
Geometric Sciences in Action: from Geometric Statistics to Shape Analysis
Centre International de Rencontres Mathématiques, Lumini, Marseille France
27 – 31 May, 2024

Organizing Committee:

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Conference Schedule

Monday, May 27, 2024

- **08:30 – 09:00** The organizers
Welcome address and information about the conference

Geometric Statistics and Stochastic Geometric Mechanics

- **09:00 – 09:30** Erlend Grong, University of Bergen, Norway
Score matching and sub-Riemannian bridges
- **09:45 – 10:15** Huiling Le, University of Nottingham, UK.
Stein's method on certain stratified spaces.
- **10:30 – 11:00** Coffee Break
- **11:00 – 11:30** Hans Munthe-Kaas, UiT The Norwegian Arctic University, Norway
Connection algebras: between algebra, geometry and computations.
- **11:45 – 12:15** Karen Habermann, University of Warwick, UK
Long-time existence of Brownian motion on configurations of two landmarks
- **12:30 – 15:00** Lunch Break
- **15:00 – 15:30** Victor Panaretos, EPFL Lausanne, CH
Geometrical Statistics in the Bures-Wasserstein Space
- **15:45 – 16:15** Stephan Huckemann, Georg-August-Universität Göttingen, DE
A Lower Bound for Estimating Fréchet Means
- **16:30 – 17:00** Coffee Break

Software Presentations I

- **17:00 – 17:30** Luis Perreira, Università degli Studi di Milano-Bicocca, IT & UC Santa Barbara, USA
Geomstat, a Python package for Riemannian geometry in machine learning
- **17:30 – 18:00** Mathieu Carriere, Université Côte d'Azur and Inria, FR
Topological Data Analysis with the Gudhi library

Tuesday, May 28, 2024

Diffeomorphic Methods and Applications to Medical Imaging and Biology

- **09:00 – 09:30** Laurent Younes, Johns Hopkins University, USA
A scale space framework for diffeomorphic shape analysis.
- **09:45 – 10:15** Barbara Gris, Sorbonne Université, FR
TBA
- **10:30 – 11:00** Coffee Break
- **11:00 – 11:30** Nicolas Charon, University of Houston, USA
Unbalanced optimal transport under path constraints in measure space
- **11:45 – 12:15** Francois-Xavier Vialard, Univ. Gustave Eiffel, FR
Synthetic nonnegative cross-curvature lifts to the Wasserstein space.

- **12:30 – 15:00** Lunch Break
- **15:00 – 15:30** Boris Khesin, University of Toronto, CA
The Madelung transform as a momentum map.
- **15:45 – 16:15** Alice Le Brigant, University Paris 1, FR
The L^p -Fisher-Rao metric and α -connections.
- **16:30 – 17:00** Coffee Break
- **17:00 – 19:00** Poster Competition

Wednesday, May 29, 2024

Curves and Surfaces I

- **09:00 – 09:30** Eric Klassen, Florida State University, USA
Elastic Metrics on Spaces of Euclidean Curves: Theory and Algorithms.
- **09:45 – 10:15** Barbara Tumpach, University of Lille, FR
Totally geodesic submanifolds in the manifold SPD of symmetric positive-definite real matrices.
- **10:30 – 11:00** Coffee Break
- **11:00 – 11:30** Peter Michor, University of Vienna, AT
Symplectic Structures on the space of space curves.
- **11:45 – 12:15** Stephen C. Preston, Brooklyn College, USA
Isometric immersions and the waving of flags
- **12:30 – 15:00** Lunch Break
- **15:00 – 19:00** Social Afternoon: Hiking in Luminy

