straightness, asymptotic straightness, and bending.

Precisely, if the reference curve is straight or asymptotic straight, the essential spectrum is preserved. While dealing with bent tubes, such geometry produces a spectrum below the first eigenvalue. All the results confirm the literature for the Laplacian operator. The results are obtained via a very delicate analysis since the nonlinearity given by the p-Laplacian operator adds different types of difficulties with respect to the linear situation.

These results are contained in a work written jointly with D. Krejčiřík.

3. **Cristian Cazacu**

Title: *Hardy inequalities for magnetic p-Laplacians*

Abstract: We establish improved Hardy inequalities for the magnetic p-Laplacian due to adding nontrivial magnetic fields. We also prove that for Aharonov-Bohm magnetic fields the sharp constant in the Hardy inequality becomes strictly larger than in the case of a magnetic-free p-Laplacian. We also post some remarks with open problems.

This is based on a joint work with D. Krejčiřík, N. Lam and A. Laptev.

4. **Andrew Comech**

Title: *Virtual levels of linear operators in Banach spaces. Application to Schrodinger operators in 2D*

Abstract: We consider the relation of the limiting absorption principle and virtual levels of operators in Banach spaces and apply the developed theory to compute the LAP estimates for the resolvent of a (non-selfadjoint) Schrödinger operator in 2D near threshold in the absence of virtual levels. We consider both continuous and discrete cases.

The talk is based on a joint research with Nabile Boussaid, Université Franche-Comté (Besançon). A review of some of the results is available here:

[1] Nabile Boussaid, Andrew Comech: Limiting absorption principle and virtual levels of operators in Banach spaces, Ann. Math. Quebec 46 (2022), 161–180

`https://link.springer.com/article/10.1007/s40316-021-00181-7`

`https://arxiv.org/abs/2109.07108`

5. **Nils Dencker**

Title: *The generic instability of differential operators*

Abstract: It came as a surprise when Hans Lewy in 1957 presented a nonvanishing smooth complex vector field on $\mathbb{R}^3$ that is not solvable anywhere. After all, the classical Cauchy-Kowalevskaya Theorem shows that any analytic PDE is solvable in the analytic category. Interestingly, the Lewy vector field is essentially the tangential Cauchy-Riemann operator on the boundary of a strictly pseudoconvex domain in $\mathbb{C}^2$. Hörmander showed in 1960 that almost all linear PDE are not solvable by proving that the nongeneric vanishing of the Poisson bracket is necessary for solvability. This bracket condition on PDE has many consequences for the kernel, the range and the spectral stability of linear PDE and the stability of the quasilinear Cauchy problem.

A fifty year development lead to the proof of the Nirenberg-Treves conjecture: that principal type differential operators are solvable if and only if condition (Ψ) on the principal symbol is satisfied. This is a condition on the sign changes of the imaginary part of the principal symbol on the bicharacteristics of the real part. For nonprincipal type differential operators, a condition similar to (Ψ) on the refined principal symbol (including the subprincipal symbol) is necessary for solvability. This condition is also sufficient for solvability of nonprincipal type differential operators having real principal symbols vanishing of second order.

6. **Tom ter Elst**

Title: *Dynamic boundary conditions for divergence form operators with Hölder coefficients*

We consider a second-order elliptic operator in divergence form with merely Hölder continuous coefficients on a bounded domain Ω with $C^{1,\kappa}$-boundary Γ and Wentzell boundary conditions of the type $\mathrm{Tr}\ Au = \beta\, \partial_\nu u + \alpha\ \mathrm{Tr}\ u$ on Γ. For strictly positive bounded measurable β we prove maximal regularity on $L_p(\Omega) \times L_p(\Gamma)$ for all $p \in (1, \infty)$, the generation of a holomorphic C_0-semigroup with angle $\frac{\pi}{2}$ for all $p \in [1, \infty)$ and also the generation of a holomorphic C_0-semigroup with angle $\frac{\pi}{2}$ on $C(\overline{\Omega})$.

In the proof we use the Dirichlet-to-Neumann operator $\mathcal{N}$ on the boundary Γ and show that $-\beta\mathcal{N}$ generates a C_0-semigroup on $L_2(\Gamma)$ which extends consistently to a C_0-semigroup on $L_p(\Gamma)$ and the latter semigroup is holomorphic with angle $\frac{\pi}{2}$ for all $p \in [1, \infty)$. Moreover, the semigroup extends to a holomorphic C_0-semigroup on $C(\Gamma)$ with angle $\frac{\pi}{2}$ and it has Poisson kernel bounds on the right half-plane.

