RECENT ADVANCES IN GEOMETRIC ANALYSIS
CIRM, 6-10 NOVEMBER 2023

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9h00		Mini Course 1.2 D. Stern	Mini Course 2.2 I. Mondello	F. Schulze	V. Agostiniani
9h30					
10h00	Welcome Coffee			A. Pigati	L. Sarnataro
10h30		Coffee break	Coffee break		Coffee break
11h00	A. Michelat	D. Tewodrose	A. Malchiodi	Coffee break	M. Ahmedou
11h30				A. Waldron	
12h00	D. Martino	A. Metras	A. Diana		
12h30					
13h00					
13h30					
14h00	Mini Course 1.1 D. Stern	Mini Course 2.1 I. Mondello		M. Pegon	
14h30					
15h00				Coffee break	
15h30	Coffee break	Coffee break		D. Stantejsky	
16h00	M. del Mar Gonzalez	A. Hassannezhad			
16h30				M. Nahon	
17h00	D-H. Seo	M. Karpukhin			
17h30	L. Lavoyer				
18h00					

Mini-Courses

Mini Course 1: Daniel Stern
Title: Variational methods for harmonic maps, and applications to spectral geometry and minimal surfaces
Abstract: I will survey recent progress on the existence and regularity theory for harmonic maps from arbitrary closed manifolds to large classes of positively curved targets, with special emphasis on a natural family of sphere-valued harmonic maps which turns out to be intimately related to isoperimetric problems in spectral geometry, based on joint work with M. Karpukhin. In the case of two-dimensional domains, I will discuss applications of these techniques to the existence, regularity, and stability of metrics maximizing Laplace or Steklov eigenvalues on surfaces, highlighting some of the key ingredients in forthcoming work with Karpukhin, Kusner, and McGrath, in which these methods are employed to produce new families of minimal surfaces in B^3 and S^3 with prescribed topology.

Mini Course 2: Ilaria Mondello
Title: Limits of manifolds with a Kato bound on Ricci curvature
Abstract: Starting from Gromov pre-compactness theorem, a vast theory about the structure of Gromov-Hausdorff limits of manifolds with a lower bound on the Ricci curvature has been developed thanks to the work of J. Cheeger, T.H. Colding, M. Anderson, G. Tian, A. Naber, W. Jiang. Nevertheless, in some situations, for instance in the study of geometric flows, a lower bound on the Ricci curvature represents a too restrictive assumption. It is then important to understand how to weaken the hypothesis on the Ricci curvature, and how the structure of the limit spaces is affected. In this mini-course I will present the main tools and the main results of a series of papers in collaboration with G. Carron and D. Tewodrose, where we developed a vast regularity theory for limits of manifolds whose Ricci curvature satisfy a Kato bound, inspired by Kato potentials in the Euclidean space.

1 hour talks

Virginia Agostiniani
Title: A PDE proof of the Riemannian Penrose inequality
Abstract: We present a simple proof of the Riemannian Penrose inequality which builds on the analysis of the level sets of p-harmonic functions.

Mohameden Ahmedou
Title: An abstract framework for the critical point theory at infinity
Abstract: In this talk we report on some new progress in setting up an abstract framework, under which Morse theoretical methods can be applied to some non compact variational problem by computing the difference of topology induced by the so called "critical points at infinity". As application we show how to apply such a method to the Nirenberg problem on spheres, recovering all existence results available in the literature. This is a joint work with Thomas Bartsch (Giessen University).

Maria Del Mar Gonzalez
Title: Spectral properties of Levy Fokker-Planck equations
Abstract: We study the spectrum of a fractional Laplacian equation with drift in suitable weighted spaces. This operator arises when studying the fractional heat equation in self-similar variables. We show, in the radially symmetric case, compactness, and then calculate the eigenfunctions in terms of Laguerre polynomials. The proofs involve conformal geometry, Mellin transform and complex analysis methods.

