Aggregation-Diffusion Equations & Collective Behavior:
Analysis, Numerics and Applications

Equations d'agrégation-diffusion et comportement collectif:
Analyse, schémas numériques et applications

8 – 12 April 2024

Centre International de Rencontres Mathématiques, Marseille

As part of the Chaire Jean Morlet: Aggregation-Diffusion and Kinetic Equations, Collective Behavior Models and Applications

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Program

Day	Monday	Tuesday	Wednesday	Thursday	Friday
08:50–09:00	Opening				
09:00–09:45	G. Savaré	F. Santambrogio	D. Peurichard	Y. Yao	R. L. Frank
09:45–10:30	J. Maas	O. Tse	V. Calvez	D. Slepčev	M. Schmidtchen
10:30–11:00	**Coffee Break**				
11:00–11:45	J. Skrzeczkowski	G. Estrada-Rodriguez	S. Fagioli	R. Fetecau	A. Arnold
11:45–12:30	D. Poyato	B. Volzone	M. Delgadino	M. Winkler	E. Carlen
12:30–12:40	**Lunch**				**Closing**
12:40–14:00					
14:00–14:45	D. Matthes	P. Laurençot	S. Daneri	Free Afternoon	
14:45–15:30	A. Świerczewska-Gwiazda	A. Lanar	B. Wennberg		
15:30–16:00	**Coffee Break**				
16:00–16:45	F. Hoffmann	M. Jacobs			
16:45–17:00	Poster presentation				
17:00–19:00	Poster session				

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Contents

Talks

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Aggregation-Diffusion Equations & Collective Behavior:
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All relative entropies for general nonlinear Fokker-Planck equations

Anton Arnold
(Vienna University of Technology)

Abstract

We shall revisit the entropy method for quasilinear Fokker-Planck equations with confinement to deduce exponential convergence to the equilibrium. Even for prototypical examples like the porous-medium equation, only one relative entropy has been known so far - the Ralston-Newman entropy, which is the analog of the logarithmic entropy in the linear case.

We shall give a complete characterization of all admissible relative entropies for each quasilinear Fokker-Planck equation. In particular we find that fast-diffusion equations with power-law nonlinearities admit only one entropy, while porous medium equations give rise to a whole family of admissible relative entropies (similar to linear Fokker-Planck equations). These additional entropies then imply also new moment-control estimates on the porous-medium solution. This is a joint work with Jose Carrillo and Daniel Matthes.

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Lane formation and aggregation spots in foraging ants

Maria Bruna
(Univ. Cambridge)

Abstract

We consider a system of interacting particles as a model for a foraging ant colony, where each ant is represented as an active Brownian particle. The interactions among ants are mediated through chemotaxis, aligning their orientations with the upward gradient of a pheromone field. Unlike conventional models, our study introduces a parameter that enables the reproduction of two distinctive behaviours: the conventional Keller-Segel collapse and the formation of travelling clusters without relying on external constraints such as food sources or nests. We consider the associated mean-field limit of this system and establish the analytical and numerical foundations for understanding these particle behaviours.

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Modeling crowds of bacteria: transition from swarming to rippling in Myxococcus xanthus collectives

Vincent Calvez
(CNRS)

Abstract

I will present a joint work with Jean-Baptiste Saulnier (Marseille), Michèle Romanos (Lyon) and Tâm Mignot (Marseille). We revisited the modeling of the so-called rippling phase of the soil bacteria Myxococcus xanthus lifecycle. The emergence of counter-propagating waves has attracted great attention from the mathematical and biophysical communities in the past two decades. Combining recent biological insights, high-resolution microscopy, trajectory analysis, kinetic modeling and numerical simulations, we proposed an integrated scenario for the collective behavior of this fascinating micro-organism, in which traffic congestion seems to play a key role.

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Stability for functional inequalities and the critical mass Keller-Segel equation

Eric A. Carlen
(Rutgers University)

Abstract

The critical mass Keller-Segel equation displays a perfect balance between the aggregating effects of a chemical attractant and the dispsersive effects of diffusion. For an initial mass and any smaller than the critical mass, the diffusion wins, and the solutions disperse to infinity. For a critical mass any larger, the attractant wins, and the solutions collapse to a point mass in finite time. At the critical mass, there is perfect balance, and the solutions tend to non-trivial steady states. This was proved, and the basins of attraction were studied, in joint work with Blanchet and Carrillo. Later, the rate of approach to equilibrium was studied in joint work with Figalli. Various functional inequalities are central to this, in particular, stability estimates for the logarithmic Hardy-Littlewood-Sobolev inequaity and certian Gagliardo-Nirenberg-Sobolev inequalities. This talk presents new results on these stability estimates, including a new sharp stability estimate for the logarithmic Hardy-Littlewood-Sobolev inequality, and these are applied to give improved estimates on the rate of convergence to the steady states.

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Convergence results for deterministic particle approximations via energy bounds

Sara Daneri
(GSSI)

Abstract

In this talk we will consider a general class of aggregation-diffusion equations on unbounded domains in one space dimension, both in absence and in presence of a nonlinear mobility. We will present a strategy which allows to show convergence of the deterministic particle approximation scheme for all bounded densities of bounded energy, introduced in collaboration with E. Radici and E. Runa.

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Generative Adversarial Networks: Dynamics and Mode Collapse

Matias Delgadino
(UT-Austin)

Abstract

Generative Adversarial Networks (GANs) was one of the first Machine Learning algorithms to be able to generate remarkably realistic synthetic images. In this presentation, we delve into the mechanics of the GAN algorithm and its profound relationship with optimal transport theory. Through a detailed exploration, we illuminate how GAN approximates a system of PDE, particularly evident in shallow network architectures. Furthermore, we investigate the phenomenon of mode collapse, a well-known pathological behavior in GANs, and elucidate its connection to the underlying PDE framework through an illustrative example.