This is joint work with T. Binz (Darmstadt).

7. **Francesco Ferraresso**

Title: *Essential spectrum of dissipative Maxwell systems*

Abstract: Electromagnetic waves propagating through conductive media tend to lose part of their energy; from a mathematical point of view, conductivity makes the underlying Maxwell operator non-selfadjoint. I will discuss some recent results about the essential spectrum of the dissipative Maxwell system in unbounded domains of $\mathbb{R}^3$.

Assuming that the tensors ε, μ tend to real multiples of the identity matrix at infinity, and that σ vanishes at infinity, the essential spectrum is characterised as the union of two parts, one related to the behaviour of divergence-free vector fields 'at infinity'; the other to the loss of ellipticity of a suitably defined $\operatorname{div} p(\omega)\nabla$ operator.

In domains with finitely many cylindrical straight ends, the essential spectrum further decomposes according to the geometry. This is proved assuming that ε, μ are asymptotically constant along each cylinder, with possibly different limits at infinity (which is the easiest case of anisotropy at infinity).

Time permitting, I will show that additional essential spectrum can be generated by imposing $\delta^{\nu\times}$-interactions along a surface (a.k.a. a Faraday layer).

Based on joint work with S. Bögli, M. Marletta, and C. Tretter.

8. **Borbala Gerhat**

Title: *Powers of discrete Laplacians and Hardy-type inequalities*

Abstract: We study the existence of non-trivial lower bounds for positive powers of the discrete Dirichlet Laplacian on the half line. Unlike in the continuous setting where both $-\Delta$ and $(-\Delta)^2$ admit a Hardy-type inequality, their discrete analogues exhibit a different behaviour. While the discrete Laplacian is subcritical,

its square is critical and the threshold where the criticality of $(-\Delta)^\alpha$ first appears turns out to be $\alpha = 3/2$. We provide corresponding (non-optimal) Hardy-type inequalities in the subcritical regime. Moreover, for the critical exponent $\alpha = 2$, we employ a remainder factorisation strategy to derive a discrete Rellich inequality on a suitable subspace (with a weight improving upon the classical Rellich weight).

Based on joint work with D. Krejčiřík and F. Štampach.

9. **Luka Grubišić**

Title: *Model order reduction of the resolvent using subspace embeddings with random Khatri–Rao products*

Abstract: We present a contour integration type eigensolver which is adapted to the Kronecker (tensor) product structure of an operator. We can think of such operators as those whose eigenvectors can be represented as short sums of functions with separable variables (loosely said this is the Khatri-Rao product structure of an eigenvector). An example of a contour integration method is the integral algorithm of WJ Beyn. A key part of a contour integration eigensolver is the evaluation of the resolvent for many integration nodes and applying it on a basis of a test space. We are going to refer to this part of the algorithm as sampling of the resolvent . By this we mean sampling both in the spectral parameter as well as in the environment space. We speed up the resolvent sampling by reformulating the problem as a computation of the solution of the multi term operator Sylvester equation with the leading dimension being the square root of the original dimension. Solutions of such equations can be provably represented by short sums of functions with separated variables. Further, we build a reduced order model of the resolvent by projecting the generating operator onto a random subspace spanned by a basis of vectors which have a structure of a Khatri–Rao product of random vectors. This structure of the space on which the resolvent will be projected is best suitable to utilize the tensor structure of the operator and the main contribution is to prove that basis vectors with this product random structure are still sufficiently well spread in the whole environment space so that we can provide competitive bounds on a probability of failure of the algorithm to compute approximations of spectral elements. Typical prototypes for the application of this algorithm are Schrödinger operators with potentials which are representable as short sums of functions with separable variables.

This is a joint work with Daniel Kressner, Zvonimir Bujanovic and Hei Yin Lam.

10. **Amru Hussein**

Title: *Non-self-adjoint scattering on graphs*

Abstract: Scattering theory for self-adjoint operators is a well-developed tool of mathematical physics. Here we consider a model case for non-self-adjoint operators: Laplacians on star graphs – that is on finitely many copies of the half line – subject to non-self-adjoint couplings at the origin. It is discussed for which couplings existence and completness of the wave operators holds, in which cases this can be expected and in which not. The wave operators provide if existent and complete – as in the self-adjoint case – a representation of the absolutely continuous part of the operator.