Thursday, May 30, 2024

Graphs, Networks and Stratified Spaces

- **09:00 – 09:30** Facundo Memoli, Ohio State University, USA
The exact determination of Gromov-type distances between spheres.
- **09:45 – 10:15** Tom Needham, Florida State University, USA
Geometry and Topology of Spaces of Structured Matrices.
- **10:30 – 11:00** Coffee Break
- **11:00 – 11:30** Blanche Buet, Université Paris Saclay, FR
Flagfolds: multi-dimensional varifolds to handle discrete surfaces
- **11:45 – 12:15** Karl-Theodor Sturm, University of Bonn, DE.
Synthetic bounds for sectional and Ricci curvature on metric spaces
- **12:30 – 15:00** Lunch Break
- **15:00 – 15:30** Ezra Miller, Duke, USA.
What is a Gaussian on a singular space?
- **15:45 – 16:15** Victor-Emmanuel Brunel, Inria, FR
Estimation of generalized barycenters in metric spaces.
- **16:30 – 17:00** Coffee Break

Software Presentations II

- **17:00 – 17:30** Emmanuel Hartman, Florida State University, USA
Elastic shape analysis of surfaces with second-order Sobolev metrics.
- **17:30 – 18:00** PyKeops, Jean Feydy, Inria, FR
KeOps: a transparent accelerator for geometric programs.
- **18:00 – 18:30** Gefan Yang
JaxGeometry

Friday, May 31, 2024

Probabilities & Shapes

- **09:00 – 09:40** Kathrin Welker, University of Freiberg, DE
Optimization of piecewise-smooth shapes.
- **09:50 – 10:30** TBA
- **10:30 – 11:00** Coffee Break
- **11:00 – 11:40** Irene Kaltenmark, Université Paris Cité, FR
Curves and surfaces. Partial matching in the space of varifolds.
- **11:50 – 12:30** Anuj Srivastava, Florida State University, USA
On Statistical Inferences Involving Shape Data.
- **12:30 – 14:00** Lunch Break
- **14:00 – 14:30** Josua Sassen, ENS Paris-Saclay, FR
Low-dimensional Product Submanifolds of the Space of Discrete Shells.
- **14:30 – 15:00** Anna Calissano, Imperial College, UK
Understanding Cardiac Fibrosis with Random Fields.
- **15:00 – 15:30** Coffee Break
- **15:30 – 16:00** James Benn, Inria, FR
The Geometry of Right-Invariant Metrics on Lie Groups.
- **16:00 – 16:30** Libby Baker, University of Copenhagen, DK
TBA

Book of Abstracts

James Benn

Title: The Geometry of Right-Invariant Metrics on Lie Groups

Abstract: This talk reviews the geometry of a right invariant metric on a (possibly infinite dimensional) Lie Group. New results on the geodesic equations and Riemannian exponential map will be presented, along with some applications to conjugate points and geodesic stability. It is known that the classical BCH formula for diffeomorphism groups does not hold due to the local surjectivity failure of the intrinsic group exponential map; this talk will close with a discussion how our results on Riemannian exponential maps of right-invariant Sobolev type metrics could be used to obtain a Riemannian BCH-type formula for the diffeomorphism group.

Victor-Emmanuel Brunel

Title: Estimation of generalized barycenters in metric spaces

Abstract: TBA

Blanche Buet

Title: Flagfolds: multi-dimensional varifolds to handle discrete surfaces

Abstract: We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well. While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We performed numerical computations of mean curvature and Gaussian curvature on point clouds in \mathbb{R}^3 to illustrate this approach. Though flexible, varifolds require the knowledge of the dimension of the shape to be considered. By interpreting the product of the Principal Component Analysis, that is the covariance matrix, as a sequence of nested subspaces naturally coming with weights according to the level of approximation they provide, we are able to embed all d-dimensional Grassmannians into a stratified space of covariance matrices. Building upon the proposed embedding of Grassmannians into the space of covariance matrices, we generalize the concept of varifolds to what we call flagfolds in order to model multi-dimensional shapes.

