Frontiers in Interacting Particle Systems, Aggregation-Diffusion Equations and Collective Behavior

Frontières dans des systèmes de particules en interaction, équations d'agrégation-diffusion et comportement collectif

24 – 28 June 2024

Centre International de Rencontres Mathématiques, Marseille

As part of the Chaire Jean Morlet: Aggregation-Diffusion and Kinetic Equations, Collective Behavior Models and Applications

Organised in partnership with

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Program

Day	Monday	Tuesday	Wednesday	Thursday	Friday
08:50–09:00	Opening				
09:00–09:45	Y. Yao	H. Murakawa	A. Jüngel	A. Korba	A. Trescases
09:45–10:30	Y. Yao	H. Murakawa	A. Jüngel	A. Korba	A. Korba
10:30–11:00	**Coffee Break**				
11:00–11:45	H. Murakawa	Y. Yao	A. Jüngel	A. Jüngel	A. Korba
11:45–12:30	H. Murakawa	Y. Yao	T. Mignot	A. Jüngel	A. Korba
12:30–12:40	**Lunch**				**Closing**
12:40–14:00					
14:00–14:45	H. Murakawa	Y. Yao	Free Afternoon	M. Tomasevic	
14:45–15:30	M. Merkel	V. Calvez		P.-E. Jabin	
15:30–16:00	**Coffee Break**			**Coffee Break**	
16:00–16:45	Poster session	P.-E. Jabin		P.-E. Jabin	
16:45–17:00		P.-E. Jabin		P.-E. Jabin	
17:00–18:00					

Contents

Talks

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The mean-field limit of non-exchangeable integrate and fire systems

Pierre-Emmanuel Jabin
(Penn State University , USA)

Abstract

We investigate the mean-field limit of large networks of interacting biological neurons. The neurons are represented by the so-called integrate and fire models that follow the membrane potential of each neuron and captures individual spikes. However we do not assume any structure on the graph of interactions but consider instead any connection weights between neurons that obey a generic mean-field scaling. We are able to extend the concept of extended graphons, introduced in Jabin-Poyato-Soler, by introducing a novel notion of discrete observables in the system. This is a joint work with D. Zhou.

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Multispecies populations: interacting particles, cross diffusion, and entropies

Ansgar Juengel
(TU-Wien, Austria)

Abstract

This mini-course provides an introduction to models for multi-species populations. The aim is to understand the interactions between the species from a diffusive viewpoint. We consider moderately interacting stochastic particle systems and derive diffusion-aggregation and cross-diffusion equations in the mean-field limit. The cross-diffusion systems possess an entropy structure, which can be exploited for a global existence analysis. If all eigenvalues of the diffusion matrix have positive real parts, we obtain gradient bounds for the population densities, leading to global weak solutions. If the real parts of some eigenvalues vanish, the system becomes mixed hyperbolic-parabolic, and we obtain global dissipative measure-valued solutions only. Numerical experiments show that the populations may completely segregate in such systems.

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Wasserstein gradient flows and applications to sampling in machine learning

Anna Korba
(ENSAE/ CREST, France)

Abstract

Sampling is a fundamental task in Machine Learning. For instance in Bayesian Machine Learning, one has to sample from the posterior distribution over the parameters of a learning model, whose density is known up to a normalizing constant. In other settings such as generative modelling, one has to sample from a distribution from which some samples are available (e.g. images). The task of sampling can be seen as an optimization problem over the space of probability measures. The mathematical theory providing the tools and concepts for optimization over the space of probability measures is the theory of optimal transport. The topic of this course will be the connection between optimization and sampling, more precisely, how to solve sampling problems using optimization ideas. The goal of the first part of the course will be to present two important concepts from optimal transport: Wasserstein gradient flows and geodesic convexity. We will introduce them by analogy with their euclidean counterpart that is well known in optimization. The goal of the second part will be to show how these concepts, along with standard optimization techniques, enable to design, improve and analyze various sampling algorithms. In particular. we will focus on several interacting particles schemes that achieve state-of-the-art performance in machine learning.