11. **Perry Kleinhenz**

Title: *Stability and optimal backward uniqueness of second order equations with unbounded damping*

Abstract: Energy decay rates for damped second order systems can be obtained, via semigroup theory, by proving resolvent estimates for the associated stationary operator, which is non-self-adjoint. In this talk, I will discuss how, for unbounded damping, abstract control estimates can be used to produce the desired resolvent estimates. As applications new decay rates are obtained for the damped wave equation with singular damping, damped linearized water waves and Euler-Bernoulli plates. I will also discuss finite-time extinction of solutions and a sharp condition on the degree of unboundedness that rules this out. This is joint work with Ruoyu P.T. Wang.

12. **Annemarie Luger**

Title: *Quasi-Herglotz functions and non-selfadjoint operators*

Abstract: Self-adjoint operators (more precisely, resolvents of self-adjoint operators) are closely related to Herglotz-Nevanlinna functions. A (possibly complex) linear combination of such functions is called *Quasi-Herglotz function* and hence these are naturally related to a class of non-self-adjoint operators.

Despite the simple definition Quasi-Herglotz functions have interesting properties and appear at surprisingly many places. We give an overview and relate to recent work of Christian Emmel about non-self-adjoint extensions of a symmetric operator.

This talk is based on joint work with Mitja Nedic as well as results by Christian Emmel.

13. **Ivica Nakić**

Title: *Spectrum of operators on equilateral metric graphs*

Abstract: It is well known that the spectrum of the Laplacian on an equilateral metric graph is essentially determined by the spectrum of the corresponding Laplacian matrix. Similar result of this type is also known for transport equations on metric graphs and in some other specific cases. In this talk we consider a class of operators on equilateral metric graphs with the property that operators acting on the edges coincide. Using ideas from control theory, I will show that the spectrum of these operators is also directly connected with the spectrum of the corresponding Laplacian matrix.

This is a joint work with Marjeta Kramar Fijavž.

14. **Tho Nguyen Duc**

Title: *Dislocated Dirac operator*

We are going to discuss about spectral properties of the one-dimensional Dirac operator perturbed by a discontinuous potential, for $m \geq 0$,

$$\mathscr{L}_m = \begin{pmatrix} m & -\partial_x \\ \partial_x & -m \end{pmatrix} + \begin{pmatrix} i\,\mathrm{sgn}(x) & 0 \\ 0 & i\,\mathrm{sgn}(x) \end{pmatrix}.$$

We will show that the pseudospectrum is trivial when $m = 0$ and non-trivial when $m > 0$, despite the operator being non-normal in both scenarios. We also discuss about the location of eigenvalues of the operator perturbed by some complex potentials.

The talk is based on the joint work with Lyonell Boulton and David Krejčiřík.

15. **Juan Manuel Perez Pardo**

Title: *On the stability of non-autonomous Schrödinger equations and applications to Quantum Control*

Abstract: I will present a stability result of the non-autonomous Schrödinger equation for Hamiltonians with constant form domain. The sharp estimates obtained improve previous results in the literature. As an application I will show how one can prove some approximate controllability results for infinite dimensional quantum control problems.

This is joint work with A. Balmaseda and D. Lonigro.

16. **Yehuda Pinchover**

Title: *The space of Hardy-weights for quasilinear equations: Maz'ya-type characterization and sufficient conditions for existence of minimizers*

Abstract: Let $p \in (1, \infty)$ and $\Omega \subset \mathbb{R}^N$ be a domain. Let $A := (a_{ij}) \in L^\infty_{\mathrm{loc}}(\Omega; \mathbb{R}^{N\times N})$ be a symmetric and locally uniformly positive definite matrix. Set

$$|\xi|_A^2 := \sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j, \quad \xi \in \mathbb{R}^N,$$

and let V be a given potential in a certain local Morrey space. We assume that the energy functional

$$Q_{p,A,V}(\phi) := \int_\Omega [|\nabla\phi|_A^p + V|\phi|^p]\mathrm{d}x$$

is nonnegative in $W^{1,p}(\Omega) \cap C_c(\Omega)$.

We introduce a generalized notion of $Q_{p,A,V}$-capacity and characterize the space of all Hardy-weights for the functional $Q_{p,A,V}$, extending Maz'ya's well known characterization of the space of Hardy-weights for the p-Laplacian. In addition, we provide various sufficient conditions on the potential V and the Hardy-weight

g such that the best constant of the corresponding variational problem is attained in an appropriate Beppo Levi space.

This talk is based on a joint work with Ujjal Das.