Asma Hassannezhad
Title: Steklov eigenvalues of negatively curved manifolds
Abstract: The geometry and topology of negatively curved manifolds are subtly reflected in a geometric bound for the Laplace eigenvalues, a connection that has been explored since the 1980's. Among these results, we can mention the spectral gap of the Laplacian in dimensions of at least three and higher-order Cheeger-type and Buser-type inequalities on negatively curved surfaces. Building upon these foundational studies, we investigate the Steklov eigenvalues of pinched negatively curved manifolds with totally geodesic boundary. These eigenvalues are associated with a first-order elliptic pseudodifferential operator known as the Dirichlet-to-Neumann operator. We discuss how the results for Laplace eigenvalues can be extended to Steklov eigenvalues. In particular, we show a spectral gap for the Steklov problem in dimensions of at least three. This talk is based on joint work with Ara Basjmaian, Jade Brisson, and Antoine Métras.

Mikhail Karpukhin
Title: New embedded minimal surfaces in 3-sphere and 3-ball via eigenvalue optimisation.
Abstract: The study of optimal upper bounds for Laplace eigenvalues on closed surfaces under area constraint is a classical problem of spectral geometry. It is particularly interesting due to the fact that optimal metrics (if exist) correspond to branched minimal surface in n-dimensional sphere. In general, determining whether this surface exists, is embedded, and finding the dimension of the sphere are very challenging problems, where very few results are known. In the present talk we will discuss how one can use group action to resolve these issues and, as a result, construct examples of embedded minimal surfaces in the 3-sphere. Similar results will be outlined for Steklov eigenvalues that correspond to free boundary minimal immersions in the unit 3-ball. Based on a joint work in progress with R. Kusner, P. McGrath and D. Stern.

Andrea Malchiodi
Title: Critical points of the Moser-Trudinger functional
Abstract: It is known that Sobolev functions of class $W_0^{1,2}$ on two-dimensional bounded domains satisfy critical embedding properties of exponential type. In 1971 Moser obtained a sharp inequality controlling the integrability of $F(u) := \int \exp(u^2)$ if $\int |\nabla u|^2 \leq 4\pi$. On the other hand, $F(u)$ is unbounded above if $\int |\nabla u|^2 > 4\pi$. We consider critical points of F constrained to any sphere $\{\int |\nabla \cdot|^2 = \beta\}$, with $\beta > 0$ arbitrary, showing that they always exist if the domain is non contractible. We also compute the Leray-Schauder degree of the constrained Euler-Lagrange equation. This is joint work with F. De Marchis, L. Martinazzi and P. D. Thizy.

Alexis Michelat
Title: Morse Index Stability Beyond Minimal Surfaces: Da Lio-Gianocca-Rivière's Theory
Abstract: The Morse index of a critical point of a Lagrangian L is the dimension of the maximal vector space on which the second derivative D^2L is negative-definite. In the classical theory of Hilbert spaces, one shows that the Morse index is lower semi-continuous, while the sum of the Morse index and nullity (the dimension of the Kernel of the differential operator associated to the second derivative) is upper semi-continuous.

Last year (arXiv:2212.03124), Francesca Da Lio, Matilde Gianocca, and Tristan Rivière (ETH Zürich) developed a new method to show upper semi-continuity results in geometric analysis—that they applied to conformally invariant Lagrangians in dimension 2 (which include harmonic maps).

In this talk, we will explain how to apply this method to the Willmore energy (work in collab- oration with Tristan Rivière) and—intrinsic or extrinsic—biharmonic maps in dimension 4. Each problem comes with its own set of technical difficulties that one needs to overcome with different though universal in nature methods.

Mickaël Nahon
Title: Sharp Quantitative Stability of the Dirichlet spectrum near the ball
Abstract: Let $\Omega \subset \mathbb{R}^n$ be an open set with same volume as the unit ball B and let $\lambda_k(\Omega)$ be the k-th eigenvalue of the Laplacian with Dirichlet condition of Ω. Suppose $\lambda_1(\Omega)$ is close to $\lambda_1(B)$, how close is $\lambda_k(\Omega)$ to $\lambda_k(B)$? We establish quantitative bounds with sharp exponents depending on the multiplicity of $\lambda_k(B)$ through the study of a perturbed shape optimization problem, in particular we prove the persistence of the ball as minimizer for a large class of spectral functionals which are small perturbations of the fundamental eigenvalue. This is a joint work with Dorin Bucur, Jimmy Lamboley and Raphaël Prunier.

