Shapes and shades of Analysis:
in depth and beyond

CONFERENCE PROGRAM

Monday, April 29

8:50–9:00 Opening

9:00–9:40 **Nikolai Nikolski,** *Schatten classes of Sobolev embeddings, and how fast oscillate L^2 bases?*

9:50–10:30 **Stanislav Smirnov,** *Coulomb gas, lattice models and potential theory*

COFFEE BREAK

11:00–11:40 **Eero Saksman,** *Approximation in the Zygmund and Hölder classes*

11:50–12:30 **Adem Limani,** *The shift operator in the Bloch space*

LUNCH at 12:30

15:00–15:40 **Konstantin Dyakonov,** *Extremal problems in BMO and VMO involving the Garsia norm*

15:50–16:20 **Anna Kononova,** *Zero sets with random arguments for Fock-type spaces*

COFFEE BREAK

16:50–17:30 **Omar El Fallah,** *Extremal functions of Dirichlet spaces*

17:40–18:10 **Andreas Hartmann,** *An analytic approach to estimating the solutions of Bézout's polynomial identity*

18:20–18:50 **Polina Perstneva,** *Counterexamples to the typical behaviour of elliptic measures on planar domains*

DINNER at 19:30

Tuesday, April 30

9:00–9:40 **Sergei Treil,** *The matrix* A_2 *conjecture fails, or* $3/2 > 1$

9:50–10:30 **Philippe Jaming,** *Lower bounds of* L^1 *norms of non-harmonic trigonometric polynomials*

COFFEE BREAK

11:00–11:40 **Anton Baranov,** *Geometry of reproducing kernels in Hilbert spaces of analytic functions*

11:50–12:30 **Dmitry Chelkak,** *Ising model on planar graphs, discrete surfaces in* $\mathbb{R}^{2,1}$*, and generalized analytic functions*

LUNCH at 12:30

15:00–15:40 **Alexandru Aleman,** *Invariant subspaces of generalized differentiation and Volterra operators*

15:50–16:20 **Brett Wick,** *The compactness of multilinear Calderón-Zygmund operators*

COFFEE BREAK

16:50–17:30 **Artur Nicolau,** *Analytic Mappings of the unit disk which almost preserve hyperbolic area*

17:40–18:10 **Karine Fouchet Isambard,** *Fourier coefficients of powers of a finite Blaschke product and application to Schäffer's question*

18:20–18:50 **Ioann Vasilyev,** *The Beurling and Malliavin Multiplier Theorem in Several Dimensions*

DINNER at 19:30

Wednesday, May 1

9:00–9:30 **Roman Bessonov,** *Stability properties of Schur's algorithm for analytic functions*

9:40–10:10 **Avner Kiro,** *Integer-valued polynomials satisfying a growth constraint*

COFFEE BREAK

10:40–11:10 **Adi Glücksam,** *Multi-fractal spectrum of planar harmonic measure*

11:20–11:50 **Yulia Kuznetsova,** *Invariant means on quantum weakly almost periodic compactifications*

12:00–12:30 **Harald Woracek,** *Universality limits for power bounded measures*

LUNCH at 12:30

FREE TIME, DISCUSSIONS

DINNER at 19:30

Thursday, May 2

9:00–9:40 **Håkan Hedenmalm,** *Hyperbolic Fourier series and the Klein-Gordon equation*

9:50–10:30 **Aleksei Kulikov,** *Time-frequency localization operator and its eigenvalues*

COFFEE BREAK

11:00–11:40 **Alon Nishry,** *The range of unbounded random analytic functions in the unit disk*

11:50–12:30 **Alexei Poltoratski,** *Pointwise convergence of the non-linear Fourier transform*

LUNCH at 12:30

14:30–15:10 **Nikolai Makarov,** *Faber transform*

15:20–15:50 **Kristian Seip,** *The norm of the backward shift operator on* H^1 *is* $\frac{2}{\sqrt{3}}$

COFFEE BREAK

16:20–17:00 **Mikhail Sodin,** *Sasha Borichev (a short sketch)*

17:30 CONCERT

CONFERENCE DINNER ("Bouillabaisse")
at 19:30

Friday, May 3

9:00–9:40 **Alexander Volberg,** *Quantum and classical low degree learning via a dimension-free Remez inequality*