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Nonlocal interaction kernels inference in nonlinear gradient flow equations

Gissell Estrada-Rodriguez
(U. Politecnica de Catalunya)

Abstract

When applying nonlinear aggregation-diffusion equations to model real life phenomenon, a major challenge lies on the choice of the interaction potential. Previous numerical and theoretical studies typically required predetermination of terms and the goal is often to reproduce the observed dynamics qualitatively, not quantitatively. In this talk, we address the inverse problem of identifying nonlocal interaction potentials in nonlinear aggregation-diffusion equations from noisy discrete trajectory data. Our approach involves formulating and solving a regularised variational problem, which requires minimising a quadratic error functional across a set of hypothesis functions. A key theoretical contribution is our novel stability estimate for the PDEs, validating the error functional ability in control- ling the 2-Wasserstein distance between solutions generated using the true and estimated interaction potentials. We demonstrate the effectiveness of the methods through various 1D and 2D examples showcasing collective behaviours.

[1] J. A. Carrillo, G. Estrada-Rodriguez, L. Mikolas, and S. Tang Sparse identification of nonlocal interaction kernels in nonlinear gradient flow equations via partial inversion. Submitted. ArXiv:2402.06355, 2024.

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Aggregation-Diffusion model for opinion formation on networks

Simone Fagioli
(Univ. L'Aquila)

Abstract

We study a system of nonlocal aggregation cross-diffusion PDEs that describe the evolution of opinion densities on a network. The PDEs are coupled with a system of ODEs that describe the time evolution of the agents on the network. Firstly, we apply the Deterministic Particle Approximation (DPA) method to the aforementioned system in order to prove the existence of solutions under suitable assumptions on the interactions between agents. Later on, we present an explicit model for opinion formation on an evolving network. The opinions evolve based on both the distance between the agents on the network and the 'attitude areas,' which depend on the distance between the agents' opinions. The position of the agents on the network evolves based on the distance between the agents' opinions. The goal is to study radicalization, polarization, and fragmentation of the population while changing its open-mindedness and the radius of interaction.

Joint work with Gianluca Favre.

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Ground states for aggregation-diffusion energies on Riemannian manifolds

Razvan Fetecau
(Simon Fraser University)

Abstract

We consider a free energy functional on Riemannian manifolds of negative curvature, and investigate the existence of its global minimizers. The energy functional consists of two components: an entropy (or internal energy) and an interaction energy modelled by an attractive potential. The two components have competing effects, as they favour spreading by linear diffusion and blow-up by nonlocal attractive interactions, respectively. We find necessary and sufficient conditions for ground states to exist, in terms of the behaviours of the attractive potential at infinity and at zero. As a key tool in our analysis, we develop a new logarithmic Hardy-Littlewood-Sobolev inequality on Cartan-Hadamard manifolds.

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Minimizers for an aggregation model with attractive-repulsive interaction

Rupert L. Frank
(Ludwig Maximilian University of Munich)

Abstract

We show that the uniform distribution on the surface of a sphere is the minimizer among probability measures for the interaction potential $(1/a)|r|^a - (1/b)|r|^b$, which is repulsive at short distances and attractive at large distances. Our result is valid in any dimension in the regime $2 \leq a \leq 4, b(a) \leq b \leq 2$ and $b < a$, where $b(a)$ is an explicitly given function. The result is optimal in the sense that for $2 \leq a \leq 4$ with $b < a$, the uniform distribution on the surface of a sphere is not a minimizer when b does not belong to $[b(a), 2]$.

The talk is based on joint work with R. Matzke.

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Dynamics of Strategic Agents and Algorithms as PDEs

Franca Hoffmann
(Caltech)

Abstract

We propose a PDE framework for modeling the distribution shift of a strategic population interacting with a learning algorithm. We consider two particular settings; one, where the objective of the algorithm and population are aligned, and two, where the algorithm and population have opposite goals. We present convergence analysis for both settings, including three different timescales for the opposing-goal objective dynamics. We illustrate how our framework can accurately model real-world data and show via synthetic examples how it captures sophisticated distribution changes which cannot be modeled with simpler methods.

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Nonlocal Approximation of Slow and Fast Diffusion

Matt Jacobs
(Purdue University)

Abstract

Motivated by recent work on the approximation of diffusion equations by deterministic interacting particle systems, we develop a nonlocal approximation for a range of linear and nonlinear diffusion equations. and prove convergence of the method in the slow, linear, and fast diffusion regimes. A key ingredient of our approach is a novel technique for using the 2-Wasserstein and dual Sobolev gradient flow structures to recover the density-pressure duality relation in the nonlocal-to-local limit. Due to the general class of internal energy densities that our method is able to handle, an intriguing byproduct of our result is a deterministic sampling method that can be applied to a wider class of distributions than the classical log-convex setting.

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Aggregation-diffusion equations: concentration and small-scale behaviour

Alexandre Lanar
(U. Lyon 1)

Abstract

Aggregation-diffusion equations are involved in modelling many phenomena, in particular in astrophysics and in biology (chemotaxis). The most well-known example is the one of the Keller-Segel (KS) system. Here we consider a class of KS-type models with solutions which explose in the zero-diffusion limit. We characterise in a sharp way their behaviour (concentration, Lebesgue norms) in the small-diffusion regime in the radially symmetric case. We will compare our results with previous analogous ones for scalar conservation laws. The research has been done in collaboration with P. Biler, G. Karch (Wroclaw) and P. Laurençot (Toulouse).