Anna Calissano

Title: Understanding Cardiac Fibrosis with Random Fields

Abstract: TBA

Mathieu Carrière

Title: Topological Data Analysis with the Gudhi library

Abstract: TBA

Nicolas Charon

Title: Unbalanced optimal transport under path constraints in measure space

Abstract: We will present a variation of the unbalanced optimal transport model and Wasserstein Fisher-Rao metric on positive measures, in which one imposes additional affine integral equality constraints. This is motivated by multiple examples from mathematics and applied mathematics that naturally involve comparing and interpolating between two measures in particular subspaces or in which one enforces some constraints on the interpolating path itself. Building from the dynamic formulation of the Wasserstein Fisher-Rao metric, we introduce a class of constrained problems where the interpolating measure at each time must satisfy a given stationary or time-dependent constraint in measure space. We then specifically derive general conditions under which the existence of minimizing paths can be guaranteed, and then examine some of the properties of the resulting models and the metrics that are induced on measures. We will further hint at the potential of this approach in various specific situations such as the comparison of measures with prescribed moments, the unbalanced optimal transport under global mass evolution or obstacle constraints, and emphasize some connections with the construction of Riemannian metrics on the space of all convex shapes in an Euclidean space. We shall conclude with a few remaining unsolved/open questions.

Jean Feydy

Title: KeOps: a transparent accelerator for geometric programs

Abstract: TBA

Barbara Gris

Title: TBA

Abstract: TBA

Erlend Grong

Title: Score matching and sub-Riemannian bridges

Abstract: We discuss how to simulate bridge processes by conditioning a stochastic process on a manifold whose generator is a hypo-elliptic operator. This operator is, up to a drift-term, the sub-Laplacian of a bracket-generating sub-Riemannian structure, meaning in particular that it has positive smooth density everywhere. The logarithmic gradient of this density is called the score, and we show that it is needed to describe the generator of the bridge process. We therefore discuss several methods for how we can estimate the score using a neural network, with examples.

The results are from a joint work with Stefan Sommer (Copenhagen) and Karen Habermann (Warwick).

Karen Habermann

Title: Long-time existence of Brownian motion on configurations of two landmarks

Abstract: In computational anatomy and, more generally, shape analysis, the Large Deformation Diffeomorphic Metric Mapping framework models shape variations as diffeomorphic deformations. An important shape space within this framework is the space consisting of shapes characterised by $n \geq 2$ distinct landmark points in \mathbb{R}^d . In diffeomorphic landmark matching, two landmark configurations are compared by solving an optimization problem which minimizes a suitable energy functional associated with flows of compactly supported diffeomorphisms transforming one landmark configuration into the other one. The landmark manifold Q of n distinct landmark points in \mathbb{R}^d can be endowed with a Riemannian metric g such that the above optimization problem is equivalent to the geodesic boundary value problem for g on Q . Despite its importance for modeling stochastic shape evolutions, no general result concerning long-time existence of Brownian motion on the Riemannian manifold (Q, g) is known. I will present joint work with Philipp Harms and Stefan Sommer on first progress in this direction which provides a full characterization of long-time existence of Brownian motion for configurations of exactly two landmarks, governed by a radial kernel.

Emmanuel Hartmann

Title: Elastic shape analysis of surfaces with second-order Sobolev metrics

Abstract: We present a set of numerical methods for Riemannian shape analysis of 3D surfaces within the setting of invariant (elastic) second-order Sobolev metrics. More specifically, we address the computation of geodesics and geodesic distances between unregistered surfaces represented as 3D meshes with potentially varying sampling or mesh structures. Building on this, we present tools for the statistical shape analysis of sets of surfaces, including methods for estimating Karcher means and performing tangent PCA on shape populations, and for computing parallel transport along paths of surfaces. The numerical framework is open access available at https://github.com/emmanuel-hartman/H2_SurfaceMatch.

Irene Kaltenmark

Title: Curves and surfaces. Partial matching in the space of varifolds.