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Mechanisms of cellular unmixing in stem cell aggregates

Matthias Merkel
(Turing Center for Living Systems, Center for Theoretical Physics, Aix-Marseille University)

Abstract

Most animals display one or more body axes (e.g. head-to-tail). In our work, we demonstrate that their formation can be promoted by large-scale tissue flows. We studied aggregates of mouse stem cells, called gastruloids, which are initially spherically symmetric, but later form an axis defined by the polarized patterns of specific proteins. We show that advection of cells with tissue flows contribute substantially to the overall polarization, and that these flows are driven by interface and surface tension differences.

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Mathematics of cell sorting: modelling, analysis and applications

Hideki Murakawa
(Faculty of Advanced Science and Technology, Ryukoku University, Japan)

Abstract

Cell-cell adhesion and cell sorting processes are essential in organ formation during embryonic development and in maintaining multicellular structure. In order to deal with cell-cell adhesion and cell sorting of aggregations of large number of cells with complicated shapes, such as aggregations of neurons in brain, it is convenient to consider cell population dynamics models. Here, we propose a nonlocal advection-diffusion system as a possible mathematical model for such phenomena. Numerical experiments illustrate that the proposed model is able to replicate different types of phenomena observed in cell sorting experiments. Moreover, we give analytical results about approximations to the system. Furthermore, we give applications to medical science.

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Steady states and dynamics of the aggregation-diffusion equation

Yao Yao
(NUS, Singapore)

Abstract

The aggregation-diffusion equation is a nonlocal PDE that arises in the collective motion of cells. Mathematically, it is driven by two competing effects: local repulsion modelled by nonlinear diffusion, and long-range attraction modelled by nonlocal interaction. In this course, I will discuss several qualitative properties of its steady states and dynamical solutions. Using continuous Steiner symmetrization techniques, we show that all steady states are radially symmetric up to a translation. (joint with Carrillo, Hittmeir and Volzone). Once the symmetry is known, we further investigate whether steady states are unique within the radial class, and show that for a given mass, the uniqueness/non-uniqueness of steady states is determined by the power of the degenerate diffusion, with the critical power being $m = 2$. (joint with Delgadino and Yan). I'll also discuss some properties on the long-time behavior of aggregation-diffusion equation with linear diffusion (joint with Carrillo, Gomez-Castro and Zeng), and global-wellposedness if Keller-Segel equation when coupled with an active advection term (joint with Hu and Kiselev).

Posters

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Pattern formation of precursor films

Tharindi Thathsarani Amarathunge Achchige
(University of South Australia)

Abstract

Precursor films formed by ionic liquids have excellent lubrication properties for high-end applications. These precursor films form different types of patterns on different substrates and a stable precursor film depends on the type of ionic liquid and the surface of interaction while the effectiveness of the lubricating layer depends on the pattern they form. This study develops a mathematical model using concepts of systems of interacting particles and thin-film type equations considering potentials relevant to ionic liquids. This approach is based on a study by Falco, C., Baker, R., and Carrillo, J. for pattern formation of biological cell colonies. We expect that the parameter in the thin-film type equation reflects a relationship between the number/ size of the liquid patches and the interaction radius of ionic liquid molecules. Furthermore, pair correlation function is used to identify characteristic length scales of spatial patterns in experimental images of precursor films and these measures will be linked with the model parameters. This new mathematical model will be able to screen ionic liquid substrate pairs that form precursor films of desired patterns for targeted lubrication applications.

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Understanding Microscopic Biological Interactions and their Impact on Macroscopic Struct

Minakshi Ashok
(California Institute of Technology)

Abstract

Cells are crisscrossed by long filaments known as the cytoskeleton, forming exquisite structures such as the football-shaped spindle that moves chromosomes in dividing cells. How do particle-like interactions of filaments and other proteins result in such emergent structure, despite the decay-esque effects of diffusion? This question has motivated the study of simplified experimental systems, where bipedal motor proteins consume energy to exert forces on filaments at the microscopic scale, leading to rich patterns at the mesoscale, such as vortices and asters. The a-priori prediction of mesoscopic pattern formation is difficult due to complex microscopic interactions. We are interested in: (1) obtaining the continuum dynamics of an appropriate set of stochastic differential equations describing particle interactions, (2) using function estimation techniques to recover the functional forms of unknown interaction terms from our experimental data, (3) studying the long-term asymptotics for different initial conditions and model parameter choices. In particular, we aim to make a rigorous connection between microscopic interactions at the filament and motor level and macroscopic behavior at the continuum level. We are working on demonstrating the above techniques on a computational toy model, intending to apply the methodology to biological data in the future. Furthermore, as these systems are of great interest from a non-equilibrium thermodynamics perspective, we are interested in analysing the mechanisms through which the energy injected by motors is dissipated.