17. **Matias Ruiz**

Title: *Spectral analysis of the material-independent modes for the Helmholtz equation*

Abstract: In this talk I will discuss the spectral analysis of a family of nonself-adjoint spectral problems defined by a homogeneous Helmholtz equation when using the permittivity/permeability as the eigenvalue. The study of such problems is motivated by the modal decomposition approach in computational electromagnetism, wherein the electromagnetic field scattered by a nanocavity subject to radiation losses (i.e. in an open system), can be described as an infinite sum using as basis functions the eigenfunctions of a spectral problem defined by the unforced Maxwell's equations. Owing to radiation conditions, this linear spectral problem is non-self-adjoint, which is at the source of numerical and theoretical difficulties. Furthermore, the problem is non-standard in that its eigenvalues both diverge and accumulate at multiple finite points. I will present a rigorous spectral analysis of these modes and show their completeness in $H^1(D)$, where D is the domain occupied by the resonant nanocavity. I will also show that they define a Riesz basis in some particular geometric configurations.

18. **Nico Michele Schiavone**

Title: *Uniform resolvent estimates, smoothing effects and spectral stability for the Heisenberg sublaplacian*

We present uniform resolvent estimates for the sublaplacian and its fractional power on the Heisenberg group. The proof is based on Hardy's type inequalities due to Garofalo-Lanconelli and D'Ambrosio, and the method of weakly conjugate operator due to Hoshiro and Boutet de Monvel–Măntoiu. As applications, we obtain some global-in-time smoothing effects for the Schrödinger equation, and spectral stability for the Schrödinger operator perturbed by complex-valued potentials on the Heisenberg group.

19. **František Štampach**

Title: *A Borg-Marchenko type theorem for Schrödinger operators with complex potentials*

The classical Borg-Marchenko uniqueness theorem for a self-adjoint Schrödinger operator H on a half-line states that the spectral measure of H determines uniquely the potential of H. If the potential is allowed to take complex values, H is not self-adjoint and it is no longer clear what the spectral measure of H should be. Instead of the spectral measure, we propose a definition of the so-called spectral data of H consisting of two objects - a measure (the spectral measure of $|H|$) and a phase function. The definition is inspired, in a sense, by the polar decomposition formula for H and is motivated by our previous similar work on complex Jacobi matrices. We discuss properties of the spectral data of H and, in particular, we show that they determine uniquely the potential. The talk is based on a recent joint work with Alexander Pushnitski.

20. **Alim Sukhtayev**

Title: *Spectral decomposition and decay to grossly determined solutions for a simplified BGK model*

Abstract: For a simplified 1D BGK model we show that H^1 solutions decay exponentially in L^2 to a subclass of the class of grossly determined solutions as defined by Truesdell and Muncaster in the context of Boltzmann's equation. In the process, we determine the spectrum and generalized eigenfunctions of the associated non-selfadjoint linearized operator and derive the associated generalized Fourier transform and Parseval's identity. Notably, our analysis makes use of rigged space techniques originating from quantum mechanics, as adapted by Ljance and others to the nonselfadjoint case.

This is joint work with Kevin Zumbrun.

21. **Carsten Trunk**

Title: *Essential spectra of Sturm–Liouville operators and their indefinite counterpart*

Abstract: We show new perturbation results and invariance of essential spectra in terms of the real, locally integrable coefficients p, q, r for general Sturm-Liouville differential expressions of the form

$$\frac{1}{r}\left(-\frac{\mathrm{d}}{\mathrm{d}x}p\frac{\mathrm{d}}{\mathrm{d}x}+q\right)$$

with a.e. positive functions p and r. If one allows sign changes of the weight function r, then the situation changes dramatically but one keeps control over the essential spectrum and some properties of the isolated point spectrum can still be proved.

This talk is based on a joint works with J. Behrndt (Graz), G. Teschl (Vienna), and P. Schmitz (Ilmenau).

22. **Sergey Tumanov**

Title: *On Molchanov's criterion for the compactness of the resolvent for the non-selfadjoint Sturm–Liouville operator*

Abstract: Consider complex-valued $q \in L_{1,loc}(\mathbb{R}_+)$ and the differential expression

$$l(y) = -y'' + qy \tag{1}$$

and lineals

$$D = \{y \in L_2(\mathbb{R}_+) \,|\, y, y' \in AC_{loc}(\mathbb{R}_+),\ l(y) \in L_2(\mathbb{R}_+)\},$$
$$D_0 = \{y \in D \,|\, y(0) = y'(0) = 0,\ \exists x_0 > 0\ \forall x \geq x_0\ y(x) = 0\},$$
$$D_U = \{y \in D \,|\, U(y) = 0\},$$

where U is some form of boundary conditions at $x = 0$:

$$U(y) = Ay(0) + By'(0), \quad A, B \in \mathbb{C},\ |A| + |B| > 0.$$

Let us define differential operators $L_0 \subset L_U$ in $L_2(\mathbb{R}_+)$ on the corresponding domains $D_0 \subset D_U$ by differential expression (1). Refer to L_0 as *minimal operator.*