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Global existence and boundedness for chemotaxis models with local sensing

Philippe Laurençot
(Laboratoire de Mathématiques (LAMA), CNRS & Université Savoie Mont Blanc)

Abstract

Global existence of classical solutions is investigated for a chemotaxis model with local sensing for a general class of mobility functions. In contrast to the classical Keller-Segel chemotaxis model, no finite blowup occurs but the formation of singularities is possibly shifted to infinity. In addition, some classes of mobility functions for which solutions are bounded are identified. Joint works with Jie Jiang, Wuhan et Yanyan Zhang, Shanghai.

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Anisotropic transport-information inequalities

Jan Maas
(IST Austria)

Abstract

We prove upper bounds on the L^∞-Wasserstein distance between strongly log-concave probability densities and log-Lipschitz perturbations. In the simplest setting, such a bound amounts to a transport-information inequality involving the L^∞-Wasserstein metric and the relative L^∞-Fisher information. We show that this inequality can be sharpened significantly in situations where the involved densities are anisotropic. Our proof is based on probabilistic techniques using Langevin dynamics. As an application of these results, we generalise a recent result by Calvez, Poyato, and Santambrogio on the rate of convergence in Fisher's infinitesimal model from dimension 1 to arbitrary dimensions. This is joint work with Ksenia Khudiakova (ISTA) and Francesco Pedrotti (ISTA).

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Diffusive transport: geodesics, convexity, and gradient flows

Daniel Matthes
(TUM)

Abstract

We introduce an — apparently novel — bi-directional variant of martingale transport from stochastics. In the spirit of the Benamou-Brenier formula for the L2-Wasserstein distance, we define the diffusive transport distance between two probability densities as the infimum of the action (momentum square over mass) along connecting curves; however, the usual continuity equation is replaced by a similar equation with two derivatives (instead of one) on the „momentum“. We prove that the thus obtained divergence is actually a metric on the space of probability measures over I. The metric is complete, geodesic, and topologically equivalent to weak convergence. Then we discuss the representation of certain evolution equations as gradient flows, including the heat and the fourth order DLSS equations, along with a variety of more "exotic" equations. For several of these PDEs, we prove uniform geodesic semi-contractivity. Finally, we present a spatial discretization of the DLSS equation that is adapted to the newly discovered gradient flow structure. That discretization turns out to preserve various additional properties of DLSS, like gradient descent in the Wasserstein metric, contractivity in the Hellinger metric, and monotonicity of significant Lyapunov functionals. This is joint work with Andre Schlichting (Uni Münster), Eva-Maria Rott (TU München), and Giuseppe Savare (Uni Pavia).

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From a nonlocal mean-field to a porous medium system without self-diffusion for multispecies interacting systems

Diane Peurichard
(INRIA Paris)

Abstract

Systems describing the long-range interaction between individuals have attracted a lot of attention in the last years, in particular in relation with living systems. These systems are quadratic, written under the form of transport equations with a nonlocal self-generated drift. We establish the localisation limit, that is the convergence of nonlocal to local systems, when the range of interaction tends to 0. These theoretical results are sustained by numerical simulations. The major new feature in our analysis is that we do not need diffusion to gain compactness, at odd with the existing literature. The central compactness result is provided by a full rank assumption on the interaction kernels. In turn, we prove existence of weak solutions for the resulting system, a cross-diffusion system of quadratic type.

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Mean-field limits of non-exchangeable multi-agent systems on hypergraphs

David Poyato
(U. Granada)

Abstract

Multi-agent systems are ubiquitous in Science, and they can be regarded as large systems of interacting particles with the ability to generate large-scale self-organized structures from simple local interactions rules between each agent and its neighbors. It is well known that the precise architecture of the network plays a crucial role in the resulting emergent behavior of the full group. In most real-life applications, connections are not based on uniform all-to-all couplings, but on highly heterogeneous couplings, sometime dense, but often sparse.

I will start the talk by reviewing some recent literature about the rigorous derivation of macroscopic PDE-based models approximating the microscopic ODE-based multi-agent system as the number of individuals tends to infinity. The strategy to tackle this problem is often based on a careful combination of methods ranging from analysis of PDEs and stochastic analysis, to kinetic equations and graph theory. In many cases, interactions among agents are considered binary and they are modulated by the underlying weighted graph of connections. In this direction, I will briefly discuss a recent work in collaboration with P.-E. Jabin (Penn State University) and J. Soler (University of Granada) where the underlying graphs can be taken asymmetric, sparse, and the resulting graph limit is encoded by a so-called extended graphon (a measure-valued extension of classical graphons).

Then, I will focus on the main part of the talk, where I will present a recent work for the case of higher-order interactions, which has largely captured the attention of the applied community in the last years. In these multi-agent systems, individuals interact by groups so that a full group jointly generates a non-linear force on any given individual, and the underlying graph of connections is then replaced by a hypergraph. We show that when the interactions kernels are regular enough and the underlying hypergraphs are described by dense simplicial complexes (possibly of unbounded order), then the mean-field limit is determined by a limiting Vlasov-type equation, where the resulting mean-field force admits infinitely-many orders of interactions for the first time in the literature.

This is based on a joint work with N. Ayi and N. Pouradier-Duteil (Sorbonne Université).