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Kinetic description and convergence analysis of genetic algorithms for global optimization

Giacomo Borghi
(Heriot-Watt University, Edinburgh)

Abstract

Genetic Algorithms (GA) are a class of metaheuristic global optimization methods inspired by the process of natural selection among individuals in a population. Despite their widespread use, a comprehensive theoretical analysis of these methods remains challenging due to the complexity of the heuristic mechanisms involved. In this poster we show how to take a first step towards a mathematical understanding of GA by showing how their behavior for a large number of individuals can be approximated through a kinetic model. This allows us to prove the convergence of the algorithm towards a global minimum under mild assumptions on the objective function for a popular choice of selection mechanism.

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Study of a class of triangular starvation driven cross-diffusion systems

Elisabetta Brocchieri
(University of Graz)

Abstract

Cross-diffusion systems are nonlinear parabolic systems, modelling the evolution of densities or concentrations of multicomponent populations in interaction. The poster aims to present the existence, regularity and uniqueness results for a general class of triangular reaction-cross-diffusion systems, coming out of the study of starvation-driven behavior for two species in competition. This study involves the use of an equivalent system appearing in non-divergence form, for which existence can be obtained thanks to a fixed point argument. This is a joint work with L. Desvillettes and H. Dietert.

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Well-posedness and Behavior of the Transformer PDE

Valerie Castin
(ENS PSL)

Abstract

Transformers have revolutionized machine learning by representing each data point as a set of particles, called tokens, which interact as they go through the model via the self-attention mechanism. By taking the infinite depth limit and then the mean-field limit on tokens, we model the Transformer architecture as a PDE on probability measures, and study its well-posedness for several variants of self-attention. This provides an interesting insight on the regularity of these different definitions of self-attention. Further, we study both theoretically and numerically the long-time asymptotics of the PDE when the initial condition is Gaussian, showing the emergence of very different behaviors according to the chosen definition of self-attention.

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Multi-scale modeling of Snail-mediated response to hypoxia in tumor progression

Giulia Chiari
(Politecnico di Torino)

Abstract

The poster focuses on tumor cell migration within the microenvironment. In particular, we investigate the influence of hypoxia with a multi-scale model to analyze the pivotal role of Snail protein expression in the cellular responses. Numerical simulations of the model are performed in biologically relevant scenarios to provide insights into the role of the multiple tactic terms, the impact of Snail expression on cell proliferation, and the emergence of hypoxia-induced migration patterns. Moreover the model's reliability in measuring the impact of Snail transcription on cell migratory potential is shown via quantitative comparison with experimental data.

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Overdamped limit for chemotacting active Brownian particles

Oscar de Wit
(University of Cambridge)

Abstract

We study a link between a Keller-Segel model with singular drift and an active Brownian particle (ABP) model in which the particles align their orientation with the upward gradient of a pheromone concentration field. Taking the interaction strength of the ABP model to infinity leads to a formal overdamped limit of this model towards a singular Keller-Segel model. We show numerical evidence for this overdamped limit with stochastic simulations at the particle level and with simulations for the time-dependent and steady state mean-field limit partial differential equations. We also provide some analytical foundations for the overdamped limit.

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Mean field system for pulse-coupled oscillators: long time asymptotics versus blowup

Xu'an Dou
(Peking Univesity)

Abstract

Pulse-coupled oscillators describe particle systems where the interaction is in a particular pulsatile manner. They have many applications in science and engineering, including fireflies, neurons and wireless networks. Yet their unique singular coupling mechanism renders them challenging to analyze. We introduce a novel reformulation of the mean-field system for pulse-coupled oscillators. It is based on writing a closed equation for the inverse distribution function associated to the probability density of oscillators with a given phase in a suitable time scale. This new framework allows to show a hidden contraction/expansion of certain distances leading to a full clarification of the long-time behavior, existence of steady states, rates of convergence, and finite time blow-up of classical solutions for a large class of monotone phase response functions. In the process, we get insights about the origin of obstructions to global-in-time existence and uniform in time estimates on the firing rate of the oscillators. This is a joint work with José Carrillo, Pierre Roux and Zhennan Zhou.

