Stochastic and Deterministic Analysis of Irregular Models
Winter school
8th – 12th January 2024

Contents of the booklet

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- Flandoli, Franco – Scuola Normale Superiore, Pisa.
- Jourdain, Benjamin – Ecole des Ponts et Chaussées.

Speakers

- Catellier, Rémi – Université Côte-d'Azur
- Ciotir, Ioana – INSA Rouen
- Eddahbi, Mhamed – King Saud University
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- Issoglio, Elena – Università di Torino
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- Neuenkirch, Andreas – Universität Mannheim
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Poster session

- Ben Lahbib, Abdelaziz – Université Mohammed V de Rabat
- Bouhmady, Achraf – Université Mohammed V de Rabat
- Louriki, Mohammed – Cadi Ayyad University
- Machado Hernández, Álvaro Enrique – University of Campinas

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- Planas, Gabriela – University of Campinas
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- Olivera, Christian – University of Campinas
- Richard, Alexandre – Université Paris-Saclay
- Russo, Francesco – ENSTA Paris
- Tomašević, Milica – CNRS, Ecole Polytechnique

I Timetable

TIME	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
08:30		Cruzeiro - 1	Cruzeiro - 2	Jourdain - 2	Jourdain - 3
09:00	Flandoli - 1				
10:00		Coffee break + posters	Coffee break	Coffee break + posters	Coffee break
10:30	Coffee break		Eddhabi		Catellier
11:00	Jourdain -1	Flandoli - 2	Ohashi	Flandoli - 3	Cruzeiro - 3
11:45			Lê		
12:30					
	Lunch	Lunch	Lunch	Lunch	Lunch
15:30	Neves	Ouknine	Gess	Rosenzweig	
16:15	Zurcher	Batista	Lucertini	Knorst	
16:30	Marino	Visconti	Trevisani	Bourdais	
16:45	Coffee break	Coffee break	Coffee break	Coffee break	
17:15	Neuenkirch	Ciotir	Issoglio	Löcherbach	
18:00	Anzeletti	Espitia	Hakiki	Weinberger	
18:15	Madry	Londoño	Robinson	Cavallazzi	

II Lectures

1 Stochastic Lagrangian flows in Hydrodynamics

Ana Bela Cruzeiro – Instituto Superior Técnico, Lisboa

1. Arnold's approach to Hydrodynamics

Arnold's Lagrangian description of the Euler equation as a geodesic in the measure-preserving group of diffeomorphisms.

2. Stochastic Lagrangian flows on the group of diffeomorphisms.

A generalization of Arnold's results for describing Navier-Stokes Lagrangian flows, namely stochastic flows associated with the (deterministic) Navier-Stokes equation.

3. Stochastic flows and forward-backward systems

A characterization of the Navier-Stokes stochastic flows of last section as solutions of (infinite-dimensional) forward-backward stochastic systems.

4. Stochastic Lagrangian flows for stochastic Navier-Stokes equations.

A variational Lagrangian formulation for randomly perturbed Navier-Stokes equation.

5. Generalized stochastic flows.

An extension of Brenier's generalized flows to the Navier-Stokes case.

6. Dissipative systems through stochastic variational principles on Lie groups

Dissipative systems on Lie groups and study their variational characterization. Navier-Stokes equations can be considered a particular case.

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2 Mean field results in fluid mechanics

Franco Flandoli – Scuola Normale Superiore, Pisa

Fluid mechanics is rich in mean field results like those of point vortex approximation in 2D. Deviation from the mean field is also a next step of great interest. We illustrate these facts with the example of particle aggregation in a turbulent fluid [9], a problem of interest for initial rain formation or planet formation in stellar dust disks. The particles have inertia, measured by the so-called Stokes number. When Stokes is large, the mean field theory describes reality well and produces physical laws coherent with experiments [1], [3], [9]. But when Stokes is small, the mean field is not sufficient and a complete solution is still debated [2], [3], [4], [7], [8], [10], [11]. Rigorous elements of the theory [8], [9], [6], [5] and heuristics about the Physics will be given.

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3 Discretization of Stochastic Differential Equations with additive Brownian noise and $L^q - L^\rho$-drift coefficient

Benjamin Jourdain – Ecole des Ponts et Chaussées

We will consider the discretization of the stochastic differential equation

$$X_t = X_0 + W_t + \int_0^t b(s, X_s) ds, \ t \in [0, T],$$

where the drift coefficient $b : [0, T] \times \mathbb{R}^d \to \mathbb{R}^d$ is measurable and satisfies the integrability condition : $\|b\|_{L^q([0,T],L^\rho(\mathbb{R}^d))} < \infty$ for some $\rho, q \in (0, +\infty]$ such that

$$\rho \geq 2 \text{ and } \frac{d}{\rho} + \frac{2}{q} < 1. \tag{1}$$

Krylov and Röckner [3] established strong existence and uniqueness under this condition.