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Estimates on the JKO scheme for aggregation-diffusion equations (only linear diffusion and nice kernels, please)

Filippo Santambrogio
(U. Lyon 1)

Abstract

When a parabolic PDE is a gradient flow in the Wasserstein space W_2 one can study it via the so-called JKO scheme. This is the case for instance of the linear Fokker-Planck equation. Some quantities such as suitable L^p, Sobolev, Lipschitz or Holder norms can be estimated along the iterations via various tools (five-gradients-inequality, flow interchange, Monge-Ampère equation...) studied in the last years. After recalling these results and the man techniques for the linear case, I will explain under which assumptions these results can be extended to the case where the potential energy is replaced with an interaction energy. This includes some non-trivial Sobolev estimates for the parabolic-elliptic Keller-Segel system obtained in a joint work with Di Marino some years ago.

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Lagrangian, Eulerian and Kantorovich formulations of multi-agent optimal control problems

Giuseppe Savaré
(Bocconi University)

Abstract

We discuss the natural Lagrangian and Eulerian formulations of multi-agent deterministic optimal control problems, analyzing their relations with a novel Kantorovich formulation. We exhibit some equivalence results among the various representations and compare the respective value functions, by combining techniques and ideas from optimal transport, control theory, Young measures and evolution equations in Banach spaces. We further exploit the connections among Lagrangian and Eulerian descriptions to derive consistency results as the number of particles/agents tends to infinity. (In collaboration with Giulia Cavagnari, Stefano Lisini and Carlo Orrieri)

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Gradient Flow Solutions For Porous Medium Equations with Nonlocal Lévy-type Pressure

Markus Schmidtchen
(U. Dresden)

Abstract

We study a porous medium-type equation whose pressure is given by a nonlocal Lévy operator associated to a symmetric jump Lévy kernel. The class of nonlocal operators under consideration appears as a generalisation of the classical fractional Laplace operator. For the class of Lévy-operators, we construct weak solutions using a minimising movements.

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New compactness estimates for aggregation-diffusion equations

Jakub Skrzeczkowski
(U. Oxford)

Abstract

I will present two results dealing with the passage to the limit in aggregation-diffusion equations where obtaining standard compactness estimates is difficult. The first result, obtained in collaboration with C. Elbar and B. Perthame, concerns the kinetic derivation of the degenerate Cahn-Hilliard equation from a certain nonlocal partial differential equation (PDE). The challenge here is that all necessary a priori estimates can only be obtained for the nonlocal quantities, providing almost no information about the limiting solution itself. We introduce a novel condition on the kernel that allows us to exploit the available nonlocal a priori estimates.

The second result, obtained in collaboration with J. A. Carrillo and Y. Salmaniw, concerns the existence (and uniqueness) of solutions to aggregation-diffusion equations where the kernel is only bounded and integrable, for instance, a characteristic function of a ball or a cube. Here, we take advantage of the gradient flow structure in a novel way, utilizing the dissipation of free energy and equiintegrability to control the gradient of the solution. This second work is particularly important in ecology, where the case of a characteristic function of a cube is widely used as a toy model to study the dynamics of populations.

Gradient flows for sampling and their deterministic interacting particle approximations

Dejan Slepcev
(Carnegie Mellon University)

Abstract

Motivated by the task of sampling measures in high dimensions we will discuss a number of gradient flows in the spaces of measures, including the Wasserstein gradient flows of Maximum Mean Discrepancy and Hellinger gradient flows of relative entropy, the Stein Variational Gradient Descent and a new projected dynamic gradient flows. For all the flows we will consider their deterministic interacting-particle approximations. The talk is highlight some of the properties of the flows and indicate their differences. In particular we will discuss how well can the interacting particles approximate the target measures.

The talk is based on joint works wit Anna Korba, Lantian Xu, Sangmin Park, Yulong Lu, and Lihan Wang.

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Cahn-Hillard and Keller-Segel systems as high-friction limits of gas dynamics

Agnieszka Świerczewska-Gwiazda
(University of Warsaw)

Abstract

Several recent studies considered the high-friction limit for systems arising in fluid mechanics. Following this approach, we rigorously derive the nonlocal Cahn-Hilliard equation as a limit of the nonlocal Euler-Korteweg equation using the relative entropy method. Applying the recent result about relations between non-local and local Cahn-Hilliard, we also derive rigorously the large-friction nonlocal- to-local limit. The result is formulated for dissipative measure-valued solutions of the nonlocal Euler-Korteweg equation which are known to exist on arbitrary intervals of time. This approach provides a new method to derive equations not enjoying classical solutions via relative entropy method by introducing the nonlocal effect in the fluid equation. During the talk I will also discuss the high-friction limit of the Euler-Poisson system.

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Large population limit of interacting particle systems for population dynamics

Oliver Tse
(TU Eindhoven)

Abstract

Spatial models for population dynamics have been pivotal in understanding how individuals and communities interact with their environments, shaping landscapes and influencing ecological and epidemiological processes. In this talk, I will discuss the derivation of effective nonlocal (possibly multi-component) systems of population dynamics from interacting particle systems on configuration space.

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Some recent results on nonlinear aggregation-diffusion equations with Riesz kernels

Bruno Volzone
(Politecnico di Milano)

Abstract

In this talk we will first give an overview about some results obtained in the papers [1]-[2] concerning the characterization of the stationary states for a nonlinear variant of the parabolic-elliptic Patlak-Keller-Segel of the form

$$\partial_t \rho = \Delta \rho^m + \nabla \cdot (\rho \nabla (W * \rho)), \tag{1}$$

being $W \in C^1(\mathbb{R}^d \setminus \{0\})$ a Riesz kernel aggregation, namely $W(x) = c_{d,s}|x|^{2s-d}$ for $s \in (0, d/2)$, in the assumptions of dominated diffusion, i.e. when *i.e.* for $m > 2 - (2s)/d$. In particular, all stationary states of the model are shown to be radially symmetric decreasing and uniquely identified with global minimizers of the associated free energy functionals. In the second part of the talk we will discuss the recent results established in the joint paper [3], in which an addition of a quadratic diffusion term in equation (1) produces a more precise competition with the aggregation term for small s, as they have the same scaling if $s = 0$. We characterize the asymptotic behavior of the stationary states behavior as s goes to zero. Finally, we establish the existence of gradient flow solutions to the evolution problem by applying the JKO scheme.