Abstract: The matching of analogous shapes is a central problem in computational anatomy. However, inter-individual variability, pathological anomalies or acquisition methods sometimes challenge the assumption of global homology between shapes. In this talk, I will present an asymmetric data attachment term characterizing the inclusion of one shape in another. This term is based on projection on the nearest neighbor with respect to the metrics of varifold spaces. Varifolds are representations of geometric objects, including curves and surfaces. Their specificity is to take into account the tangent spaces of these objects and to be robust to the choice of parametrization. This new data attachment term extends the scope of application of the pre-existing methods of matching by large diffeomorphic deformations (LDDMM). The partial registration is indeed induced by a diffeomorphic deformation of the source shape. The anatomical (topological) characteristics of this shape are thus preserved. This is a joint work with Pierre-Louis Antonsanti and Joan Glaunès.

Boris Khesin

Title: The Madelung transform as a momentum map

Abstract: We discuss various ramifications of D.Fusca's observation that the inverse of the Madelung transform between compressible fluids and wave functions can be understood as a momentum map. This is the momentum map for the group of diffeomorphisms extended by smooth functions which is acting on wave functions regarded as half-densities.

Eric Klassen

Title: Elastic Metrics on Spaces of Euclidean Curves: Theory and Algorithms

Abstract: This talk is concerned with shape analysis of curves in Euclidean space. A one-parameter family of first-order Sobolev metrics on the shape space of immersed Euclidean curves was introduced in 2007 by Mio et al. Intuitively, the parameter gives the relative penalty on bending as opposed to stretching while deforming curves. We refer to this family of metrics as "elastic metrics". The elastic metrics are defined first on the space of parameterized curves; they are then shown to be parameterization invariant so that they descend to metrics on the shape space. In order to apply these metrics to data analysis, it is necessary to compute geodesics, first in the space of parameterized curves and then (by optimally registering two curves) in the shape space. For one member of this family (in the case of planar curves), Younes et al., in 2008, introduced a simplifying transformation that locally flattened the space of parameterized curves and thereby made it easy to compute geodesics. In 2010, Srivastava et al. introduced a similar simplifying transformation for a different member of the family of elastic metrics. This metric has come to be called the SRVF (square root velocity function) metric. It extends to all absolutely continuous curves in R^n for all $n > 0$. This transformation completely flattens the space of parameterized curves, turning it into a Hilbert space, in which the geodesics are straight lines and, hence, trivial to compute. In order to compute geodesics in shape space, the optimal registration of two curves (with respect to the SRVF) was usually approximated using dynamic programming. In 2015, Lahiri et al. developed an algorithm that yielded the precise optimal registration between PL curves. This is useful, since absolutely continuous curves can be approximated by PL curves.

In this talk, we show that all of the algorithms described above for the SRVF metric can be extended to every member of the family of elastic metrics. To be precise, every elastic metric can be extended to the space of all absolutely continuous curves in R^n , geodesics between parameterized curves can be computed quickly and precisely, optimal registration can be approximated by dynamic programming, and the precise algorithm of Lahiri et al. can be extended to all elastic metrics. When analyzing data consisting of curves, this provides the freedom to choose whichever elastic metric is the most useful for the given data, instead of restricting oneself to the SRVF.

Stephan Huckeman

Title: A Lower Bound for Estimating Fréchet Means

Abstract: Fréchet means are generalizations of the Euclidean expected value and are hence among the most popular nonparametric statistics for non-Euclidean data. In fact the umbrella of generalized Fréchet means nicely encompasses various other non-Euclidean statistics, for instance principal components geodesic, principle nested spheres, principle flows, barycentric subspaces and entire flags composed of these. They can suffer, however, from unprecedented non-Euclidean behavior, like smeariness and stickiness, which may manifest even for considerable high sample sizes. Here we focus on the effect of smeariness, namely how closeness of distributions with unique Fréchet means near distributions with nonunique means affects the estimation of the former. It turns out that, independent of sample size, it is not possible to uniformly estimate Fréchet means below a precision determined by the diameter of the set of nonunique Fréchet means nearby". Illustrating the relevance of our lower bound, examples of extrinsic, intrinsic, Procrustes, diffusion and Wasserstein means showcase either deteriorating constants or slow convergence rates of empirical Fréchet means for samples near the regime of nonunique means. This is joint work with Shayan Hundrieser and Benjamin Eltzner

Huiling Le

Title: Stein's method on certain stratified spaces

Abstract: TBA

Facundo Mémoli

Title: The exact determination of Gromov-type distances between spheres.