Marc Pegon
Title: An isoperimetric problem involving the competition between the perimeter and a nonlocal perimeter
Abstract: In this talk, I will present an isoperimetric problem in which the perimeter is replaced by $P - \gamma P_\varepsilon$, where $\gamma \in (0, 1)$, P stands for the classical perimeter and P_ε is a nonlocal energy which converges to the perimeter as ε vanishes. This problem is derived from Gamow's liquid drop model for the atomic nucleus in the case where the repulsive potential is sufficiently decaying at infinity and in the large mass regime. I will discuss the existence, and characterization of minimizers for small ε.

Alessandro Pigati
Title: Regularizing the mean curvature flow in codimension two with the abelian Yang-Mills-Higgs energy
Abstract: Inspired by Ilmanen's regularization of mean curvature flow using the Allen-Cahn energy, which is a successful approximation for hypersurfaces, we discuss a way to regularize it in codimension two, using the abelian Yang-Mills-Higgs model used in superconductivity and studying the singular set where energy concentrates as a suitable scaling parameter goes to zero. After discussing earlier results in the static setting, we will look at their parabolic analogue, and we will compare this with other previously known ways to regularize the mean curvature flow. We will also see a couple of simple variational applications.

Felix Schulze
Title: Generic regularity for minimizing hypersurfaces in dimensions 9 and 10
Abstract: Let Γ be a smooth, closed (compact and boundaryless), oriented, $(n-1)$-dimensional submanifold of $\mathbb{R}^{n+1}$. The Plateau problem asks if among all smooth, compact, oriented hypersurfaces $M \subset \mathbb{R}^{n+1}$ with $\partial M = \Gamma$, does there exist one with least area? Foundational results in geometric measure theory can be used to show that for $n+1 \leq 7$ there is a smooth, compact, oriented, area-minimizing hypersurface M solving the problem. In higher dimensions smooth minimizers can fail to exist but it is nevertheless known that away from a closed set sing$M \subset \mathbb{R}^{n+1} \setminus \Gamma$ of Hausdorff dimension $\leq n-7$, there is a minimizer M which is a smooth hypersurface with boundary Γ. A fundamental result of Hardt-Simon shows that the singularities (necessarily isolated points) for $n+1 = 8$ can be eliminated by a slight perturbation of the boundary, Γ, thus yielding solutions to the original problem. It has been a longstanding conjecture that similar results hold in higher dimensions. We show that for $n+1 = 9, 10$ singularities can still be perturbed away as well. This is joint work with O. Chodosh and C. Mantoulidis.

Dominik Stantejsky
Title: Minimizers of the Ginzburg-Landau Energy with Planar Boundary Anchoring.
Abstract: In this talk, I will present recent results on minimizers of the Ginzburg-Landau energy in the singular limit subject to strong anchoring tangential boundary condition. Through a reflection method, we are able to study regularity of minimizers close to the boundary. We also obtain results on the type and location of defects that can occur, such as boundary "boojums" and interior vortices. The talk is based on joint work with L. Bronsard and A. Colinet.

David Tewodrose
Title: Critical metrics of eigenvalue functionals via subdifferential
Abstract: I will present a joint work with Romain Petrides (Université Paris Cité) where we propose a general approach to study critical metrics of functionals F from U to $\mathbb{R}$, $F(g) = F(S_g)$, where U is an open set of Riemannian metrics on a given smooth manifold, S_g is a set of eigenvalues depending on g and F is a locally Lipschitz function. At the core of our approach is Clarke's notion of subdifferential. This covers well-known cases, like Laplace and Steklov eigenvalues, and provides new possibilities for other situations.

Alex Waldron
Title: A report on 2D harmonic map flow
Abstract: Harmonic map flow has been studied extensively in the critical dimension but key open questions remain. A notorious counterexample of Topping can be weighed against a conjecture (also due to Topping) that for real-analytic target metrics, the body map at a finite-time singularity has only removable discontinuities. Uniqueness of subsequential limits at infinite time is an equally subtle question. I'll discuss recent progress on continuity of the body map when the target is compact Kähler with nonnegative holomorphic bisectional curvature (joint with C. Song, to appear in JDG), as well as forthcoming work on Lojasiewicz inequalities and quantitative convergence when the target is S^2.