References

[1] J. A. Carrillo, F. Hoffmann, E. Mainini, B. Volzone. *Ground States in the Diffusion-Dominated Regime*, Calc. Var. Partial Differ. Equ. 57, No. 5, Paper No. 127, 28 p. (2018).

[2] H. Chan, M. González, Y. Huang, E. Mainini, B. Volzone. *Uniqueness of entire ground states for the fractional plasma problem.*, Calc. Var. Partial Differ. Equ. 59, No. 6, Paper No. 195, 41 p. (2020).

[3] Y. Huang, E. Mainini, J. L. Vázquez, B. Volzone. *Nonlinear aggregation-diffusion equations with Riesz potentials*, arXiv:2205.13520 [math.AP] (2022)

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Kac models with inclusion and exclusion

Bernt Wennberg
(Chalmers)

Abstract

We have previously analysed a Kac model with a hard core exclusion principle, where only particle configurations satisfying $|x_j - x_k| > \epsilon_n$ are allowed, with ϵ_n depending on the number of particles n. This mimics dynamics of Fermions in the quantum case, and raises interesting questions concering the limit as $n \to \infty$. In this talk I will discuss variants of this model, and present resent results concering a "Bosonic" version of the first model.

It is work in progress together with Eric Carlen

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The role of cross-degeneracies in reaction-diffusion driven structure evolution

Michael Winkler
(U. Padeborn)

Abstract

Simple models for nutrient-oriented bacterial migration are compared. In particular, the potential of certain cross-degenerate diffusion mechanisms to adequately describe experimentally observed phenomena related to emergence and stabilization of structures are discussed. Resulting mathematical challenges are described and possible approaches outlined, both at levels of basic existence theories and the stage of qualitative analysis.

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Suppression of chemotactic blow-up by active advection

Yao Yao
(N.U. Singapore)

Abstract

Chemotactic blow up in the context of the Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that when the Keller-Segel equation is coupled with passive advection, blow-up can be prevented if the flow possesses mixing or diffusion-enhancing properties, and its amplitude is sufficiently strong. In this talk, we consider the Keller-Segel equation coupled with an active advection, which is an incompressible flow obeying Darcy's law for incompressible porous media equation and driven by buoyancy force. We prove that in contrast with passive advection, this active advection coupling is capable of suppressing chemotactic blow up at arbitrary small coupling strength: namely, the system always has globally regular solutions. (Joint work with Zhongtian Hu and Alexander Kiselev).

Posters

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Weak-strong uniqueness and high-friction limit for Euler-Riesz systems

Nuno Alves
(University of Vienna)

Abstract

In this work, we employ the relative energy method to obtain stability results for Euler-Riesz systems. The first stability result establishes a weak-strong uniqueness principle, where dissipative weak solutions are compared with strong solutions. Then, in the high-friction limit, we demonstrate the convergence towards a gradient flow. In both cases, we assume that the initial data satisfies the conditions required for the existence of strong solutions. The main technical challenge in our analysis is addressed using a specific case of a Hardy-Littlewood-Sobolev inequality for Riesz potentials. This is a joint work with José A. Carrillo.

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Asymptotic Smoothing and Pullback Attractor for the KdV-Burgers Equation

Nesibe Ayhan
(University of Graz / Department of Mathematics and Scientific Computing)

Abstract

The long-time behavior of solutions to the nonautonomous KdV-Burgers equation on the real line $u_t + u_{xxx} + uu_x + \lambda u - \mu u_{xx} = g(x,t)$ is studied in the phase space $L^2(\mathbb{R})$. We first obtain the existence of a pullback attractor $\{A\mu(t)\}$ for each $\mu \geq 0$ in $L^2(\mathbb{R})$ and then prove an optimal asymptotic smoothing effect for the solution, i.e., the pullback attractor is in fact compact in $H^3(\mathbb{R})$. These results are proven by using the energy equation method together with a splitting of the solutions and exploiting the dispersive regularization property of this equation in appropriate Bourgain spaces. We also show the upper semi-continuity of the pullback attractors $\{A\mu\}_{\mu\in[0,1]}$ at $\mu = 0$ and finally show that the pullback attractor $\{A\mu(t)\}$ has a finite fractal dimension.

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Coupled Gradient Flows for Strategic Non-Local Distribution Shift

Lauren Conger
(Caltech)

Abstract

We propose a novel framework for analyzing the dynamics of distribution shift in real-world systems that captures the feedback loop between learning algorithms and the distributions on which they are deployed. The coupled partial differential equation model captures fine-grained changes in the distribution over time by accounting for complex dynamics that arise due to strategic responses to algorithmic decision-making, non-local endogenous population interactions, and other exogenous sources of distribution shift. We consider two common settings in machine learning: cooperative settings with information asymmetries, and competitive settings where a learner faces strategic users. For both of these settings, when the algorithm retrains via gradient descent, we prove asymptotic convergence of the retraining procedure to a steady-state, both in finite and in infinite dimensions, obtaining explicit rates in terms of the model parameters. To do so we derive new results on the convergence of coupled PDEs that extends what is known on multi-species systems. Empirically, we show that our approach captures well-documented forms of distribution shifts like polarization and disparate impacts that simpler models cannot capture.