Let $n \in \mathbb{N}^*$, $h = \frac{T}{n}$ and $t_k = kh$ for $k \in [\![0, n]\!]$. Since there is no smoothing effect in the time variable, we introduce a sequence $(U_k)_{k\in[\![0,n-1]\!]}$ independent from $(X_0, (W_t)_{t\geq 0})$ of independent random variables which are respectively distributed according to the uniform law on $[kh, (k+1)h]$. The resulting scheme Euler is initialized by $X_0^h = X_0$ and evolves inductively on the regular time-grid $(t_k = kh)_{k\in[\![0,n]\!]}$ by:

$$X^h_{t_{k+1}} = X^h_{t_k} + W_{t_{k+1}} - W_{t_k} + b_h\left(U_k, X^h_{t_k}\right) h, \tag{2}$$

where b_h is some truncation of the drift function b. When b is bounded, one of course chooses $b_h = b$. Then the order of weak convergence in total variation distance is $1/2$, as proved in [1]. It improves to 1 up to some logarithmic correction under some additional uniform in time bound on the spatial divergence of the drift coefficient. In the general case (1), we will see that for suitable truncations b_h, the difference between the transition densities of the stochastic differential equation and its Euler scheme is bounded from above by $Ch^{\frac{1}{2}\left(1-\left(\frac{d}{\rho}+\frac{2}{q}\right)\right)}$ multiplied by some centered Gaussian density, as proved in [2].

We will also try to address the order of strong convergence $1/2 - \varepsilon$ with $\varepsilon > 0$ arbitrarily small established by Lê and Ling in [4] for the semi-discrete scheme

$$dX^h_t = dW_t + b_h\left(t, X^h_{\tau^h_t}\right) dt, \ t \in [0, T] \text{ where } \tau^h_t = \left\lfloor \frac{t}{h} \right\rfloor h.$$

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III Talks

1 Monday 8th, January

Homogenization of Schrödinger equations. Extended Effective Mass Theorems for non-crystalline matter.

Wladimir Neves – Universidade Federal do Rio de Janeiro

This talk concerns the homogenization of Schrödinger equations for non-crystalline matter, that is to say the coefficients are given by the composition of stationary functions with stochastic deformations. Two rigorous results of so-called effective mass theorems in solid state physics are obtained: a general abstract result (beyond the classical stationary ergodic setting), and one for quasi-perfect materials (i.e. the disorder in the non-crystalline matter is limited). The former relies on the double-scale limits and the wave function is spanned on the Bloch basis. Therefore, we have extended the Bloch Theory which was restricted until now to crystals (periodic setting). The second result relies on the Perturbation Theory and a special case of stochastic deformations, namely stochastic perturbation of the identity.

Spatial Sojourn Time of Stochastic Wave Equation. *(Short talk)*

Jérémy Zurcher – Université de Lille

We consider in this talk the 1D linear stochastic wave equation, whose solution is taken in the mild sense. We are interested in the spatial sojourn time associated. We fix the time, a level λ and an observation window $A > 0$. Then the spatial sojourn time is the length of the subset of $[-A, A]$ where the solution at time t is above the level λ. In 2013, Pham proved in On The Rate Of Convergence For Central Limit Theorems Of Sojourn Times Of Gaussian Fields that this process converges in law toward a Gaussian random variable when the window observation A goes to infinity, using tools from Stein-Malliavin method. Our work with C. A. Tudor is still to prove convergence theorem, but when the time t does depend on A, in particular in a polynomial dependence. We proved that no matter the polynomial dependence between t and A, the spatial sojourn time still converges in distribution as A goes to infinity, but not necessarily to a Gaussian random variable. We can moreover, thanks to the Stein-Malliavin theory, estimate the rate of convergence. With Ciprian Tudor.

About the regularity of degenerate non-local Kolmogorov operators under Lévy perturbations. *(Short talk)*

Lorenzo Marino – ENSTA Paris

We study the effects of a Lévy perturbation to an Ornstein-Uhlenbeck operator of the form:

$$\mathcal{L}_\alpha + \langle Ax, D_x\rangle - \partial_t, \qquad \text{on } (0,T)\times\mathbb{R}^N. \tag{3}$$

Here, $\mathcal{L}_\alpha$ is a possibly degenerate operator acting on the subspace $(B\mathbb{R}^N)$ which is diffusive, i.e. $\mathrm{Tr}(BD_x^2)$, if $\alpha = 2$ or an α-stable operator if $\alpha \in (0,2)$, under an hypoelliptic framework, when the matrices A, B on $\mathbb{R}^N \times \mathbb{R}^N$ satisfy the Kalman rank condition. In particular, we show that various estimates already known for this class of operators, such as for example the anisotropic Schauder ([Lunardi'97], [Marino'21]) or Sobolev estimates ([Bramanti-Cupini-Lanconelli-Priola'10], [Huang-Menozzi-Priola'19]), are still valid, and with the same *constants*, for the perturbed operator:

$$\mathcal{L}_\alpha + \langle Ax, D_x \rangle - \partial_t + \mathcal{L}_t, \tag{4}$$

where $\mathcal{L}_t$ is a (possibly) asymmetric, time dependent Lévy operator characterized by an integrable in time Lévy triplet $(b(t), a(t), \nu_t)$. Importantly, we do not impose any hypoelliptic condition on the perturbation that indeed may oscillates between being non-degenerate and degenerate on some time intervals. Our approach relies on the stochastic perturbative technique based on Poisson processes introduced by [Krylov-Priola'17] in the purely diffusive setting and then generalized in [Marino-Menozzi-Priola'22] to the hypoelliptic framework. Independently from the constants preservation, we emphasise that the L^p-estimates for the perturbed operator in (4) seem to be new and to the best of our knowledge, the question to obtain the indicated stability results by a "purely analytic" proof remains open.