9:50–10:30 **Pascal Thomas,** *Sharp Invertibility in Quotient Algebras of* H^∞

COFFEE BREAK

11:00–11:40 **Joaquim Ortega-Cerdà,** *Linear statistics of determinantal point processes and norm representations*

11:50–12:30 **Alexander Pushnitski,** *A Borg-Marchenko type uniqueness theorem for complex potentials*

LUNCH at 12:30

ABSTRACTS

Alexandru Aleman
University of Lund

Invariant subspaces of generalized differentiation and Volterra operators

Abstract: The problem of describing the invariant subspaces of the usual Volterra operator on $L^2[0,1]$ goes back to Gelfand (1938) and this direction contains many ingenious ideas. Invariant subspaces for differentiation on C^∞ were studied much later by Korenblum and myself and in subsequent work with Baranov and Belov. I intend to give a presentation of some of these ideas and then continue with a more abstract setting consisting of an unbounded operator D with a compact quasi-nilpotent right inverse V.

It turns out that under certain general conditions one can prove similar results for a large class of examples (for D) containing Schrödinger operators and many other canonical systems.

This is a report about joint work with Alex Bergman.

Anton Baranov
Saint Petersburg State University

Geometry of reproducing kernels in Hilbert spaces of analytic functions

We will give a short survey of Alexander Borichev's work on sampling and interpolation in spaces of analytic functions and on related geometric properties of systems of reproducing kernels. In particular, we discuss a surprising construction of Fock-type spaces admitting Riesz bases of normalized reproducing kernels due to A. Borichev and Yu. Lyubarskii as well as several of its consequences. Another group of questions is related to the so-called spectral synthesis problem which was solved in our joint work with Yu. Belov and A. Borichev for the Paley-Wiener, de Branges and Bargmann-Fock spaces. This line of research culminated in the solution (by Borichev and Belov) of the Newman-Shapiro spectral synthesis problem which remained open for more than 50 years.

Roman Bessonov
Saint Petersburg State University

Stability properties of Schur's algorithm for analytic functions

We prove a sharp stability estimate for Schur's iterates of contractive analytic functions in the open unit disk. We then apply this result and solve the discrete integrable nonlinear Schrödinger equation (Ablowitz-Ladik equation, AL) on the integer lattice, $\mathbb{Z}$, by means of the nonlinear Fourier transform for all initial data in $\ell^2(\mathbb{Z})$. From an ideological point of view, the algorithm of solution is similar to the widely used scheme "linear Fourier transform solves linear Schrödinger equation in one time step". Indeed, our algorithm uses a variant of the nonlinear Fourier transform in place of the usual Fourier transform and evaluates the solution of AL equation "in one time step". Its main feature is that it works for all $\ell^2(\mathbb{Z})$ initial data. Estimates of the rate of convergence of the algorithm follow from the stability estimate for Schur's iterates of analytic functions.

Joint work with Pavel Gubkin (St. Petersburg).

Dmitry Chelkak
University of Michigan

Ising model on planar graphs, discrete surfaces in $\mathbb{R}^{2,1}$, and generalized analytic functions

It is well known that fermionic observables in the critical Ising model on (e.g.) the square grid are discrete holomorphic; a careful analysis thereof, in particular, led to the famous Smirnov's proof of the convergence of interfaces to SLE curves as well as to many other results on the convergence of correlations functions. However, these ideas remained restricted to a very limited class of weighted planar graphs until recently.

We discuss a general framework of the so-called s-embeddings of planar graphs carrying the nearest-neighbor Ising model. This construction represents a given graph as a discrete surface in the Minkowski space $\mathbb{R}^{2,1}$, and the small mesh size limits of fermionic observables become generalized analytic - or massive holomorphic - functions, with the mean curvature of the limiting surface playing the role of the mass. This raises a number of natural questions of various kinds - both in the discrete and in the continuum - many of which remain open despite the recent progress.

Konstantin Dyakonov
ICREA and Universitat de Barcelona

Extremal problems in BMO and VMO involving the Garsia norm

We study the bounded functions f on the circle with the property that the function's uniform norm, $\|f\|_\infty$, agrees with its Garsia norm, $\|f\|_G$. Also, we discuss the geometry of the unit balls in BMO and in VMO, both of them endowed with the Garsia norm $\|\cdot\|_G$. Finally, we arrive at an amusing "geometric" characterization of inner functions.