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Convergence of the Cucker-Smale model via a probabilistic method

Adrien Cotil
(INRAE Montpellier)

Abstract

In this poster, we discuss the flocking phenomenon for the Cucker-Smale model in continuous time on a general oriented and weighted graph with a general communication function. We present a new approach to studying this problem, based on a probabilistic interpretation of the solutions, using the duality between the Cucker-Smale and Kolmogorov equations. Moreover, we highlight how flocking is related to the convergence in total variation of a certain Markov jump process. Using Doeblin-type techniques, we refine previous results on the minimal case where the graph admits a unique closed communication class.

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On an inhomogeneous coagulation model with a differential sedimentation kernel

Iulia Cristian
(University of Bonn)

Abstract

We study an inhomogeneous kinetic equation that contains a transport term in the spatial variable modeling the sedimentation of clusters. We prove local existence of mass conserving solutions for a class of coagulation kernels for which in the space homogeneous case instantaneous gelation (i.e., instantaneous loss of mass) occurs.

Our result holds true in particular for two types of kernels which are relevant in applications. These include sum-type kernels of homogeneity greater than one and the so-called differential sedimentation kernel which is essentially a sum-type kernel with homogeneity greater than one that in addition vanishes on the diagonal.

For the first type of kernel mentioned, it is established that solutions do not exist in the space homogeneous case, while the latter type has been used to describe rain initiation times or the behavior of air bubbles in water.

This is based on a joint work with B. Niethammer (U. Bonn) and J. J. L. Velázquez (U. Bonn).

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Bounds and Limiting Minimizers for a Family of Interaction Energies

Cameron Davies
(University of Toronto)

Abstract

In the study of attractive-repulsive particle dynamics, many researchers have studied power-law interaction energies of the form

$$\mathcal{E}_{a,b}[\mu] = \iint_{\mathbb{R}^n \times \mathbb{R}^n} |x-y|^a/a + |x-y|^b/b d\mu(x) d\mu(y).$$

We introduce a new perspective, and study the infimal energy

$$E_{a,b} = \inf_{\mu} \mathcal{E}_{a,b}[\mu]$$

as a function on the parameter space $a > b > 0$. In this poster, I will explain this perspective, and some of its implications, including an a priori lower bound on the energy.

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From cell population models to Hele-Shaw type dynamics: the rate of convergence

Tomasz Dębiec
(University of Warsaw)

Abstract

The incompressible limit of nonlinear diffusion equations of porous medium type has attracted a lot of attention in recent years, due to its ability to link the weak formulation of cell population models to free boundary problems of Hele–Shaw type. Although a vast literature is available on this singular limit, little was known on the convergence rate of the solutions. In this work, we compute the convergence rate in a negative Sobolev norm and, upon interpolating with BV-uniform bounds, we deduce a convergence rate in a range of Lebesgue spaces.

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Mean field equation of Pulse-coupled oscillators: Reformulation and Consequences

Xu'an Dou
(Peking University)

Abstract

Pulse-coupled oscillators describe particle systems where the interaction is in a particular pulsatile manner. They have many applications in science and engineering, including fireflies, neurons and wireless networks. Yet their unique singular coupling mechanism renders them challenging to analyze. At the mean-field level, a partial differential equation for pulse-coupled oscillators, which characterizes the nonlinear interaction at a continuous level, has been derived formally in physics literature 30 years ago. However, it still lacks a systematic mathematical treatment.

In this work, we study the mean-field equation from a novel reformulation, which reveals several hidden structures of the equation, and facilitates the study on its basic properties, including well-posedness, blow-ups and long time convergence to the steady state. Joint work with José Carrillo, Pierre Roux and Zhennan Zhou.

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Competing effects in fourth-order aggregation-diffusion equations

Alejandro Fernández-Jiménez
(University of Oxford)

Abstract

On a recent work from Falcó, Baker, and Carrillo, they describe a new local model for cell-cell adhesion for two species,

$$\begin{cases} \partial_t \rho &= -\mathrm{div}\left(\rho \nabla\left(\kappa \Delta \rho + \alpha \Delta \eta + \beta \rho + \omega \eta\right)\right), \\ \partial_t \eta &= -\mathrm{div}\left(\eta \nabla\left(\alpha \Delta \rho + \Delta \eta + \omega \rho + \eta\right)\right). \end{cases}$$

Here, we focus on the one species model and study the problem,

$$\partial_t \rho = -\,\mathrm{div}(\rho \nabla(\Delta \rho)) - \Delta \rho^m.$$

We are able to express this equation with a gradient flow formulation. Therefore, using the JKO scheme we can prove existence of (very) weak solutions for $1 \leq m < 2 + \frac{2}{d}$ (where d is the dimension). With the same technique we can extend the result to the system.

The poster presents joint work with J.A. Carrillo, A. Esposito, and C. Falcó.

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Estimating the distribution of fitness effects in a structured population of cells

Guillaume Garnier
(Inria, Sorbonne Université)

Abstract

All cells evolve through mutation. These mutations can have a beneficial or deleterious effect on cells. In this work, we model cell evolution using a mutation equation, close to the fragmentation growth equation, and we look for conditions on the mutation kernel that would explain why the population eventually reaches a stationary state.