Time permitting, we will also present some applications of the above results to the weak well-posedness of SDEs driven by oscillating multi-fractional noises such as:

$$dX_t \,=\, AX_t dt + BdZ_t + \sigma(t,x)dZ'_t,$$

for two stable processes $(Z_t, Z'_t)_{t\geq 0}$ with possibly different α-stability index and a continuous diffusive matrix σ in $\mathbb{R}^N \otimes \mathbb{R}^N$ which may vanish on some regions of $(0,T) \times \mathbb{R}^N$.

SDEs driven by fractional Brownian motion: dependence on the Hurst parameter.

Andreas Neuenkirch – Universität Mannheim

We show that the Mandelbrot-van Ness representation of fractional Brownian motion is almost surely smooth in the Hurst parameter H. This dependence result is transferred to the solution of a stochastic differential equation driven by fractional Brownian motion if the stochastic differential equation is one-dimensional or if $H > 1/2$. In the multidimensional case and $H \in (1/3, 1/2]$ we use rough path theory to make sense of the differential equations. However, despite it being possible to lift fractional Brownian motion as well as its derivative in H to a rough path via the limit of dyadic approximations, they cannot be lifted jointly in the same way. Nevertheless, we obtain that the solution to a rough stochastic differential equation driven by fractional Brownian motion is locally Lipschitz continuous in H.

This talk is based on the PhD thesis of Stefan Koch, which I supervised.

Density of the solution to SDEs with distributional drift and fractional Brownian noise. *(Short talk)*

Lukas Anzeletti – TU Wien

We look at SDEs with additive fractional Brownian noise and distributional (time-dependent) drift. Well-posedness of such equations has been thoroughly studied in recent years, generally known as a phenomenon called "regularization by noise". We prove existence of a density of the solution to such equations and investigate the properties thereof. In particular, the density has a certain regularity and Gaussian tails. This is used to extend results on existence of a solution to a convolution-type McKean-Vlasov equation. Based on joint work with Lucio Galeati, Alexandre Richard and Etienne Tanré.

Some recent progress on the small noise question of singular SDEs. *(Short talk)*

Lukasz Madry – Université Paris-Dauphine

In this talk I'll briefly present results on the small noise limit of singular SDEs of type $dx = b(x)dt + dw_t$, where $b(x)$ is a singular power law function and w_t is fBm. Thanks to our Markovianization scheme, inspired by the works of Panloup-Richard 20' we can construct a sequence of stopping times that allows us to bypass the lack of strong Markov property and establish sub-exponential rate of convergence to the extremal solution. Our proof is pretty robust - time permitting, I'll also talk about ongoing work on the extension of our method to the case of singular dispersing particle system and multidimensional case in the setting of Delarue-Maurelli, with some numerical simulations. This is based on the results from my PhD thesis under supervision of Paul Gassiat.

2 Tuesday 9th, January

Reflected and Doubly RBSDEs with Irregular Obstacles and a Large Set of Stopping Strategies.

Youssef Ouknine – Cadi Ayyad University

We introduce a new formulation of reflected BSDEs and doubly reflected BSDEs associated with irregular obstacles.

The first part of our work extends the classical optimal stopping problem to a broader set of stopping systems, namely the set of split stopping times, allowing for irregular payoff processes ξ and a general filtration. The use of split stopping times enables a flexible representation of financial contracts and derivatives that rely on multiple conditions or triggers, and stochastic processes featuring jumps and other discontinuities.

We show that the value family can be aggregated by an optional process v, which is characterized as the Snell envelope of the reward process ξ over split stopping times. Using this, we prove the existence and uniqueness of a solution Y to irregular reflected BSDEs. In the second part of the paper, motivated by the classical Dynkin game with completely irregular rewards considered by Grigorova et al. (2018), we generalize the previous equations to the case of two reflecting barrier processes.