Omar El-Fallah
Mohammed V University in Rabat

Extremal functions of Dirichlet spaces

Let μ be a positive finite Borel measure on the unit circle $\mathbb{T}$ of the complex plane, and let

$$\mathcal{D}(\mu) = \{f \in \mathrm{hol}(\mathbb{D}) : \quad \int_{\mathbb{D}} |f'(z)|^2 P[\mu](z) dA(z) < \infty\}$$

be the associated Dirichlet space. In this talk we discuss Dirichlet spaces $\mathcal{D}(\mu)$ for measures μ such that the potential

$$V_\mu(\zeta) = \int_{\mathbb{T}} \frac{d\mu(\xi)}{|\xi - \zeta|^2}$$

is log integrable on $\mathbb{T}$. We will focus on the regularity of extremal functions and their applications.

Karine Fouchet Isambard
Aix-Marseille Université

Fourier coefficients of powers of a finite Blaschke product and application to Schäffer's question

Understanding the asymptotic behaviour of the norms of powers of certain functions in Banach spaces is crucial in various areas of function theory, e.g. J-P. Kahane (1956), G. W. Hedstrom (1966), D. J. Newman (1969), M. Blyudze and S. Shimorin (1996).

Given a finite Blaschke product B we prove asymptotically sharp estimates on the ℓ^∞-norm of the sequence of the Fourier coefficients of B^n as n tends to ∞. This norm decays as $n^{-1/N}$ for some $N \geq 3$. Furthermore, for every $N \geq 3$ we produce explicitly finite Blaschke products B with decay $n^{-1/N}$. As an application we construct a sequence of $n \times n$ invertible matrices T with arbitrary spectrum in the unit disk and such that the quantity

$|\det T| \cdot \|T^{-1}\| \cdot \|T\|^{1-n}$ grows as a power of n, which is motivated by Schäffer's question on norms of inverses.

The talk is based on a joint work with A. Borichev and R. Zarouf.

Adi Glücksam
Northwestern University

Multi-fractal spectrum of planar harmonic measure

In this talk, I will define various notions of the multi-fractal spectrum of harmonic measures and discuss finer features of the relationship between them and properties of the corresponding conformal maps. Furthermore, I will describe the role of multifractal formalism and dynamics in the universal counterparts.

This talk is based on a joint work with I. Binder.

Andreas Hartmann
Université de Bordeaux

An analytic approach to estimating the solutions of Bézout's polynomial identity

The aim of this talk is to revisit Bézout's theorem on minimal polynomial solutions R and S of the identity $AR+BS=1$ where A and B are given polynomials. We are in particular interested in estimating the norms of R and S depending on a suitable separation of the zeros of A and B and when A and B are bounded in norm by 1. This will be achieved *via* an analytic approach essentially based on Cauchy's formula. Bézout's identity is related to Carleson's famous corona theorem which considers such an identity for functions in the space H^∞ of uniformly bounded holomorphic functions on the unit disk. Recall that in Carleson's theorem the solutions are controlled by $C\log(1/\delta)/\delta^2$, where δ is a lower bound of $|A|+|B|$ on the unit disk (even for general functions A and B in the ball of H^∞), whereas we obtain an estimate in C/δ^2 where δ is now a lower bound of $|A|+|B|$ in the complex plane.

Our results also yield estimates of the norm of the inverse to the Sylvester matrix.

This is joint work with Emmanuel Fricain, William T. Ross and Dan Timotin.

Håkan Hedenmalm
Royal Institute of Technology

Hyperbolic Fourier series and the Klein-Gordon equation

We introduce the concept of hyperbolic Fourier series, which can be used to represent uniquely a distribution on the extended real line in a series which resembles Fourier series. Those hyperbolic Fourier series are intimately connected with the Klein-Gordon equation, in $1+1$ dimensions. We discuss the implications for the interpolation theory of solutions to the Klein-Gordon equation.

Philippe Jaming
Université de Bordeaux

Lower bounds of L^1 norms of non-harmonic trigonometric polynomials

The aim of this talk is to present several results concerning the L^1 norm of non-harmonic trigonometric polynomial. The first result is a quantitative version of a result of Nazarov, inspired by McGehee, Pigno and Smith' solution of the Littlewood conjecture: when $T > 1$ and $\lambda_{j+1} - \lambda_j \geq 1$

$$C(T) \sum_{j=1}^{N} \frac{|a_j|}{j+1} \leq \frac{1}{T} \int_{-T/2}^{T/2} \left| \sum_{j=1}^{N} a_j e^{2i\pi\lambda_j t} \right| \mathrm{d}t.$$

Here $C(T)$ is an explicit constant that is independent of N and the a_j's.