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Dynamics of growth, collision, and cell division in epithelial monolayers.

Carles Falco
(University of Oxford)

Abstract

Although tissues are typically studied in isolation, such situations rarely occur in biology, as cells, tissues, and organs coexist and interact across various scales, shaping both form and function. In this study, we adopt a quantitative approach that combines recent experimental data, mathematical modelling, and Bayesian parameter inference to describe the dynamics of freely expanding and colliding epithelial monolayers. Two simple and extensively studied continuum models are employed, where cells move either randomly or in response to population pressure gradients. Following appropriate calibration, both models successfully replicate the primary features of individual tissue expansions. However, our findings demonstrate that when tissues are not isolated and interactions become relevant, assuming random cell motion can lead to unrealistic behaviour. In such cases, a model that considers population pressure from different cell populations proves more suitable and facilitates comparison with experimental measurements. Additionally, we investigate the dynamics of cell division within epithelial monolayers and demonstrate how a combination of minimal modelling and Bayesian inference can capture mechanical checkpoints in cell-cycle progression.

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Aggregation-Diffusion with saturation

Alejandro FernandezJimenez
(University of Oxford)

Abstract

On this poster we will focus on the aggregation-diffusion equation with saturation

$$\frac{\partial \rho}{\partial t} = \mathrm{div}\left(\mathrm{m}(\rho)\nabla(U'(\rho) + V)\right).$$

Here, U is convex, $V(x) \geq 0$ is a given potential regular enough, and $\mathrm{m}(\rho)$ is a non-linear mobility such that $\mathrm{m}(0) = \mathrm{m}(\alpha) = 0$, $\mathrm{m} > 0$ in $(0, \alpha)$. We study existence and long-time behaviour of the solutions using a regularised version of the problem, and its gradient flow structure. Finally, we also consider a finite-volumes scheme constructed for this equation by Bailo, Carrillo, and Hu and we connect it with the analysis results.

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Analysis of a Poisson-Nernst-Planck System with Steric Effect

Peter Hirvonen
(TU Wien)

Abstract

This poster presents an analysis of a modified Poisson-Nernst-Planck system with so-called steric effect for an arbitrary number of species, modeling ion transport in a small domain. The additional term appearing in the equations was suggested by Lin and Eisenberg, aiming to include ion sizes, which are not taken into account in the original Poisson-Nernst-Planck equations. Global existence of weak solutions is proved, using the boundedness-by-entropy method. Furthermore, the weak-strong uniqueness property is shown by means of a relative entropy inequality. Finally, topics for further research on this system are outlined.

https://conferences.cirm-math.fr/3050.html

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On collective dynamics over domains with boundaries

Hui Huang
(University of Graz)

Abstract

Systems of interacting particles are extensively utilized to model the collective behaviors of organisms and social aggregations. In this poster, we explore the collective dynamics within domains that include boundaries, which are prevalent in realistic physical settings. For instance, boundaries can manifest as environmental obstacles, such as rivers or the ground. Our research contributions include establishing the mean-field limit for the particle system under a reflecting boundary condition, as well as analyzing the zero-diffusion limit transition from the aggregation-diffusion model to the plain aggregation model. Additionally, this particle system framework can also be applied to develop a consensus-based optimization method for constrained optimization problems.

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Plaque formation in modelling Multiple Sclerosis

Megha Kamath
(Universita de L'Aquila)

Abstract

We study the pattern formation in a chemotaxis system with non-linear degenerate diffusion arising in modelling the Multiple Sclerosis disease. A variant of the disease, called Balo MS, is characterised by a distinct pattern of concentric layers, alternating between areas of plaques and no plaques. Numerical simulations on the model are used to realise the patterns. Further, we study the linearised stability of spatially homogeneous stationary states with dependence on the chemo-attractive parameter.