Rate of convergence of N-player Games of Moderate Interactions to the associated MFG PDEs. *(Short talk)*

Alexandre Batista de Souza – UniCamp

We consider the N-player game where each player $i = 1, ..., N$ has position given by

$$dX_t^{N,i} = \Big(\alpha^{N,i}(t) + b\big(X_t^{N,i}, \frac{1}{N}\sum_{j=1}^{N} V^N(X_t^{N,i} - X_t^{N,j})\big)\Big)dt + dW_t^{N,i},\ t \in [0,T].$$

where $\alpha^N = (\alpha^{N,1}, ..., \alpha^{N,N})$ is a vector of strategies, b is deterministic and $W^{N,i}$ are independent Wiener processes. Each player acts to minimize the cost of their strategies. This setting is considered in [Flandoli-Ghio-Livieri'22], where they prove the minimum is attained by $\alpha^*(t,x) \doteq -\nabla u(t,x)$. The mollified empirical measure $p^N \doteq V^N * S^N$, where $S_t^N \doteq \frac{1}{n}\sum_i \delta_{X_t^{N,i}}$ converges to the density p, where (u,p) is the solution of the associated MFG system of PDEs

$$\begin{cases} -\partial_t u - \frac{1}{2}\Delta u - b(x,p(t,x)) \cdot \nabla u + \frac{1}{2}|\nabla u|^2 = f(x,p(t,x)), & (t,x) \in [0,T) \times \mathbb{R}^d, \\ \partial_t p - \frac{1}{2}\Delta p + \operatorname{div}[p(t,x)(-\nabla u(t,x) + b(x,p(t,x)))] = 0, & (t,x) \in (0,T] \times \mathbb{R}^d, \\ p(0,\cdot) = p_0(\cdot), \quad u(T,\cdot) = g(\cdot), & x \in \mathbb{R}^d. \end{cases}$$

Using the semigroup approach (see [Olivera-Richard-Tomasevic'23]), we prove the following rate of convergence. For every $\epsilon > 0$, $m \geq 1$, and $q \in [1,\infty)$, we have

$$\| \|p^N - p\|_{T,L^q(\mathbb{R}^d)} \|_{L^m(\Omega)} \leq C \| \|p^N(0,\cdot) - p_0\|_{L^q(\mathbb{R}^d)} \|_{L^m(\Omega)} + CN^{-\rho+\epsilon},$$

with

$$\rho \doteq \min\left(\frac{\beta}{d}\gamma \wedge \gamma', \frac{1}{2}(1 - \beta(1+\theta_q))\right),\ \theta_q \doteq \left(1 - \frac{2}{q}\right) \vee 0$$

and γ, γ' are the Hölder regularity of $\alpha = -\nabla u$ and all p^N, respectively.
$\beta \in (0, \frac{1}{2})$ expresses the moderate interaction. This is a joint work with Josué Knorst and Christian Olivera.

Optimality conditions for parabolic stochastic optimal control problems with boundary controls. *(Short talk)*

Piero Visconti – INSA Rouen

Optimality conditions are provided for a class of control problems driven by a Wiener process, which amount to a stochastic maximum principle in differential form. The control is considered to act on the drift and the volatility, both of which may be unbounded operators, which allows us to consider SPDEs with control and/or noise on the boundary. By the factorization method, a regularizing property is established for the state equation which is then employed to prove, by duality, a similar result for the backward time costate equation. The costate equation is understood in the sense of transposition. Finally, the cost is shown to be Gâteaux differentiable and its derivative is represented in terms of the costate, the optimality condition is deduced using results of set valued analysis.

The stochastic fast logarithmic equation in $\mathbb{R}^d$ with multiplicative Stratonovich noise.

Ioana Ciotir – INSA Rouen

This paper is concerned with the existence and uniqueness of the solution for the stochastic fast logarithmic equation with Stratonovich multiplicative noise in $\mathbb{R}^d$ for $d > 2$. It provides an answer to a critical case (morally speaking, corresponding to the porous media operator ΔX^m for $m = 0$) left as an open problem in the paper Barbu-Röckner-Russo treating stochastic porous media equations in unbounded domains. We face several technical difficulties related both to the degeneracy properties of the logarithm and to the fact that the problem is treated in an unbounded domain. Firstly, the order in which the approximations are considered is very important and different from previous methods. Since the estimates are leading only to weak convergence of the relevant terms, identifying the limit of the nonlinear part of the equation is more fastidious. Secondly, the energy estimates needed in the last step can only be achieved with a well-chosen Stratonovich-type rectification of the noise.

This a presentation is based on a joint work with Reika Fukuizumi and Dan Goreac.

Invariant measures for stochastic nonlinear partial differential equations in the space of almost periodic functions. *(Short talk)*

Claudia Espitia – UniCamp

In this talk we present some results about two stochastic partial differential equations generalizing some previous results to a more general class of oscillatory solutions. More specifically, we treat stochastic conservation laws and stochastic degenerate parabolic-hyperbolic equations considering a wider notion of periodic solutions. For these equations, we study the well-posedness and the long-time behavior of almost periodic solutions under the assumption of Lipschitz continuity of the flux and the viscosity functions, and some non-degeneracy conditions. As a main objective, for each equation we show the existence and uniqueness of an invariant measure in a separable subspace of the space of Besicovitch almost periodic functions.

These results correspond to a joint work with Prof. H. Frid and Prof. D. Marroquin.

Stochastic Euler equations with multiplicative noise. *(Short talk)*

Juan D. Londoño – UniCamp

We analyze a Lagragian formulation of the stochastic incompressible Euler equations with multiplicative noise on a domain $\mathbb{T}^d$. First, we discuss how to establish the equivalence between the Lagrangian and classical formulations for the stochastic incompressible Euler equations. The proof relies on the Itô-Wentzell-Kunita formula and employs various stochastic analysis techniques. Next, we demonstrate a local existence result for the Lagrangian formulation within appropiate Sobolev Spaces.