It is unknown if the condition $T > 1$ is necessary. However, when $\lambda_{j+1} - \lambda_j \to 0$, as $j \to +\infty$, we will show that the result is valid for any $T > 0$ but with a non-explicit constant $C(T)$.

This is joint work with K. Kellay, S. Saba and Y. Wang.

Avner Kiro
Horizon Rock Development

Integer-valued polynomials satisfying a growth constraint

Integer-valued polynomials (IVPs) are algebraic polynomials which take integer values on the integers. We consider IVPs that satisfy an exponential growth condition on the natural numbers. Elkies and Speyer, answering a question by Dimitrov on Math Overflow, showed that there is a growth threshold, such that there are infinitely many IVPs with growth rate above the threshold and only finitely many IVPs below that threshold. We estimate the number of IVPs with a growth rate above the threshold, for polynomials of large degree. In addition, we consider a more general problem, with a not necessarily symmetric growth condition on the integers.

Based on a joint work in progress with Alon Nishry.

Anna Kononova
Tel Aviv University

Zero sets with random arguments for Fock-type spaces

We address the following question. Consider a sequence $\left\{\lambda_n e^{i\theta_n}\right\}$, where $\{\lambda_n > 0\}$ is a fixed sequence of positive numbers tending to infinity, and $\theta_n \in [0, 2\pi]$ are independent random variables. Will this random sequence be a zero set of a function in a given weighted Fock-type space

$$\mathcal{F}^p_\varphi := \left\{ f - \text{entire}, \int_{\mathbb{C}} |f(z)|^p \, e^{-p\varphi(|z|)} \mathrm{d}m(z) < \infty \right\}?$$

We will discuss some results in this direction.

Aleksei Kulikov
Tel Aviv University

Time-frequency localization operator and its eigenvalues

Given a measurable set $U \subset \mathbb{R}$, we define the projection onto U as $P_U : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ given by $(P_U f)(x) = f(x)\chi_U(x)$. Similarly, for the set $V \subset \mathbb{R}$ we define the Fourier projection onto V as $Q_V = \mathcal{F}^{-1} P_V \mathcal{F}$, where $\mathcal{F}$ is the Fourier transform. The operator $S_{U,V} = P_U Q_V P_U$ is called time-frequency localization operator, associated with U and V.

In general $S_{U,V}$ is a non-negative definite operator of norm at most 1. However, if both U and V have finite measure then $S_{U,V}$ is also a Hilbert–Schmidt operator. In particular, it is compact and thus it has a sequence of eigenvalues $1 > \lambda_1(U, V) \geq \lambda_2(U, V) \geq \ldots > 0$.

In this talk, we will focus on the case when both U and V are intervals. In this case the eigenvalues depend only on the product of length of the intervals $c = |U||V|$, so we have a sequence $1 > \lambda_1(c) > \lambda_2(c) > \ldots > 0$. It turns out that these eigenvalues exhibit a phase transition: first $\approx c$ of them are very close to 1, then there are $\approx \log c$ intermediate ones, and the remaining eigenvalues decay to zero extremely fast. We will overview the known results on the behaviour of eigenvalues in these three regimes, and also discuss a new exponential upper bound for $1 - \lambda_n(c)$ when $n < (1 - \varepsilon)c, \varepsilon > 0$.

Yulia Kuznetsova
Université de Franche-Comté

Invariant means on quantum weakly almost periodic compactifications

Analysis may take quite topological shapes. A compactification of a given topological group can give it new properties, such as, the most interesting for this talk, the existence of an invariant mean. The road to take is to pass to the weakly almost periodic (WAP) compactification, and on the way to observe the class of WAP functions, having connections to representations and applications in PDEs. I am interested in the operator-algebraic version of this theory, for what one can call also quantum groups. The very non-commutative definition of the WAP compactification has met several difficulties, and especially, invariant means have been inaccessible until recently. I will show how a proper view on this setting can lead to fixed point theorems and to existence of invariant means, all within the context of quantum groups.