We say that q satisfies the *Molchanov condition* if for any $a > 0$:

$$\lim_{x\to+\infty} \int_x^{x+a} |q(\xi)|\, d\xi = +\infty,$$

Theorem 1. *For the minimal operator L_0 in $L_2(\mathbb{R}_+)$ to have an extension with a compact resolvent, it is necessary that q satisfies the Molchanov condition.*

Definition 1. *We say that q satisfies $\mathbb{R}_-$–condition if for all sufficiently large $x > x_0 \geq 0$ the values of $q(x)$ lie in the sector $\alpha \leq \arg(q(x) - q_0) \leq \beta$ for some $-\pi < \alpha \leq \beta < \pi$ and $q_0 \in \mathbb{C}$.*

Definition 2. *We call the potential q sectorial if q satisfies $\mathbb{R}_-$–condition with $\beta - \alpha < \pi$.*

Theorem 2. *Let the potential q be sectorial. Then the operator L_U has a compact resolvent if and only if q satisfies Molchanov's condition.*

The condition $\beta - \alpha < \pi$ cannot be weakened:

Theorem 3. *There exist potentials q taking imaginary values $q(x) \in i\mathbb{R}$ for all $x \in \mathbb{R}_+$ such that $|q| \to +\infty$ as $x \to +\infty$, but the minimal operator L_0 has no extensions with compact resolvents.*

In this case $\beta - \alpha = \pi$, and obviously such potentials satisfy the Molchanov condition. Moreover L_U with Dirichlet boundary conditions $U(y) = y(0)$ has a bounded non-compact resolvent, at least in the left half-plane.

The following theorem gives a sufficient condition for the compactness of the resolvent when q satisfies $\mathbb{R}_-$–condition with $\beta - \alpha > \pi$. In this case, the sectoriality property of the operators themselves is lost; in particular, the numerical range of L_U may cover the entire complex plane.

Theorem 4. *Let for some $x_0 > 0$ for all $x \geq x_0 > 0$ $|q(x)| \geq 1$ and additionally:*

- *$q \in AC_{loc}[x_0, +\infty)$,*
- *for some $0 < \varkappa < \pi$*

$$-\pi + \varkappa < \arg q(x) < \pi - \varkappa, \quad x \geq x_0,$$

- *for some* $0 < \delta < 1$

$$\left| \frac{q'(x)}{q^{3/2}(x)} \right| < 4\delta \sin \frac{\varkappa}{2}, \quad x \geq x_0.$$

Then for the compactness of the resolvent L_U *it is sufficient that for any* $a > 0$*:*

$$\lim_{x \to +\infty} \int_x^{x+a} |q(x)|^{1/2}\, dx = +\infty.$$

23. **Rongwei Yang**

Title: *Pluri-harmonic solutions to the Maxwell's equations and Yang-Mills equations*

Abstract: This talk provides a view of Maxwell's equations and Yang-Mills equations from the perspective of complex analysis, operator theory, and C^*-algebras. Based on pluri-harmonic differential forms, we will present a new class of instanton (self-dual or anti-self-dual) solutions to the equations. It is a joint work with Marius Beceanu and Sachin Munshi.

Posters

1. **Aitor Balmaseda**

Title: *A quantum particle in a box with moving walls: dynamics and controllability*

Abstract: On this talk, the dynamics of a quantum particle confined in a 1-dimensional box with moving walls is explored. We will first consider the well-posedness and the stability of the Schrödinger equation, and discuss the possibility of controlling the state of the particle by appropriately moving the walls of the box. We will show global approximate controllability results by moving either one or both of the box's walls.

2. **Adrien Chaigneau**

Title: *Numerical insights on the generalized Steklov problem on planar domains*

Abstract: We present numerical results on the generalized Steklov problem (and the related Dirichlet-to-Neumann operator) for the modified Helmholtz equation and focus on the relation between its spectrum and the geometric structure of the domain. We address two distinct aspects: (i) the asymptotic behavior of eigenvalues for polygonal domains; and (ii) the localization and exponential decay of Steklov eigenfunctions away from the boundary for smooth shapes and in the presence of corners. We discuss applications of the obtained results in the theory of diffusion-controlled reactions and formulate conjectures with relevance in spectral geometry. Finally, we also discuss some open questions in the non-self-adjoint setting.