3 Wednesday 10th, January

Existence of solutions singular stochastic differential equations with jumps.

Mhamed Eddahbi – King Saud University

We are focused on solving stochastic differential equations driven by jump processes (SDEJs) and measurable drifts, which may exhibit quadratic growth. Our approach involves employing a space transformation and utilizing Itô-Krylov formula to eliminate the singular component of the drift. This transformation allows us to obtain SDEJs devoid of the singular drift, making them solvable under certain standard conditions. Subsequently, we apply the inverse transformation, which has been proven to be a one-to-one mapping, enabling us to find solutions to the original equation. In this context, we will explore several illustrative examples. This work constitutes a natural extension to the realm of jump processes, expanding upon the seminal 1984 result of J.F. Le Gall concerning SDEs driven by Brownian motion.

Isometries for stochastic integrals driven by fractional Brownian motion.

Alberto Ohashi – Universidade de Brasilia and Rice University

In this talk, we will present the exact expression of the L^2-norm of the symmetric-Stratonovich stochastic integral driven by a multi-dimensional fractional Brownian motion B with parameter $\frac{1}{4} < H < \frac{1}{2}$ and $\frac{1}{2} < H < 1$. Our main result is a complete description of a Hilbert space of integrand processes which realizes the L^2-isometry where none regularity condition in the sense of Malliavin calculus is imposed. The Hilbert space is characterized in terms of a random Radon σ-finite measure on $[0, T]^2$ off diagonal which can be characterized as a product of a non-Markovian version of the stochastic Nelson derivatives. As a by-product, we present the exact explicit expression of the L^2-norm of the pathwise rough integral in the sense of Gubinelli for $\frac{1}{4} < H < \frac{1}{2}$ and Young for $\frac{1}{2} < H < 1$.

Sewing methods in differential equations.

Khoa Lê – University of Leeds

The sewing lemma is originated from Lyons' rough path theory in 1998. Since then, the lemma had re-appeared in other occasions together with some improvements and new applications. The development can be seen through the works of Gubinelli 2004, Feyel - de La Pradelle 2006, Davie 2007 and through its recent stochastic extension. We will review the sewing lemmas via a unified perspective. Some recent applications to McKean-Vlasov equations and singular SDEs are discussed to illustrate the method.

From fluctuations in conservative systems, to large deviations, to PDEs with irregular coefficients.

Benjamin Gess – Universität Bielefeld

Fluctuations are ubiquitous in non-equilibrium systems. While their mean behavior in hydrodynamic limits is described by partial differential equations, the fluctuations around this mean can be described in terms of large deviations estimates. The analysis of such large deviations has led to the so-called macroscopic fluctuation theory, a general framework for non-equilibrium statistical mechanics. In this talk, we will survey this link between hydrodynamic limits, large deviations, and PDEs for the example of the zero range process. We will demonstrate how the task of proving large deviations estimates leads to the analysis of PDEs with irregular coefficients in critical spaces and to Gamma convergence. Finally, by developing a nonlinear extension of the DiPerna-Lions theory of renormalized solutions, we solve an open problem in the proof of a full large deviation principle for the zero-range process.

Optimal regularity for degenerate Kolmogorov equations with rough coefficients and applications to SDEs' uniqueness. *(Short talk)*

Giacomo Lucertini – Università di Bologna

We present recent results on the existence and optimal regularity of the solution for degenerate linear Kolmogorov equations. We consider a class of operators satisfying a parabolic Hörmander condition and with coefficients that are Hölder continuous in space and only measurable in time. We define intrinsic Hölder spaces exploiting the geometry induced by the operator, then we establish optimal Schauder estimates for the solution of the associated Cauchy problem. These results can be applied to study the weak and strong uniqueness for a class of degenerate SDEs with rough coefficients: we investigate the case when the coefficients of the equation are α-Hölder continuous, with $\alpha \in]1/3, 2/3[$, that is an open problem. This presentation is based on joint works with Professors Stefano Pagliarani and Andrea Pascucci, together with an ongoing collaboration with Professors Stéphane Menozzi and Stefano Pagliarani.

Degenerate SDEs with drift in anisotropic negative Besov spaces. *(Short talk)*

Davide Trevisani – Universidade da Coruña

Under the weak Hörmander condition, the Kolmogorov operator associated with a degenerate SDE induces an anisotropic structure in space. Its fundamental solution, indeed, is Hölder continuous with respect to the initial condition, but this regularity changes with the direction of the increment. Accordingly, in the context of singular degenerate SDEs, it is natural to consider equations with drift in a certain anisotropic Besov space.

In this presentation we outline the steps to establish well-posed for this kind of SDEs in terms of a proper martingale problem. Our analysis applies when the Besov regularity of the drift is not worse than $-\frac{1}{2}$.