Abstract: TBA

Peter Michor

Title: Symplectic Structures on the space of space curves

Abstract: For $c \in \text{Imm}(S^1, \mathbb{R}^3)$ the 2-form

$$\Omega_c^{MW}(h, k) = \int_{S^1} \det(D_s c, h, k) ds,$$

where $ds = |c'(\theta)|d\theta$ and $D_s = \frac{1}{|c'(\theta)|} \partial_\theta$, induces the Marsden-Weinstein symplectic structure¹ on the shape space $\text{Imm}(S^1, \mathbb{R}^3) / \text{Diff}(S^1)$, corresponding to a Kähler structure. The Hamiltonian flow for the length functional is the binormal flow. In this talk, I will present other natural symplectic structures related to this one. Based on collaboration with Martin Bauer and Sadashige Ishida.

Ezra Miller

Title: What is a Gaussian on a singular space?

Abstract: TBA

Hans Munthe-Kaas

Title: Connection algebras: between algebra, geometry and computations.

Abstract: Butcher's B-series have been a central tool in numerical analysis for over 50 years. More recently they have been generalised to Lie-Butcher series and related series for analysing flows on Lie groups, homogeneous manifolds, symmetric spaces and other geometries. Such series are becoming increasingly important in many application areas. Their algebraic properties are intimately related to the geometry of the domain, through the algebras of the connection in the particular geometry. The term "Connection algebras" cover the special cases of pre-Lie algebras (B-series) for Euclidean geometry, post-Lie algebras for Lie groups, Lie-admissible triple algebras for symmetric spaces, as well as the case of general (non-invariant connections), relating to the work of A.V. Gavrillov.

Tom Needham

Title: Geometry and Topology of Spaces of Structured Matrices

Abstract: A finite unit norm tight frame (FUNTF) is a spanning set of unit vectors in a finite-dimensional Hilbert space such that the spectrum of singular values of an associated operator is constant. In signal processing applications, it is desirable to use FUNTFs to encode signals, as such representations are proven to be optimally robust to noise. This naturally gives rise to questions about the geometry and topology of the space of FUNTFs. For example, the conjecture that every space of FUNTFs is connected was open for 15 years, and slight variants of this problem still remain open. I will discuss recent work with Clayton Shonkwiler, where we answer several questions about random matrix theory and optimization in spaces of structured matrices, using tools from symplectic geometry and geometric invariant theory.

Victor Panaretos

Title: Geometrical Statistics in the Bures-Wasserstein Space

Abstract: TBA

Luis Peirera

Title: the Geomstats library

Abstract: TBA

Stephen C. Preston

Title: Isometric immersions and the waving of flags

Abstract: A physical flag can be modeled geometrically as an isometric immersion of a rectangle into space, with one edge fixed along the flagpole. Its motion, in the absence of gravity and wind, can be modeled as a

¹Marsden, J., and Weinstein, A. Coadjoint orbits, vortices, and Clebsch variables for incompressible fluids. *Physica D: Nonlinear Phenomena* 7, 1 (1983), 305-323.

geodesic in the space of all isometric immersions, where the Riemannian metric is inherited from the kinetic energy on the much larger space of all immersions. In this talk I will show how to derive the geodesic equation, which turns out to be a highly nonlinear, nonlocal coupled system of two wave equations in one space variable, with tension determined by solving an ODE system.

This is joint work with Martin Bauer and Jakob Moeller-Andersen.