This is a joint work with Denis S. Grebenkov.

3. **Jaroslav Dittrich**

Title: *Dirac particle in an one-dimensional box with moving wall*

Abstract: Dirac equation in an one dimensional bounded box with one moving wall is considered. Instantaneous Hamiltonians are the standard self-adjoint Dirac Hamiltonians but the time derivative in the Dirac equation is modified to guarantee the probability conservation. Another interpretation is to keep standard time derivative but modify the Hamilotonians to the non-selfadjoint ones. The resulting equation is transformed to the Dirac equation in a static box with time-dependent mass. Time-dependences of the average kinetic energy and quantum force are analyzed, the average kinetic energy remains bounded for the box length bounded from below, i.e., unlimited Fermi acceleration does not occur. The talk is based on the paper J. Dittrich, S. Rakhmanov, D. Matrasulov, Phys. Lett. A 503 (2024), 129408.

4. **Catherine Drysdale**

Title: *Taming non-Hermitian Eigenvalue Problems: Verified spectra of the imaginary cubic oscillator*

Abstract: The imaginary cubic oscillator is the fons et origo of PT-symmetric quantum mechanics and features a completely real spectrum with discrete eigenvalues. The eigenfunctions and eigenvalues have generated considerable interest; however, their closed forms remain elusive. To date, approximations of these eigenpairs are limited to the first few and are accurate only to a few digits. Computing these eigenpairs accurately remains an open problem and presents challenges due to the sheer non-normality of the operator. Through a combination of asymptotic analysis, semigroup theory, and rigorous spectral computations with error control, we efficiently compute verified eigenpairs. For example, the 100th eigenvalue is 627.6947122484365113526737029011536 (correct to 30 digits) and has a condition number of 3.2978×10^{77}. Our approach opens the door to verified spectra of more general non-Hermitian problems.

5. **Lukáš Heriban**

Title: *Non-self-adjoint relativistic point interactions*

Abstract: The one-dimensional Dirac operator perturbed by the singular potential given by $A|\delta\rangle\langle\delta|$, where A can be any 2×2 complex matrix and δ stands for the delta distribution, will be presented as a closed operator with a certain transmission condition at zero. We will find regular approximations for such not necessarily self-adjoint operator and we will perform the non-relativistic limit for both the toy model and its approximation. Moreover, we will study its spectral properties.

6. **David Kramár**

Title: *Spectral stability of quantum systems on the half-line*

Abstract: It is well-known that under certain smallness assumptions, Robin Laplacians on the half-line posses spectral stability. The smallness can be quantified in the form of an integral sufficient condition. In our recent work with David Krejčiřík, we study half-line Dirac operators on the half-line subjected to the generalized MIT boundary conditions. We establish sufficient condition for preservation of their spectra and seek connection to the Robin Laplacians. Eventually, we offer a new possible interpretation of the Robin Laplacians as non-relativistic regimes of our models.

7. **Jan Kříž**

Title: *Bound states in soft quantum waveguides and layers*

Abstract: We consider Schrödinger operator with confining potentials depending on the distance to the planar curve or spatial surface. For special geometries we localise the essential spectrum and establish the existence of discrete eigenvalues. This is a joint work with David Krejčiřík and Sylwia Kondej.

8. **Ondřej Kubů**

Title: *First-order superintegrability with complex magnetic fields*

Abstract: In the mathematical community, there has been a surging interest into complex magnetic fields following the experimental observation of Lee-Yang zeros [Peng et al., Phys. Rev. Lett. 114, 010601 (2015)]. In the physical community studying superintegrable systems, which are exactly solvable, these developments have so far been ignored as unphysical. In this talk, we present our classification of first-order superintegrable systems of cylindrical type. We find one new systems and three previously known are extended by choosing complex coupling constants. Rigorous mathematical analysis of these systems is challenging due to the non-Hermitian setting and lost gauge invariance and is posed as an open challenge.

9. **Romana Kvasničková**

Title: *Absence of embedded boundstates in semi-Dirac semi-metals*

Abstract: Semi-Dirac semi-metals are recently constructed nanostructures. The behaviour of these materials depends on the direction, which is caused by their exotic electronic band structure. Their behaviour is a combination of the properties of conventional zero-gap semiconductors and graphene. These nanostructures have been extensively studied in physics literature, however, the rigorous theory is missing.