Adem Limani
Universitat Autònoma de Barcelona

The shift operator in the Bloch space

Bloch functions are likely to originate from the work of Bloch, Landau and Valiron around the 1920's, independently of each other, in connection to right inverses of holomorphic mappings. In modern times, Bloch functions naturally appear in the study of boundary behaviours of conformal mappings, and enjoy some intrinsic probabilistic properties, hence maintain a crucial role in geometric function theory. From a function theoretical point of view, the space of Bloch functions conceived within the framework of Bergman spaces, play the analogue role of BMO in the classical framework of Hardy spaces. As such, several problems in operator and function theory, such as actions of various operators, may also be phrased therein. Our purpose is to consider two intimately related problems on the shift operator in the Bloch space. More precisely, we shall consider the problem of shift invariant subspaces and the problem of simultaneous approximation in the Bloch space.

This talk is partially based on joint work with Artur Nicolau.

Nikolai Makarov
Caltech

Faber transform

I will show how Faber transform can be useful in certain problems of complex analysis.

Artur Nicolau
Universitat Autònoma de Barcelona

Analytic Mappings of the unit disk which almost preserve hyperbolic area

We consider analytic self-maps of the unit disk which distort hyperbolic area of hyperbolic disks by a bounded amount. We give a number of characterizations involving angular derivatives, Lipschitz extensions, Möbius distortion, the distribution of critical points and Aleksandrov-Clark measures.

Joint work with Oleg Ivrii.

Nikolai Nikolski
Université de Bordeaux

Schatten classes of Sobolev embeddings, and how fast oscillate L^2 bases?

The "positivity phenomenon" for Bessel sequences, frames and Riesz bases (u_k) are studied in L^2 spaces over the compacts of homogeneous (Coifman-Weiss) type $\Omega = (\Omega, \rho, \mu)$. Under some relations between three basic metric-measure dimensions of Ω, we obtain asymptotics for the mass moving norms $\|u_k\|_{KR}$ (Kantorovich-Rubinstein), as well as for singular numbers of the Lipschitz and Hajłasz-Sobolev embeddings. Our main observation shows that, quantitatively, the rate of the convergence $\|u_k\|_{KR} \longrightarrow 0$ depends on an interplay between geometric doubling and measure doubling/halving exponents. The "more homogeneous" is the space, the sharper are the results.

Alon Nishry
Tel Aviv University

The range of unbounded random analytic functions in the unit disk

We consider random analytic functions with finite radius of convergence. We show that any unbounded random Taylor series with rotationally invariant coefficients have dense image in the plane. Moreover, if in addition the coefficients are complex Gaussian with regular variances, then the image is the whole complex plane. These results were known only under the assumption that the sum of the variances diverges. Our methods make use of an inherent branching process structure built into the random series.

Based on a joint work with Elliot Paquette.

Joaquim Ortega-Cerdà
University of Barcelona

Linear statistics of determinantal point processes and norm representations

We study the asymptotic behaviour of the fluctuations of smooth and rough linear statistics for determinantal point processes on the sphere and on the Euclidean space. The main tool is the generalization of some norm representation results for functions in Sobolev spaces and in the space of functions of bounded variation.

This is a joint work with Jordi Marzo and Matteo Levi.

Polina Perstneva
Université Paris-Saclay

Counterexamples to the typical behaviour of elliptic measures on planar domains

Recent developments in Geometric Measure Theory have led to the understanding that, essentially, rectifiability of the boundary of a domain is necessary and sufficient for the harmonic measure to be (qualitatively) absolutely continuous with respect to the Hausdorff measure on that boundary. Plus, for purely unrectifiable sets (e.g., the Koch snowflake, the four corners Cantor set, the Sierpinski carpet, etc), the harmonic measure is singular with respect to the boundary measure.

It is also known that all operators close to the Laplacian, which generates the harmonic measure, produce elliptic measures with the same properties as above. However, it turns out that for some unrectifiable sets on the plane, there exists an elliptic operator with a scalar coefficient whose elliptic measure's behaviour is dramatically different. We will discuss these counterexamples discovered in the last couple of years and some open problems around them.

Alexei Poltoratski
University of Wisconsin

Pointwise convergence of the non-linear Fourier transform

I will discuss a non-linear version of Carleson's theorem on the pointwise convergence of the Fourier transform along with the corresponding maximal estimates, examples and further questions.