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Uniqueness of weak solutions to the Maxwell-Stefan cross-diffusion system

Stefanos Georgiadis
(King Abdullah University of Science and Technology and TU Wien)

Abstract

Cross-diffusion systems are strongly coupled parabolic systems describing phenomena in which multiple species diffuse and interact with one another, e.g. in fluid mechanics or population dynamics. Although many methods have been developed to study existence of weak solutions to such systems, uniqueness is in general an open problem. To this degree, we study a particular cross-diffusion system, known as the Maxwell-Stefan system which describes diffusive phenomena in a multicomponent system of gases. We employ renormalized solutions and give conditions under which such solutions are unique. We, then, study the relation between weak and renormalized solutions, allowing us to identify conditions that guarantee uniqueness of weak solutions. The proof is based on an identity for the evolution of the symmetrized relative entropy. Using the method of doubling the variables we derive the identity for two renormalized solutions and use information on the spectrum of the Maxwell-Stefan matrix to estimate the symmetrized relative entropy and show uniqueness.

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Interacting particle approximation of cross-diffusion systems

Shuchen Guo
(University of Oxford)

Abstract

We derive a class of multi-species cross-diffusion systems from stochastic interacting particles. We prove existence of weak solutions of the limiting cross-diffusion system as well as the propagation of chaos by means of nonlocal approximation of the nonlinear diffusion terms, coupling methods and compactness arguments. We show that these equations capture the macroscopic behavior of the interacting particle system if the localisation parameter is chosen logarithmically with respect to the number of particles.

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Some progress in incompressible limit of chemotaxis models

Qingyou He
(LCQB, Sorbonne University)

Abstract

We consider chemotaxi modeling the competition effect between aggregation and diffusion, and study its incompressible limit of chemotaxi models even with growth term. The limiting problem is called as the Hele-Shaw problem, which builds a connection between the compressible (porous medium type) model and the incompressible (Here-Shaw) model. In this poster, I will give some recent results on the incompressible limits of chemotaxi jointed with prof. Benoit Perthame and prof Hai-Liang Li.

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Graph-based nonlocal gradient systems and their local limits

Georg Heinze
(University of Augsburg)

Abstract

I will present a model for systems of multiple nonlocally interacting species on a large class of graphs, ranging from finite graphs to continuous graphs. The model is based on the theory of metric gradient flows and provides a unified upwind-based framework, including not only concave mobilities, but also non-1-homogeneous kinetic relations. In addition to proving existence of solutions by variational methods, we numerically study the behaviour of the arising dynamics. Furthermore, we prove that such models approximate an Otto-type gradient structure inducing a system of nonlocal interaction equations in Euclidean space by means of EDP convergence. What is more, a similar approximation is possible even after introducing tensor-valued anisotropies into the limiting geometry. This poster is based on joint works with Antonio Esposito, André Schlichting, Jan-Frederik Pietschmann and Markus Schmidtchen.

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Existence of global weak solutions to a Cahn-Hilliard cross-diffusion system in lymphangiogenesis

Yue Li
(Vienna University of Technology)

Abstract

The global-in-time existence of weak solutions to a degenerate Cahn-Hilliard cross-diffusion system with singular potential in a bounded domain with no-flux boundary conditions is proved. The model consists of two coupled parabolic fourth-order partial differential equations and describes the evolution of the fiber phase volume fraction and the solute concentration, modeling the pre-patterning of lymphatic vessel morphology. The fiber phase fraction satisfies the segregation property if this holds initially. The existence proof is based on a three-level approximation scheme and a prior estimates coming from the energy and entropy inequalities. While the free energy is nonincreasing in time, the entropy is only bounded because of the cross-diffusion coupling.

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A model for non-instantaneous collisions with alignment

Carmela Moschella
(University of Vienna)

Abstract

In this poster I am going to consider a Boltzmann-type equation for the description of a collision dynamic which is not instantaneous. This new class of kinetic equations has been introduced by Kanzler, Schmeiser, and Tora to model ensembles of living agents, where the changes of state are the result of complicated internal processes, and not simple mechanical interactions. We extend their work introducing a first-order approximation to the instantaneous equation, where non-binary collisions are included. This is motivated by the fact that during an extended collision period there is a positive probability that a colliding pair is joined by additional particles. The interaction kernel is of alignment type, where the states of the particles approach each other. For this spatially homogeneous approximation, we check that the formal properties of the system are kept. Furthermore, existence and uniqueness of solutions are examined.

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Drift-Diffusion for Memristors Coupled to a Network

Tuan Tung Nguyen
(TU Wien)

Abstract

The memristor is a novel semiconductor device equipped with a memory due to the change of its electrical resistance. In this way, it may mimic the behavior of a synapse in the human brain. We analyze a model of memristors that are coupled with an electric network consisting of various electronic devices. While nonlinear drift-diffusion equations describe the motion of charged particles within the memristor, the node potentials in the network follow Kirchhoff's laws, i.e. ordinary differential equations and algebraic constraints. The coupling between them results in a system of partial differential-algebraic equations. The existence analysis employs entropy methods crucially.

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Relative entropy estimates for convolution interaction forces

Paul Nikolaev
(University of Mannheim)

Abstract

Quantitative estimates are derived, on the whole space, for the relative entropy between the joint law of random interacting particles and the tensorized law at the limiting systeme. The developed method combines the relative entropy method under the moderated interaction scaling introduced by Oeschläger, and the propagation of chaos in probability. The result includes the case that the interaction force does not need to be a potential field. Furthermore, if the interaction force is a potential field, with a convolutional structure, then the developed estimate also provides the modulated energy estimates. Moreover, we demonstrate propagation of chaos for large stochastic systems of interacting particles and discuss possible applications to various interacting particle systems, including the Coulomb interaction case.