In the pioneering work [1], Krejčiřík and Antunes analyse the spectral stability properties under small perturbations, notably sufficient and necessary conditions of existence of discrete eigenvalues.

The objective of this poster is to formulate the sufficient conditions of absence of the embedded eigenvalues in the essential spectrum. The first approach is in the spirit of the virial theorem with the background in quantum mechanics. The other approach is the method of multipliers with the background in theory of partial differential equations.

[1] D. Krejčiřík and P. R. S. Antunes (2020) Bound states in semi-Dirac semi-metals, *Phys. Lett. A* **386**, 126991.

10. **Victor Laliena**

Title: *Scattering of spin waves by one dimensional solitons*

Abstract: A spin wave is a perturbation of the magnetization of a magnet which propagates as a wave. It has two degrees of freedom that define a square integrable function of position and time, $\xi : \Omega \times \mathbb{R} \to \mathbb{R}^2$, where $\Omega \subseteq \mathbb{R}^3$ represents the space occupied by the magnet. The dynamics obeys a linear hamiltonian PDE of the form $\partial_t \xi = JK\xi$, where J is a time independent linear bounded operator with $J^* = -J$ and $J^2 = -I$, and K is a time independent linear differential operator of second order, selfadjoint in an appropiate domain of $L^2(\Omega, \mathbb{R}^2)$. If the magnet hosts a one dimensional magnetic soliton, then we have [1]

$$J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad K = \begin{pmatrix} \mathcal{D}_1 & 0 \\ 0 & \mathcal{D}_2 \end{pmatrix}, \quad \mathcal{D} = JK = \begin{pmatrix} 0 & -\mathcal{D}_2 \\ \mathcal{D}_1 & 0 \end{pmatrix},$$

where $\mathcal{D}_1$ and $\mathcal{D}_2$ are Schrödinger operators with potentials that vary only along a single direction, say z, and which are even functions of z if the center of the soliton is at $z = 0$. To study the scattering of spin waves by this soliton, the case of an infinite magnet, $\Omega = \mathbb{R}^3$, is considered. The Fourier transform in the transverse coordinates x and y leads to a one dimensional problem in which only the variable z remains. Using results from [2], it is shown here that $\tilde{\xi}$ can be expanded in terms of the bounded solutions of $(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2 - \lambda I)u = 0$ and $(\tilde{\mathcal{D}}_2\tilde{\mathcal{D}}_1 - \lambda I)u = 0$, provided that $\tilde{\xi}$ is restricted to an appropriate domain (the tilde denotes the corresponding quantities after the partial Fourier transform). Using this expansion, the scattering problem can be solved in a similar way to the scattering of quantum particles by a one dimensional potential. In particular, the reflection and tranmission amplitudes, ρ and σ, are related to phase shifts in the same way as in quantum scattering:

$$\rho = \sin(\delta_0 - \delta_1)\mathrm{e}^{\mathrm{i}(\delta_0+\delta_1+\pi/2)}, \quad \sigma = \cos(\delta_0 - \delta_1)\mathrm{e}^{\mathrm{i}(\delta_0+\delta_1)},$$

where δ_0 and δ_1 are the phase shifts corresponding to even and odd waves, respectively. The phase shifts can be obtained approximately by numerical means. The numerical computations suggest that other results from Schrödinger operators, as Levinson's theorem, hold also in this case.

This is a joint work with Javier Campo.

[1] V. Laliena, J. Campo, Adv. Electron. Matter., 2100782 (2021).

[2] R. R. D. Kemp, Canadian Journal of Mathematics, 12, 641 (1960).

11. **Edison Jair Leguizamon Quinche**

Title: *Limit point and limit circle case criteria for Sturm-Liouville problems with complex potentials on a ray*

Abstract: Motivated by the work of C.M. Bender and S. Boettcher on PT symmetric operators we study the Sturm-Liouville equation $y''(z) + q(z)y(z) = \lambda y(z)$, where z is in a ray on $\mathbb{C}$ and the potential q may be complex valued. It was shown by A.R. Sims in 1957 and later by B.M. Brown et al. in 1999 that in this situation a Weyl-type classification at infinity holds. However, the classical limit point case splits into the so-called limit point I and limit point II case. We found useful criteria for integrability properties of solutions which allow us to distinguish between limit point I, limit point II and limit circle case. We illustrate these criteria with several examples.