The probabilistic techniques employed in this proof rely deeply on analytical results about singular PDEs. Besides well-posedness, particular attention is paid to the regularity and stability of the

Kolmogorov equation. Regularity in time is equally important: on one hand, Schauder estimates delimit the boundary beyond which the drift is too irregular, and we provide these estimates for Besov spaces, akin to those valid for the heat kernel. On the other hand, the regularity along the Hörmander field plays a crucial role in the existence of the martingale problem.

This talk is based on the joint work with Stefano Pagliarani (Universitá di Bologna), Elena Issoglio (Universitá di Torino) and Francesco Russo (ENSTA Paris, Institut Polytechnique de Paris).

Degenerate McKean-Vlasov SDEs with singular coefficients.

Elena Issoglio – Università di Torino

In this talk we consider a class of degenerate SDEs with drift depending on the law density of the solution, hence of McKean-Vlasov type. These SDEs are singular because the drift is an element of a negative Besov space, and moreover they are degenerate because the Brownian noise acts only in some directions and it is zero in the other directions. These equations are interpreted in the sense of a suitable singular martingale problem, thus a key tool is the study of the corresponding singular Kolmogorov PDE and singular Fokker-Planck PDE, whose solutions naturally live in a family of anisotropic Besov spaces. Using these results, it is possible to define the notion of solution to the singular McKean equation by a martingale problem formulation, and to show its existence and uniqueness.

This talk is based on a joint work with S. Pagliarani (Bologna), D. Trevisani (A Coruna) and F. Russo (ENSTA).

Sample path properties of highly irregular Gaussian processes. *(Short talk)*

Youssef Hakiki – Cadi Ayyad University

Let X be a d-dimensional Gaussian process in $[0,1]$, where the component are independent copies of a scalar Gaussian process X_0 on $[0,1]$ with a given general variance function $\gamma^2(r) = \mathrm{Var}\,(X_0(r))$ and a canonical metric $\delta(t,s) := (\mathbb{E}\,(X_0(t) - X_0(s))^2)^{1/2}$ which is commensurate with $\gamma(t-s)$. Under a weak regularity condition on γ, denoted as $(\mathbf{C_{0+}})$, which allows γ to be far from Hölder-continuous, we prove that for any Borel set $E \subset [0,1]$, the Hausdorff dimension of the image $X(E)$ and of the graph $Gr_E(X)$ are constant almost surely. Furthermore, we show that these constants can be explicitly expressed in terms of $\dim_\delta(E)$ and d. However, when $(\mathbf{C_{0+}})$ is not satisfied, the classical methods may yield different upper and lower bounds for the underlying Hausdorff dimensions. This case is illustrated via a class of highly irregular processes known as logarithmic Brownian motion (logBm). Even in such cases, we employ a new method to establish that the Hausdorff dimensions of $X(E)$ and $Gr_E(X)$ are almost surely constant. The method uses the Karhunen-Loève expansion of X to prove that these Hausdorff dimensions are measurable with respect to the expansion's tail sigma-field, then by the 0-1 law of Kolmogorov these dimensions are almost surely constant. This is based on a joint work with Professor Frederi Viens.

Bi-causal optimal transport for SDEs with irregular coefficients. *(Short talk)*

Benjamin Robinson– University of Vienna

Many natural phenomena that exhibit randomness can be modelled by stochastic differential equations (SDEs), often having less regularity than the classical case of SDEs with Lipschitz coefficients. In such settings, we are interested in quantifying model uncertainty and the impact model choice on the value of stochastic optimisation problems. To this end, we seek an appropriate notion of distance on the space of models. In particular, we study the adapted Wasserstein distance between the laws of SDEs. This is a special case of a bi-causal optimal transport problem, in which the classical optimal transport problem is constrained to respect the flow of information inherent in stochastic processes.

Under minimal regularity assumptions on the coefficients, we show that the value of the bi-causal optimal transport problem between the laws of one-dimensional SDEs is attained by the synchronous coupling. This is the coupling induced by taking a common Brownian motion as the driving noise for each SDE. Our proof is based on a discretisation method, exploiting monotonicity properties of the resulting discrete-time processes. A key tool in our work is a transformation-based semi-implicit Euler-Maruyama scheme for SDEs whose drift coefficient may have discontinuities and exponential growth. We prove the first strong existence and uniqueness result for such SDEs, as well as strong convergence of the implicit scheme. Moreover, our results give rise to a method of efficient computation of the adapted Wasserstein distance. This is joint work with Michaela Szölgyenyi (University of Klagenfurt).

4 Thursday 11th, January

The attractive log gas: stability, uniqueness, and propagation of chaos.

Matthew Rosenzweig – Carnegie Mellon University

We consider the dynamics of a system of particles with logarithmic attractive interaction on the torus at inverse temperature beta. We show phase transitions for the stability and uniqueness of the uniform distribution. Investigating the mean-field convergence of the system by the modulated free energy method, we show uniform-in-time convergence is true for small beta, while false for large beta. In the process, we identify interesting questions concerning functional inequalities of logarithmic Hardy-Littlewood-Sobolev type and uniqueness for Kazhdan-Warner type equations.