Josua Sassen

Title: Low-dimensional Product Submanifolds of the Space of Discrete Shells

Abstract: TBA

Karl-Theodor Sturm

Title: Synthetic bounds for sectional and Ricci curvature on metric spaces

Abstract: TBA

Anuj Srivastava

Title: On Statistical Inferences Involving Shape Data

Abstract: In this talk, I will cover problems of statistical modeling or inferences involving the shapes of some functional objects. These problems of interest include: (1) shape-constrained density or curve estimation, (2) shape regression models, (3) statistical models for shape sequences, (4) statistical modeling of shape graphs, and (5) learning pose manifolds of 3D objects.

Barbara Tumpach

Title: Totally geodesic submanifolds in the manifold SPD of symmetric positive-definite real matrices

Abstract: We present necessary and sufficient conditions for a submanifold $\exp(E)$ of the manifold of symmetric positive-definite $n \times n$ real matrices $\text{SPD}(n)$ to be totally geodesic for the affine Riemannian metric. A non-linear projection on a totally geodesic submanifold $\exp(E)$ is defined. Fiber bundle decompositions of $\text{SDP}(n)$ follow, as well as corresponding decompositions of the general linear group $\text{GL}(n)$. If time permits, extensions of this work to other homogeneous spaces will be addressed. This is joint work with G. Laroitonda.

François-Xavier Vialard

Title: Synthetic nonnegative cross-curvature lifts to the Wasserstein space

Abstract: In this talk, we insist on the concept of nonnegative cross-curvature and its synthetic definition for a general cost on a product space. Then, by using a formal argument we show why one can expect that such a property should be also true for the Wasserstein space. Then, we give examples of cost satisfying this synthetic nonnegative cross-curvature, in particular a new one with the Bures-Wasserstein case. We extend the result to the case of unbalanced optimal transport and show some potential applications.

Kathrin Welker

Title: Optimization of piecewise-smooth shapes

Abstract: TBA

Laurent Younes

Title: A scale space framework for diffeomorphic shape analysis.

Abstract: We study a model, inspired by recent works of Miller, Trouvé and Tward, that develops shape comparisons in a continuous Riemannian scale space of diffeomorphisms. We provide basic inclusion properties of this scale space, geodesic equations, and propose, in particular, computational approaches to compute the reproducing kernel intervening in this equation. Preliminary experiments will be provided. This is joint work with Oscar Liu (JHU).

Benedikt Wirth

Title: Fourier discretizations of LDDMM and their convergence

Abstract: We will reconsider bandlimited LDDMM discretizations and discuss their behavior and convergence.

Posters

Shreya Arya

Title: Diffusion on Subanalytic Sets

Benjamin Beaudett

Title: Characterizing Pose Image Manifolds Using Geometry-Preserving GANs and Elasticae

Jonathan Cerqueira

Title: Discretized Sobolev metrics on curves

Théo Dumont

Title: Existence of Monge maps for the Gromov-Wasserstein problem

Florine Hartwig and Sascha Beutler

Title: Discrete geodesic calculus in the space of Sobolev curves

Erik Jansson

Title: Shape analysis on matrix groups with applications to Cryo-EM microscopy

Lars Lammers

Title: Stickiness: a Blessing or a Curse?

Levin Maier

Title: On the interaction of Hamiltonian dynamics, infinite dimensional geometry and nonlinear PDEs

Elodie Maignant and Anna Calissano

Title: Barycentric subspace analysis of a set of graphs

Mao Nishino

Title: Path constrained unbalanced optimal transport

Guillaume Olikier

Title: Projected gradient descent accumulates at Bouligand stationary points

Morten Akhøj Pedersen

Title: Swallowtail shape dynamics

Lidiya Pryymak

Title: Riemannian geometry in shape optimization

Guillaume Sériey

Title: Metamorphoses of manifold-valued images

Tom Szwagier

Title: Principal component analysis with flag manifolds

Yann Thanwerdas

Title: Invariant metrics on covariance and correlation matrices