Half an hour talks

Antonia Diana
Title: Elastic flow of curves with partially fixed boundary points: short and long–time existence results.
Abstract: Geometric gradient flows for elastic energies play an important role in mathematics and in many applications. In our work, we consider a curve with boundary points free to move on a line in $\mathbb{R}^2$ (the x–axis, for instance), which evolves by the L^2–gradient flow of the elastic energy, which is a linear combination of the Willmore and the length functional. For such planar evolution problem we study the short and long–time existence. Once we establish under which boundary conditions the PDE's system is well–posed (in our case the Navier boundary condition), employing the Solonnikov theory for linear parabolic systems in Hölder space, we show that there exists a unique flow in a maximal time interval $[0, T)$. Then, we also show that the solution is unique in a purely geometric sense. Finally, using energy methods we prove that this maximal time is actually $T = +\infty$. This is a joint work with Alessandra Pluda.

Lucas Lavoyer
Title: Ricci flow from spaces with edge type conical singularities
Abstract: In this talk, we will construct a solution to Ricci flow coming out of spaces with edge type conical singularities along a closed, embedded curve, under the additional assumption that for each point of the curve, our space is locally modelled on the product of a fixed positively curved cone and a line. We also prove curvature estimates for the solution and, for edge points, we show that the tangent flow at these points is a positively curved expanding gradient Ricci soliton solution crossed with a line.

Dorian Martino
Title: Classification of branched Willmore spheres.
Abstract: The Willmore energy is a way to merge minimal surface and conformal geometry. In 1984, Bryant proved that any smooth Willmore sphere is a conformal transformation of a minimal surface. However when studying the compactness of Willmore surfaces or the Willmore flow, branched spheres naturally arise. In this talk, we will show that Bryant's proof extends to branched spheres.

Antoine Metras
Title: Dirac eigenvalue optimization on surfaces
Abstract: When optimizing Laplace eigenvalues on a surface with respect to the metric (in a fixed conformal class), one is naturally led to a connection between critical metrics and harmonic maps to sphere. In this talk, I will present a similar result for Dirac eigenvalues on surfaces, where now the critical metric is related to some special harmonic maps to complex projective space. Joint work with M. Karpukhin and I. Polterovich.

Lorenzo Sarnataro
Title: Optimal regularity for minimizers of the prescribed mean curvature functional over isotopies.
Abstract: In this talk, I will describe the regularity theory for surfaces minimizing the prescribed mean curvature functional over isotopies in a closed Riemannian 3-manifold, which is a prescribed mean curvature counterpart of the celebrated regularity result of Meeks, Simon and Yau about minimizers of the area functional over isotopies. Whereas

for the area functional minimizers over isotopies are smooth embedded minimal surfaces, minimizers of the prescribed mean curvature functional turn out to be $C^{1,1}$ immersions which can have a large self-touching set where the mean curvature vanishes. Even though the proof broadly follows the same general strategy as in the case of the area functional, several new ideas are needed to deal with the lower regularity setting. This regularity theory plays an important role in Z. Wang-X. Zhou's recent proof of the existence of 4 embedded minimal spheres in a generic metric on the 3-sphere. The results in this talk are joint work with Douglas Stryker (Princeton).

Dong-Hwi Seo
Title: Uniqueness results for the critical catenoid.
Abstract: A free boundary minimal surface in the three-dimensional unit ball is a properly immersed minimal surface in the unit ball that meets the unit sphere orthogonally along the boundary of the surface. The topic was initiated by Nitsche in 1985, derived from studies by Gergonne, Schwarz, Courant, and Lewy. Basic examples are the equatorial disk and the critical catenoid. The equatorial disk is the only immersed free boundary minimal disk in the ball up to congruence. The critical catenoid is claimed to be the only embedded free boundary minimal annulus in the ball up to congruence. Recently, the problem has been attempted using a relationship with the Steklov eigenvalue problem. In this talk, I will describe uniqueness results for the critical catenoid under some symmetry conditions.