Alexander Pushnitski
King's College London

A Borg-Marchenko type uniqueness theorem for complex potentials

The Borg-Marchenko uniqueness theorem for a self-adjoint Schrödinger operator H on the semi-axis says that the spectral measure of H uniquely determines the potential. The main object of the talk is the class of Schrödinger operators H with complex potentials on the semi-axis. In this case, H is non-self-adjoint and it is no longer clear how to define its spectral measure. Nevertheless, I will explain that it is possible to associate with H the spectral data which uniquely determines the potential. The spectral data is inspired by the polar decomposition of H and consists of a pair (measure, function). Here measure is the spectral measure of $|H|$ and function (on the spectrum of $|H|$) represents certain "complex rotations"which trivialise in the self-adjoint case.

This is recent joint work with Frantisek Stampach (Prague).

Eero Saksman
University of Helsinki

Approximation in the Zygmund and Hölder classes

We estimate in terms of various quantities the distance of a given Zygmund functions in $\mathbb{R}^n$ to BMO-type subspaces. This generalizes an earlier result due to A. Nicolau and O. Soler i Gibert to higher dimensions.

The work is based on a joint work with O. Soler i Gibert (Barcelona).

Kristian Seip
Norwegian University of Science and Technology

The norm of the backward shift operator on H^1 is $\frac{2}{\sqrt{3}}$

We show that the norm of the backward shift operator on H^1 is $2/\sqrt{3}$, and we identify the functions for which the norm is attained.

Joint work with Ole Fredrik Brevig.

Stanislav Smirnov
Université de Genève

Coulomb gas, lattice models and potential theory

Introduction of Conformal Field Theory by Belavin, Polyakov and Zamolodchikov led to spectacular applications of complex analysis to 2D lattice models of statistical physics and 2D Field theories (which still are underappreciated in complex analysis itself). A few years earlier den Nijs and Nienhuis were able to derive many of the exponents and dimensions for 2D lattice models using Coulomb gas techniques. They postulated (unrigorously) that height functions of lattice models converge to the Gaussian Free Field. Their method is in many ways mysterious, in particular it was never formulated in the presence of a boundary.

We will discuss possible formulations and their relations to CFT, SLE and conformal invariance of critical lattice models. Interestingly, new objects in complex potential theory seem to arise.

Pascal J. Thomas
Université Paul Sabatier Toulouse

Sharp Invertibility in Quotient Algebras of H^∞

Given an inner function $\Theta \in H^\infty(\mathbb{D})$ and $[g]$ in the quotient algebra $H^\infty/\Theta H^\infty$, its quotient norm is $\|[g]\| := \inf\{\|g + \Theta h\|_\infty, h \in H^\infty\}$. We show that when g is normalized so that $\|[g]\| = 1$, the quotient norm of its inverse can be made arbitrarily close to 1 by imposing $|g(z)| \geq 1 - \delta$ when $\Theta(z) = 0$, with $\delta > 0$ small enough, (call this property SIP) if and only if the function Θ satisfies the following growth property:

$$\lim_{t\to 1} \inf \left\{ |\Theta(z)| : z \in \mathbb{D}, \rho(z, \Theta^{-1}\{0\}) \geq t \right\} = 1,$$

where ρ is the usual pseudohyperbolic distance in the disc, $\rho(z, w) := \left|\frac{z-w}{1-z\bar{w}}\right|$.

We prove that an inner function is SIP if and only if for any $\varepsilon > 0$, the set $\{z : 0 < |\Theta(z)| < 1 - \varepsilon\}$ cannot contain hyperbolic disks of arbitrarily large radius.

Thin Blaschke products provide an example of such functions. Some SIP Blaschke products fail to be interpolating (and thus aren't thin), while there exist Blaschke products which are interpolating and fail to be SIP. We also study the functions which can be divisors of SIP inner functions.

This is joint work with Alexander Borichev, Artur Nicolau, and Myriam Ounaïes.

Sergei Treil
Brown University

The matrix A_2 conjecture fails, or $3/2 > 1$

The matrix A_2 condition on the matrix weight W

$$[W]_{A_2} := \sup_I \left\| \langle W \rangle_I^{1/2} \langle W^{-1} \rangle_I^{1/2} \right\|^2 < \infty$$

where supremum is taken over all intervals $I \subset \mathbb{R}$, and

$$\langle W \rangle_I := |I|^{-1} \int_I W(s) \mathrm{d}s,$$

is necessary and sufficient for the Hilbert transform T to be bounded in the weighted space $L^2(W)$.