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Recent Developments in Consensus Based Optimization

Dohyeon Kim
(California Institute of Technology)

Abstract

Consensus-based optimization algorithms are a recent family of particle methods for solving complex non-convex optimization problems. In many application settings, the objective function is not available in closed form. Additionally, derivatives may not be available, or very costly to obtain. Consensus-Based-Optimization (CBO) makes use of the Laplace principle to circumvent the use of gradients and is well suited for black-box objectives. Most of the available analysis for this recent family of algorithms studies the corresponding mean-field descriptions of the distribution of particles. Especially convergence analysis with explicit rates is of interest to assess algorithm performance and has mostly been done on the level of the mean-field PDEs. However, all results currently in the literature connecting the discrete particle system to the mean-field regime are restricted to finite time domains. In this talk, we present recent advances regarding the CBO algorithm and its variants and discuss uniform-in-time mean field limits. We focus on second-order variants of CBO as they have numerical advantages in terms of convergence and provide a conceptual bridge to Particle Swarm Optimization (PSO), one of the most widely used particle-based optimization methods.

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Systems of particles with singular interaction under multiplicative common noise

Josue Knorst
(ENSTA-Paris)

Abstract

We consider a system of particles interacting via a singular kernel (Keller-Segel, for instance) and subject to common noise in addition to individual noises. The common noise is multiplicative, i.e., has the form of $\sigma_t(x)B_t$. The limiting equation for such a system is an SPDE, for which we prove well-posedness via classical arguments (Krylov's L^p-theory for SPDEs). We prove the propagation of chaos by classical compacity arguments.

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Critical thresholds in pressureless Euler–Poisson equations with background states

Dowan Koo
(Yonsei University)

Abstract

We investigate the critical threshold phenomena in a large class of one-dimensional pressureless Euler–Poisson (EP) equations, with non-vanishing background states. We provide a rather definitive answer for the critical thresholds in the case of repulsive EP systems with variable backgrounds. As an application, we analyze the critical thresholds for the damped EP system for cold plasma ion dynamics, where the density of electrons is given by the Maxwell-Boltzmann relation.

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In silico study of heterogeneous tumour-derived organoids response to CAR T-cell therapy

Luciana Luque
(Cancer Research UK Scotland Institute)

Abstract

Chimeric antigen receptor (CAR) T-cell therapy is a promising immunotherapy for treating cancer. This method consists in modifying the patients' T-cells to directly target antigen-presenting cancer cells. One of the barriers to the development of this type of therapies is target antigen heterogeneity. It is thought that intratumour heterogeneity is one of the leading determinants of therapeutic resistance and treatment failure. While understanding antigen heterogeneity is important for effective therapeutics, a good therapy strategy could enhance the therapy efficiency. In this poster I will introduce an agent-based model, that rationalise the outcomes of different CAR T-cells therapies strategies over heterogeneous tumour-derived organoids. Such computational approaches provide a framework to model treatment combinations in different scenarios and to explore the characteristics of successful and unsuccessful treatments.

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Constructive Krein-Rutman result for kinetic Fokker-Planck equations in a domain

Richard Medina Rodriguez
(Universite Paris Dauphine - PSL)

Abstract

In this work we present a Krein - Rutmann theory with construvtive rate of convergence to study the kinetic Fokker Planck equation in a spatial domain and with Maxwell boundary conditions. This theory generalises previous non-constructive results in domains, and follows the recent trend of applications of the De Giorgi - Nash - Moser theory for hypoparabolic equations. Moreover we prove an ultracontractivity property for the semigroup and a Harnack inequality that will allow us to construct the rate of convergence by using the Doblin-Harris theory.

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Deterministic approximations for Wasserstein distances with nonlinear mobilities and corresponding minimising movement schemes

Michele Mencarelli
(Universita de L'Aquila)

Abstract

Motivated by the formal observation that transport equations with nonlinear mobilities can be interpreted as gradient flow evolutions in the space of probability measures with respect to generalised Wasserstein distances, which differ from the classical ones by singular weight functions related to the nonlinear mobilities, we aim to show the rigorous well-posedness of the gradient flow structure in suitable approximate N-dimensional settings and study the limit behaviour as N tends to infinity. At the present stage, we can prove a Gamma-convergence result for the discrete distances towards the continuum generalised Wasserstein one, and a Wasserstein-convergence result for the minimisers of variational problems involving the discrete distances towards the corresponding continuum one. Such variational problems typically appear iterated in time-discrete minimising movement schemes which, in turn, well approximate gradient flow evolutions in the limit as the time-step vanishes. The compactness in the time-step of the minimising movement schemes in the considered discrete settings is a current work in progress. With respect to the present literature on this topic, we consider new discrete distances which are suited to recover the unique entropy solution of the original problem, independently on the regime of the potentials involved in the velocity field.