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Geodesics and Acceleration-free Curves on the Wasserstein Space

Guy Parker
(Durham University)

Abstract

We show that a differentiable function on the 2-Wasserstein space is geodesically convex if and only if it is also convex along a larger class of curves which we call 'acceleration-free'. In particular, the set of acceleration-free curves includes all generalised geodesics. We also explore some properties of acceleration-free curves.

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Measure-based approach to mesoscopic modeling of optimal transportation networks

Simone Portaro
(KAUST)

Abstract

We propose a mesoscopic modeling framework for optimal transportation networks with biological applications. The network is described in terms of a joint probability measure on the phase space of tensor-valued conductivity and position in physical space. The energy expenditure of the network is given by a functional consisting of a pumping (kinetic) and metabolic power-law term, constrained by a Poisson equation accounting for local mass conservation. We establish convexity and lower semicontinuity of the functional on appropriate sets. We then derive its gradient flow with respect to the 2-Wasserstein topology on the space of probability measures, which leads to a transport equation, coupled to the Poisson equation. To lessen the mathematical complexity of the problem, we derive a reduced Wasserstein gradient flow, taken with respect to the tensor-valued conductivity variable only. We then construct equilibrium measures of the resulting PDE system. Finally, we derive the gradient flow of the constrained energy functional with respect to the Fisher-Rao (or Hellinger-Kakutani) metric, which gives a reaction-type PDE. We calculate its equilibrium states, represented by measures concentrated on a hypersurface in the phase space.

Asymptotic Behavior of a Nonlocal Fokker-Planck Equation

Niccolò Tassi
(University of Granada)

Abstract

We study a nonlocal approximation of the Fokker-Planck equation in which we can estimate the speed of convergence to equilibrium independent of the non-locality of the equation. This uniform estimate cannot be easily obtained with standard inequalities or entropy methods, but can be obtained through the use of Harris's theorem, finding interesting links to quantitative versions of the central limit theorem in probability. The associated equilibrium also has some interesting tail and regularity properties, which we also study. This is a joint work with José Cañizo of University of Granada.

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Long time behavior of Fokker-Planck equations

Gayrat Toshpulatov
(Vienna University of Technology (TU Wien))

Abstract

We consider the large-time behavior of Fokker-Planck type equations (advection-diffusion equations). These equations describe the evolution of a cloud of particles undergoing diffusion. There is a unique equilibrium state for these equations, and solutions are supposed to converge to it as time goes to infinity. When these equations are non-degenerate (i.e., the diffusion matrix is positive definite), their large-time behavior is comprehensively studied, for example, one can use the well-known Bakry-Emery theory. However, degenerate Fokker-Planck equations are challenging and there are many open problems. We develop a method for proving the convergence of solutions to the equilibrium state for degenerate Fokker-Planck equations, and, in particular, we apply this method to the kinetic Fokker-Planck equation and the Vlasov-Poisson-Fokker-Planck system. Our method lets us obtain explicit and constructive estimates on the rate of convergence and it is based on construction of appropriate Lyapunov functionals.

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Analysis of mass-controlled reaction-diffusion systems with nonlinearities having critical growth rates

Juan Yang
(University of Graz/ Department of Mathematics and Scientific Computing & Lanzhou University / School of Mathematics and Statistics)

Abstract

We analyze semilinear reaction–diffusion systems that are mass controlled, and have nonlinearities that satisfy critical growth rates. The systems under consideration are only assumed to satisfy natural assumptions, namely the preservation of non-negativity and a control of the total mass. It is proved in dimension one that if nonlinearities have (slightly super-) cubic growth rates then the system has a unique global classical solutions. Moreover, in the case of mass dissipation, the solution is bounded uniformly in time in sup-norm. One key idea in the proof is the Hölder continuity of gradient of solutions to parabolic equation with possibly discontinuous diffusion coefficients and low regular forcing terms. When the system possesses additionally an entropy inequality, the global existence and boundedness of a unique classical solution is shown for nonlinearities satisfying a cubic intermediate sum condition, which is a significant generalization of cubic growth rates. The main idea in this case is to combine a modified Gagliardo-Nirenberg inequality and the newly developed L^p-energy method. This idea also allows us to deal with the case of discontinuous diffusion coefficients in higher dimensions, which has only recently been touched in the context of mass-controlled reaction-diffusion systems.

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A Linear Complexity Solver for Aggregation PDE

Umberto Zerbinati
(University of Oxford)

Abstract

Aggregation partial differential equations (PDEs), arise in various scientific and engineering applications, such as modeling collective behavior in biological systems, pedestrain dynamics and granular flow. We will explore how traditional convolution based scheme can be computationally costly for large-scale problems. We will also introduce a novel approach for Hilbert-Schmidt aggregation PDE based on discrete Green's function in combination with multigrid solvers. By leveraging the inherent structure of aggregation PDEs, we aim to develop efficient algorithms that can significantly reduce computational overhead while maintaining accuracy. In particular, the solver here presented is a highly scalable linear solver with cross architecture support.

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Patterns in the Keller-Segel system with density cut-off

Mingyue Zhang
(Sorbonne University)

Abstract

The Patlak-Keller-Segel system with logistic sensitivity has been widely advocated as a model which avoids over-crowding and generates complex patterns. In this presentation, we will focus on the case of a nonlinear diffusion of porous medium type with exponent m and show that the pattern formation ability. The sensitivity and the conserved total mass also plays a crucial role. We will focus specifically on the conditions for long term convergence to the constant solution, uniqueness of the steady state and on the contrary, existence of increasing steady solutions in dimension one.