It was well known since early 90s that $\|T\|_{L^2(W)} \gtrsim [W]_{A_2}^{1/2}$ for all weights, and that for some weights $\|T\|_{L^2(W)} \gtrsim [W]_{A_2}$. The famous A_2 conjecture (first stated for scalar weights) claims that the second bound is sharp, i.e.

$$\|T\|_{L^2(W)} \lesssim [W]_{A_2}$$

for all weights.

After some significant developments (and some prizes obtained in the process) the scalar A_2 conjecture was finally proved: first by J. Wittwer for Haar multipliers, then by S. Petermichl for Hilbert Transform and for the Riesz transforms, and finally by T. Hytönen for general Calderón–Zygmund operators.

However, while it was a general consensus that the A_2 conjecture is true in the matrix case as well, the best known estimate, obtained by Nazarov–Petermichl–Treil–Volberg (for all Calderón–Zygmund operators) was only $\lesssim [W]_{A_2}^{3/2}$.

But this upper bound turned out to be sharp! In a recent joint work with K. Domelevo, S. Petermichl and A. Volberg we constructed weights W such that

$$\|T\|_{L^2(W)} \gtrsim [W]_{A_2}^{3/2},$$

so the above exponent $3/2$ is a correct one.

In the talk I'll explain motivations, history of the problem, and outline the main ideas of the construction. The construction is quite complicated, but it is an "almost a theorem" that no simple example is possible.

This is joint work with K. Domelevo, S. Petermichl and A. Volberg.

Ioann Vasilyev
CY Cergy Paris Université

The Beurling and Malliavin Multiplier Theorem in Several Dimensions

The present talk is devoted to a new multidimensional version of the Beurling and Malliavin Multiplier Theorem. In more detail, we shall introduce and discuss a new sufficient condition for a function to be a Beurling and Malliavin admissible majorant in several dimensions. The main result of this talk provides a partial answer to a question posed by L. Hörmander.

Alexander Volberg
Michigan State University, Hausdorff Center for Mathematics

Quantum and classical low degree learning via a dimension-free Remez inequality

Recent efforts in Analysis of Boolean Functions aim to extend core results to new spaces, including noncommutative spaces (matrix algebras). We present here a new way to relate functions on the hypergrid (or products of cyclic groups) to their harmonic extensions over the polytorus. We show the supremum of a function over products of the cyclic group controls the supremum of function over the entire polytorus, with multiplicative constant not depending on the dimension of polytorus.

This inequality is the key ingredient that allows us to extend to new spaces a recent series of algorithms for learning low-degree polynomials on the hypercube and low-degree quantum observables on qubits. In particular, our dimension-free Remez inequality implies cyclic-group Bohnenblust-Hille-type (BH-type) estimates that seem impossible to obtain with existing techniques.

Brett Wick
Washington University in Saint Louis

The compactness of multilinear Calderón-Zygmund operators

We prove a wavelet T(1) theorem for compactness of multilinear Calderón-Zygmund (CZ) operators. Our approach characterizes compactness in terms of testing conditions and yields a representation theorem for compact CZ forms in terms of wavelet and paraproduct forms that reflect the compact nature of the operator.

This talk is based on joint work with Anastasios Fragkos and Walton Green.

Harald Woracek
Vienna University of Technology

Universality limits for power bounded measures

We investigate rescaling limits at a regularly varying scale along a chain of de Branges spaces contained isometrically in a space $L^2(\mu)$. We give a characterisation of existence of such a limit in terms of the spectral measure μ, including an explicit formula for the limit kernel in terms of confluent hypergeometric functions. This result naturally generalises previous work, mainly from the context of moment problems, about bulk universality or hard edge universality (both being particular cases of a rescaling limit for the Christoffel-Darboux kernels of the moment sequence). In this context typically the Paley-Wiener kernel or Bessel kernels appear as limits. For the proof we develop an extension of de Branges' homeomorphism between Herglotz functions and canonical systems to a certain setting within the regime of power bounded measures, and this is actually the core of the matters. The mentioned characterisation will be deduced from that, and the mentioned explicit formulae will follow on combining with results about homogeneous de Branges spaces.

The talk is based on joint work with B. Eichinger and M. Lukic.