This is a joint work with F. Leben, C. Trunk and M. Winklmeier.

12. **Abdallah Maichine**

Title: *tba*

Abstract: tba

13. **Sukrid Petpradittha**

Title: *Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials*

Abstract: The purpose of this talk is to investigate a conjecture which was stated by Demuth, Hansmann, and Katriel in 2013. We study a possible generalisation of Lieb-Thirring inequalities for eigenvalues of non-selfadjoint Schrödinger operators, with complex-valued potentials, acting on $L^2(\mathbb{R}^d)$ where $d \geq 2$. In particular, we will find the asymptotic behaviour for the discrete spectra of Schrödinger operators with a one parameter family of rapidly decaying complex-valued potentials, and present a disproof of this conjecture.

Based on a joint work with Sabine Bögli (Durham) and František Štampach (Prague).

14. **Matěj Tušek**

Title: *Non-local relativistic δ-shell interactions*

Abstract: New realizations of the Dirac operator in dimension two and three are introduced. It is shown that they may be associated with the formal expression $\mathcal{D}_0 + |F\delta_\Sigma\rangle\langle G\delta_\Sigma|$, where $\mathcal{D}_0$ is the free Dirac operator, F and G are matrix valued coefficients, and δ_Σ stands for the single layer distribution supported on a hypersurface Σ, and that they can be understood as limits of the Dirac operators with scaled non-local potentials. Furthermore, their spectral properties are analysed.

15. **Nicolas Weber**

Title: *Weakly coupled bound states of Schrödinger operators with complex potentials in one and two dimensions*

Abstract: We consider the (not necessarily self-adjoint) Schrödinger operator of the form $H_\beta = -\Delta - V_\beta$ in $L^2(\mathbb{R}^d)$, $d \in \{1, 2\}$, where $V_\beta : \mathbb{R}^d \to \mathbb{C}$ is a complex-valued potential, depending on some parameter $\beta \in \mathbb{C}$, such that $V_\beta \to 0$ as $\beta \to 0$ in an appropriate sense. We derive sufficient conditions, depending on the decay behaviour of V_β and the spatial dimension, for the existence of an eigenvalue $\lambda_\beta \in \sigma_p(H_\beta) \setminus [0, \infty)$, as $\beta \to 0$, and also obtain an asymptotic expansion for this eigenvalue. Finally, we provide conditions on V_β, under which H_β never has an eigenvalue in $\mathbb{C} \setminus [0, \infty)$, as $\beta \to 0$.

16. **Monika Winklmeier**

Title: *Spectral inclusions for perturbations of normal operators and applications*

Abstract: We consider a normal operator T on a Hilbert space H. Under various assumptions on the spectrum of T, we give bounds for the spectrum of $T + A$ where A is T-bounded with relative bound less than 1 but we do not assume that A is symmetric or normal. We give estimates in terms of hyperbolas for the localization of the spectrum of $T + A$ in the following cases: the imaginary part of the spectrum of T is bounded; it is contained in a bisector; or it has infinitely many gaps. Moreover, we give a stability result for the essential spectrum of $T + A$.

This is joint work with Javier Moreno.

17. **Michele Zaccaron**

Title: *Geometry deformations of the Maxwell operator*

Abstract: In this talk we consider the self-adjoint Maxwell operator from $L^2(\Omega)^3 \times L^2(\Omega)^3$ to itself

$$\mathcal{M} = i \begin{pmatrix} 0 & \varepsilon^{-1}\,\mathrm{curl} \\ -\mu^{-1}\,\mathrm{curl} & 0 \end{pmatrix}$$

on a bounded domain Ω of $\mathbb{R}^3$. The operator $\mathcal{M}$ is defined on

$$\mathrm{Dom}(\mathcal{M}) = \{u = (\mathbf{E}, \mathbf{H}) \in L^2(\Omega)^3 \times L^2(\Omega)^3 : \mathcal{M}u \in L^2(\Omega)^3 \times L^2(\Omega)^3, \mathbf{E} \times \nu|_{\partial\Omega} = 0\},$$

where ν is the outer unit normal to Ω. In general we assume the parameters of electric permittivity ε and magnetic permeability μ to be symmetric matrix valued functions in $L^\infty(\Omega)^{3\times3}$ satisfying an ellipticity condition. We inspect the dependence of the spectrum of $\mathcal{M}$ on the geometry of Ω. Moreover, we deal with an optimization problem for the dependence of the eigenvalues upon variation of the electric permittivity ε and we prove a corresponding Maximum Principle.