Systems of particles with singular interaction under common noise. *(Short talk)*

Josué Knorst – UniCamp

We consider a system of particles interacting via a singular kernel and subject to common noise in addition to individual noises. The common noise has the form of $\sigma_t B_t$. The limiting equation for such a system is an SPDE, for which we prove well-posedness via classical arguments (Krylov's L^p-theory for SPDEs). We prove the propagation of chaos by establishing the convergence rate of the mollified empirical measure towards the limiting density. This is done without the semigroup approach, which cannot be applied in this context.

This is a joint work with Alexandre B. de Souza and Christian Olivera.

This work is supported by FAPESP Thematic Project 2020/04426-6 and FAPESP Post-doctoral Project 2022/13413-0.

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Entropy weighted optimization problems with applications to stochastic optimal control. *(Short talk)*

Thibaut Bourdais – ENSTA Paris

We study the optimization problem

$$\inf_{\mathbb{Q}} \mathbb{E}^{\mathbb{Q}}[\varphi(X)] + H(\mathbb{Q}|\mathbb{P}), \tag{5}$$

where $\mathbb{P}$ is a probability measure on the space of cadlag trajectories $\Omega := D([0,T],\mathbb{R}^d)$, X is the canonical process on Ω, the infimum is taken over all probability measures on Ω and H is the relative entropy. This problem appears in variational representation formulas originating from the theory of large deviations. Under very mild assumptions on the measurable function φ, it admits a unique solution $\mathbb{Q}^* \propto \exp(-\varphi(X))d\mathbb{P}$. Assuming that the reference probability measure $\mathbb{P}$ is solution of a martingale problem, we characterize $\mathbb{Q}^*$ as the solution of a well-identified martingale problem by means of a generalized gradient. When $\mathbb{P}$ is the law of a Markovian diffusion, Problem (5) can be interpreted as a stochastic control problem and we prove existence of an optimal Markovian control with few assumptions on the coefficients of the underlying SDE, including the case of a solution in law of an SDE with distributional drift. We use this result to propose an alternative technique to dynamic programming to solve some stochastic optimal control problems. We reformulate the control problem as an optimization program on the space of probability measures that we regularize by splitting the minimization variables and penalizing the entropy between the two probability measures to be optimized. We show that the regularized problem provides a good approximation of the original problem when the weight of the entropy regularization term is large enough. We then provide an alternating minimization algorithm whose convergence to the infimum of the regularized problem is proved. We illustrate our approach by solving a high-dimensional stochastic control problem appearing in energy management.
This talk is based on a joint work with Nadia Oudjane (EDF R & D) and Francesco Russo (ENSTA Paris).

Mean field limits of interacting particle systems having alpha-stable jump heights.

Eva Löcherbach – Université Paris 1 Panthéon-Sorbonne

The study the convergence of systems of interacting particles driven by Poisson random measure, having mean field interactions and position dependent jump rate. Jumps are simultaneous, that is, at each jump time, all particles of the system are affected by this jump and receive a positive random jump height. This random kick is distributed according to an alpha-stable law and scaled in $N^{-1/\alpha}$, where N is the size of the system. This particular scaling implies that the limit of the empirical measures of the system is random, describing the conditional distribution of one particle in the limit system. Such limits are conditional McKean-Vlasov limits. The conditioning in the limit measure reflects the dependencies between coexisting particles in the limit system such that we are dealing with a conditional propagation of chaos property. I will spend some time to explain the explicit structure of the limit system which turns out to be the solution of a non-linear SDE driven by Poisson random measure and an independent stable process.

In a second part of the talk I discuss strong error bounds allowing us to control the rate of convergence of the finite particle system to the limit system and explain the main differences when alpha is smaller or bigger than one.

Finally I discuss the case when the jumps of the particle system do only belong to the domain of attraction of a stable law.

Stochastic Differential Equations Involving the Local Time of the Unknown Process Driven by Stable Processes. *(Short talk)*

Johanna Weinberger – Technion - Israel Institute of Technology

Stochastic differential equations involving the local time of the unknown process with Brownian forcing were thoroughly discussed by Jean-François Le Gall in his seminal paper from 1983. A special case of these equations are of the form

$$X_t = x_0 + \int_{\mathbb{R}} \ell_t^x \mu(dx) + B_t, \quad t \geq 0,$$

where B is a standard Brownian motion, $x_0 \in \mathbb{R}$, μ is a Radon measure and ℓ is the local time of X given by the Tanaka's formula. In this talk, we formulate an analogous equation in dimension $d = 1$, where B is replaced by an α-stable process L with $\alpha \in (1, 2)$ and discuss weak and strong existence and uniqueness of solutions, as well as the equivalence to singular SDEs with drift μ. To this end, we define a local time ℓ by deriving a Tanaka-type formula for semimartingales whose martingale part is an α-stable process with $\alpha \in (1, 2)$. We show that ℓ coincides with the occupation density of X for solutions to singular SDEs under mild assumptions on the drift μ. The talk is based on joint work with Leonid Mytnik.