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Shock positions for regularised reaction-diffusion equations with negative diffusivity

Thomas Miller
(University of South Australia)

Abstract

Aggregation can be modeled as negative diffusivity in reaction diffusion equations. Reaction diffusion equations with negative diffusivity can give rise to shock solutions and may need regularization terms in order to be solved numerically. However, imposing a regularization term can also impose a shock position. A shock position we are interested in is one that conserves the diffusivity across the shock, and we have been investigating a nonlinear regularization that imposes such a shock position.

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Drift-Diffusion for Memristors Coupled to a Network

Tuan Tung Nguyen
(TU Wien)

Abstract

The memristor is a novel semiconductor device equipped with a memory due to the change of its electrical resistance. In this way, it may mimic the behavior of a synapse in the human brain. We analyze a model of memristors that are coupled with an electric network consisting of various electronic devices. While nonlinear drift-diffusion equations describe the motion of charged particles within the memristor, the node potentials in the network follow the Kirchhoff law, i.e. ordinary differential equations and algebraic constraints. The coupling between them results in a system of partial differential-algebraic equations. The existence analysis employs entropy methods crucially.

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Long-time behaviour of a multi dimensional age-dependent branching process with a singular jump kernel: a model for telomere shortening in a continuous state space

Jules Olaye
(Ecole Polytechnique)

Abstract

We investigate the ergodic behaviour of a multidimensional age-dependent branching process with a singular jump kernel, motivated by studying the phenomenon of telomere shortening in cell populations. Our model tracks individuals evolving within a continuous-time framework indexed by a binary tree, characterized by age and a multidimensional trait. Branching events occur with rates dependent on age, where offspring inherit traits from their parent with random increase or decrease in some coordinates, while the most of them are left unchanged. Exponential ergodicity is obtained at the cost of an exponential normalization, despite the fact that we have an unbounded age-dependent birth rate that may depend on the multidimensional trait, and a non-compact transition kernel. These two difficulties are respectively treated by stochastically compare our model by Bellman-Harris processes, and by using a weak form of a Harnack inequality. We conclude this study by giving a biologically relevant example where the assumptions of our main result are verified, and a numerical illustration of the convergence towards a stationary profile for this example.

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An evolutionary dynamics model of the development of drug resistance in metastatic tumors

Federica Padovano
(Laboratoire Jacques-Louis Lions, Sorbonne Universite)

Abstract

We present a mathematical model of evolutionary dynamics of a metastatic tumour under chemotherapy, comprising non-local phenotype-structured PDEs for the primary tumour and its metastasis, where drug delivery is described using a physiologically-based PK model implementing a realistic delivery schedule. By means of long-time asymptotic analysis, global sensitivity analysis and numerical simulations, we explore the impact of cell migration from the primary to the metastatic site, physiological aspects of the tumour and drug dose on the development of drug resistance and treatment efficacy.

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Convergence to equilibrium for cross diffusion systems with nonlocal interaction

Christian Parsch
(Technical University of Munich, Germany; School of Computation, Information and Technology, Department of Mathematics)

Abstract

We study existence and long-term asymptotics of weak solutions to a system of non-linear diffusion-aggregation equations. The system is interpreted as a metric gradient flow on the space of pairs of probability measures, where the underlying energy contains a λ-contractive non-local interaction term and a diffusion term with small cross-diffusion, scaled by a small coupling parameter ϵ. The energy functional is λ-convex along geodesics for $\epsilon = 0$, however this property is lost for any positive ϵ. We prove that if $\epsilon > 0$ is sufficiently small, despite of the loss of geodesic convexity, the gradient flow solutions to the system still exist and converge to the unique global minimizer of the energy at a slightly smaller exponential rate $\lambda - C\epsilon$. The main idea of the proof is to split the energy functional into a convex and a small non-convex part, and to control the influence of the non-convex terms.

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Strategic geometric graphs through mean field games

Matthias Rakotomalala
(CMAP Ecole Polytechnique)

Abstract

We exploit the structure of geometric graphs on Riemannian manifolds to analyze strategic dynamic graphs at the limit, when the number of nodes tends to infinity. This framework allows to preserve intrinsic geometrical information about the limiting graph structure, such as the Ollivier curvature. After introducing the setting, we derive a mean field game system, which models a strategic equilibrium between the nodes. It has the usual structure with the distinction of being set on a manifold. Finally, we establish existence and uniqueness of solutions to the system when the Hamiltonian is quadratic for a class of non-necessarily compact Riemannian manifolds, referred to as manifolds of bounded geometry.