Quantitative weak propagation of chaos for McKean-Vlasov SDEs driven by α-stable processes. *(Short talk)*

Thomas Cavallazzi – Université Paris-Saclay

In this talk, we will deal with McKean-Vlasov SDEs driven by α-stable processes, with $\alpha \in (1, 2)$. We make Hölder-type assumptions on the coefficients, with respect to both space and measure variables. We will study the associated semigroup, acting on functions defined on the space of probability measures, through the related backward Kolmogorov PDE describing its dynamics. We will focus in particular on its regularizing properties. We will finally use the preceding results to prove quantitative weak propagation of chaos for the associated mean-field interacting particle system.

5 Friday 12th, January

Regularization by noise for rough differential equations driven by Gaussian rough paths.

Rémi Catellier – Université Côte-d'Azur

We consider the rough differential equation with drift driven by a Gaussian geometric rough path. Under natural conditions on the rough path, namely non-determinism, and uniform ellipticity conditions on the diffusion coefficient, we prove path-by-path well-posedness of the equation for poorly regular drifts.

IV Posters

Theoretical and numerical analysis of the stochastic epidemic model under time delay and non-linear incidence.

Abdelaziz Ben Lahbib – Université Mohammed V de Rabat

The disease's evolution is influenced by various models, determined by the characteristics of the disease and the specific population. In this article, we categorize the population into four classes : susceptible $S(t)$, infected $I(t)$, recovered $R(t)$, and cross-immune $C(t)$ individuals. Transitions between these groups, often seeming random, are modeled using stochastic approaches such as white noise.This article presents theoretical and numerical investigations of a stochastic SIRC epidemic model with time delay and nonlinear incidence. The existence and uniqueness of a global positive solution is proved. The Lyapunov analysis method is used to obtained sufficient conditions for the extinction and the existence of a unique stationary distribution under certain asymptions. Numerical simulation is also elaborated for the considered stochastic model to support the theoretical results.

Beyond Market Forces: Leveraging Control Theory for Optimal Pricing Strategy in restricted market.

Achraf Bouhmady – Université Mohammed V de Rabat

Price controls can have a significant impact on the pricing strategies of businesses operating in many markets. The success of a new product launch, in particular, depends heavily on its pricing policy, which can be challenging to determine when price controls are in place. In this presentation, we explore how control theory can be used to develop an optimal pricing strategy for a new product launch under price controls. Our main aim is to identify the optimal pricing policy that maximizes profit throughout the product's life cycle while accounting for price controls. Using real-world data from Apple iPhones, we demonstrate how control theory can be used to model and analyze pricing dynamics under price controls. We derive the optimal pricing policy using analytical techniques and perform a sensitivity analysis to assess the impact of variations in innovation and imitation parameters on the pricing decision and product diffusion in the market. Our findings can guide businesses in setting optimal product prices during the launch phase, even when price controls are in place. This work contributes to the literature on pricing strategies and provides valuable insights for businesses operating in markets with price controls.

Lévy Bridges with Random Length for Modelling of the Financial Information.

Mohammed Louriki – Cadi Ayyad University

The main purpose of this talk is to extend the information-based approach of Bedini-Buckdahn-Engelbert to a more general set-up. Instead of using only a Brownian bridge as an information process, we consider another important type of information process. To model the flow of information concerning the time of the bankruptcy of a company (or a state) arriving on the market, we introduce

Lévy bridges with random length, generalizing the Brownian bridge and gamma bridge information processes. Our first goal is to rigorously define a Lévy bridge with random length. Our second task is to establish the Markov property with respect to its completed natural filtration and thus with respect to the usual augmentation of the latter. The resulting conclusion is the right-continuity of completed natural filtration. Certain examples of such a process are considered.

An Itô-Wentzell formula for rough paths.

Álvaro Enrique Machado Hernández – UniCamp

In this work, we introduce an Itô-Wentzell type formula tailored for a rough path $\mathbf{X} = (X, \mathbb{X}) \in \mathcal{C}_X^{\alpha}([0,T], V)$ with α-Hölder regularity, where $\alpha \in (\frac{1}{3}, \frac{1}{2}]$. Our contribution builds upon the earlier findings of R. Castrequini and P. Catuogno concerning the Young integral, as well as those of C. Keller and J. Zhang in the context of rough paths.

Being more specific, we consider a continuous function $h : [0,T] \times U \to L(V, W)$ that satisfies certain regularity conditions. On the other hand, we consider the continuous function $g : [0,T] \times U \to W$ given by

$$g(t,x) = g(0,x) + \int_0^t h(s,x) d\mathbf{X}_s.$$

Under certain conditions imposed on these functions, we assert that for any $(Z, \partial_X Z) \in \mathcal{D}_X^{2\alpha}([0,T]; U)$, the following formula holds:

$$\begin{aligned} g(t, Z_t) &= g(0, Z_0) + \int_0^t h(r, Z_r) d\mathbf{X}_r \\ &\quad + \int_0^t Dg(r, Z_r) d_{\mathbf{X}} Z_r + \int_0^t Dh(r, Z_r) \partial_X Z_r d[\mathbf{X}]_r \\ &\quad + \frac{1}{2} \int_0^t D^2 g(r, Z_r) \partial_X Z_r \otimes \partial_X Z_r d[\mathbf{X}]_r. \end{aligned}$$