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Derivation of Ionic Euler-Poisson Equations as Hydrodynamic Limit

Sihyun Song
(Yonsei University)

Abstract

The ionic Euler-Poisson equations describe the evolution of ions in an ionized plasma. We consider the equations under power-law pressure, and show that it is derived as the hydrodynamic limit of a kinetic equation, namely the Vlasov-Poisson equation subject to the BGK operator. For a specific case, we also prove the global existence of weak solutions to the Vlasov-Poisson-BGK equation satisfying the kinetic entropy inequality, making our work completely rigorous for this case.

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Stability of solutions of the porous medium equation with growth with respect to the diffusion exponent

Zuzanna Szymanska
(ICM, University of Warsaw)

Abstract

We consider a macroscopic model for the growth of living tissues incorporating pressure-driven dispersal and pressure-modulated proliferation. Assuming a power-law relation between the mechanical pressure and the cell density, the model can be expressed as the porous medium equation with a growth term. We prove Holder continuous dependence of the solutions of the model on the diffusion exponent. The main difficulty lies in the degeneracy of the porous medium equations at vacuum. To deal with this issue, we first regularise the equation by shifting the initial data away from zero and then optimise the stability estimate derived in the regular setting.

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Modelling yeast biofilm expansion

Alex Tam
(UniSA STEM, The University of South Australia (Adelaide, Australia))

Abstract

Biofilms are communities of microbial cells and extracellular fluid. They pose major risks due to their ability to colonise and contaminate surfaces such as medical devices, pipelines, and food processing equipment. I am part of a group of researchers from Australia (Universities of Adelaide and South Australia) and the United Kingdom (Universities of Kent, Essex, Southampton) investigating yeast biofilm formation, growth, and cell death using mathematical modelling and experimentation. The mathematical modelling group uses various approaches, including agent-based models, reaction-diffusion systems, and multiphase fluid mechanical models, my poster will highlight this modelling work.

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Structure-preserving semi-convex-splitting numerical scheme for a Cahn-Hilliard cross-diffusion system in lymphangiogenesis

Boyi Wang
(TU Wien)

Abstract

A fully discrete semi-convex-splitting finite-element scheme with stabilization for a Cahn-Hilliard cross-diffusion system is analyzed. The system consists of parabolic fourth-order equations for the volume fraction of the fiber phase and solute concentration, modeling pre-patterning of lymphatic vessel morphology. The existence of discrete solutions is proved, and it is shown that the numerical scheme is energy stable up to stabilization, conserves the solute mass, and preserves the lower and upper bounds of the fiber phase fraction. Numerical experiments in two space dimensions using FreeFEM illustrate the phase segregation and pattern formation.

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Mean Field Control Problem for two dimensional Keller-Segel System

Yuchen Wang1
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Abstract

In this work, we show that by using the mean field limit theory, the optimal control of two dimensional Keller-Segel system can be obtained as a limit of the corresponding control problem of the interacting stochastic particle system. In these problems, the cost functions are chosen in the same structure which includes the mean field effect. The difficulty arises from the singularity of the fundamental solution of the two-dimensional Poisson equation. Therefore, we start with the control problem of a smoothed version of particle system. In the compactness argument, the Gamma convergence technique has been used. The important technique step is to show that the compactness of the cost functions implies strong convergence of the mean field limit. It is achieved by using a combination of the relative entropy method and the convergence in probability of the trajectory. This work is collaborated with Prof. Li Chen and Zhao Wang.

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Finite-volume approximation of cross-diffusion systems for tumor growth

Sara Xhahysa
(Technische Universitat Wien)

Abstract

We present an implicit Euler finite volume scheme for the mechanical tumor growth model proposed by Jackson and Byrne. The model comprises a cross-diffusion system with no-flux boundary conditions. The numerical scheme preserves the formal gradient-flow or entropy structure and meets the requirements for the boundedness-by-entropy method. We prove the existence of discrete solutions and the convergence of the numerical scheme.