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# About

## Organizing committee

Nicolas Champagnat (Centre Inria Nancy – Grand Est)  
Gilles Pagès (Sorbonne Université)  
Etienne Tanré (Centre Inria d’Université Côte d’Azur)  
Milica Tomašević (CNRS, École polytechnique)

## Scientific committee

Nicolas Champagnat (Centre Inria Nancy – Grand Est)  
Sylvie Méléard (École polytechnique)  
Gilles Pagès (Sorbonne Université)  
Philippe Protter (Columbia University)  
Etienne Tanré (Centre Inria d’Université Côte d’Azur)  
Milica Tomašević (CNRS, École polytechnique)

# Timetable

## Monday 4 September

9:00–9:30	<b>Registration</b>	
9:30–9:45	<b>Welcome speech</b>	
9:45–10:30	<b>Pierre-Louis Lions</b> Collège de France	Large Random Matrices and PDE's
10:30–11:00	<b>Coffee</b>	
11:00–11:45	<b>Bernard Lapeyre</b> Ecole des Ponts	How many inner simulations to compute conditional expectations with least-square Monte Carlo?
11:45–12:30	<b>Carl Graham</b> CNRS, École Polytechnique	Perfect simulation of the invariant laws of Markovian load-balancing queueing networks
12:30–14:00	<b>Lunch</b>	
14:00–14:45	<b>Philip Protter</b> Columbia University	The Cox Construction of Stopping Times and Markov Processes
14:45–15:30	<b>Christophette Blanchet-Scalliet</b> Ecole Centrale de Lyon	Gambling for resurrection and the heat equation on a triangle
15:30–16:00	<b>Coffee</b>	
16:00–16:45	<b>Emmanuel Gobet</b> École Polytechnique	Walking forward and backward in Euler schemes and random number generators
17:00–19:30	<b>Poster session with Welcome Drink</b>	
19:30–	<b>Dinner</b>	

## Tuesday 5 September

7:00–9:00	<b>Breakfast</b>	
9:00	<b>Start</b>	
9:00–9:45	<b>Sylvie Méléard</b> École Polytechnique	Exponent dynamics for branching processes
9:45–10:30	<b>Nicolas Champagnat</b> Inria Nancy	Wasserstein convergence of penalized Markov processes
10:30–11:00	<b>Coffee</b>	
11:00–11:45	<b>Fabien Panloup</b> Université Angers	Asymptotically unbiased approximation of the QSD of diffusion processes
11:45–12:30	<b>Quentin Cormier</b> Inria Saclay	Stability and metastability in mean-field equations
12:30–14:00	<b>Lunch</b>	
14:00–14:45	<b>Vlad Bally</b> Université Marne la Vallée	Construction of Boltzmann and McKean Vlasov type flows (the sewing lemma approach)
14:45–15:30	<b>Nicolas Fournier</b> Sorbonne Université	Systèmes de particules pour l'équation de Keller-Segel dans le plan.
15:30–16:00	<b>Coffee</b>	
16:00–16:45	<b>Milica Tomašević</b> CNRS, École Polytechnique	Propagation of chaos for stochastic particle systems with singular mean-field interaction of $L^q - L^p$ type
16:45–17:30	<b>Benjamin Jourdain</b> École des Ponts	Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with $L^q - L^p$ drift coefficient and additive noise
17:30–18:15	<b>Alexandre Richard</b> CentraleSupélec	Regularisation by noise for SDEs driven by fractional Brownian motion
19:30–	<b>Dinner</b>	

## Wednesday 6 September

7:00–9:00	<b>Breakfast</b>	
9:00–9:45	<b>Jean Jacod</b> Sorbonne Université	Systematic Jump Risk
9:45–10:30	<b>Damien Lambertson</b> Université Marne la Vallée	Regularity results in optimal stopping: a probabilistic approach
10:30–11:00	<b>Coffee</b>	
11:00–11:45	<b>Olivier Pironneau</b> Sorbonne Université	Numerical Analysis Of Degenerate Kolmogorov Equations of Constrained Stochastic Hamiltonian Systems
11:45–12:30	<b>Benoîte de Saporta</b> Université Montpellier	Stochastic control for medical treatment optimization
12:30–14:00	<b>Lunch</b>	
14:00–18:00	<b>Visit MUCEM</b>	
19:00–	<b>Dinner</b>	

## Thursday 7 September

7:00–9:00	<b>Breakfast</b>	
9:00–9:45	<b>Nicole El Karoui</b> Sorbonne Université	Forward Convex random fields with Applications to Convex pricing.
9:45–10:30	<b>Gilles Pagès</b> Sorbonne Université	Functional convex order for stochastic processes: a constructive (and simulable) approach
10:30–11:00	<b>Coffee</b>	
11:00–11:45	<b>Antoine Lejay</b> Inria Nancy	Walking in abrupt lands
11:45–12:30	<b>Stéphane Menozzi</b> Université Evry	Multidimensional Stable driven McKean-Vlasov SDEs with distributional interaction kernel : existence, uniqueness and propagation of chaos
12:30–14:00	<b>Lunch</b>	
14:00–14:45	<b>Étienne Pardoux</b> Université Aix-Marseille	Recent results on epidemic models
14:45–15:30	<b>François Delarue</b> Université Côte d'Azur	Rearranged stochastic heat equation
15:30–16:00	<b>Coffee</b>	
16:00–16:45	<b>Julien Claisse</b> Université Paris-Dauphine	Mean-field Optimization regularized by Fisher Information
16:45–17:30	<b>Miguel Martinez</b> Université Marne la Vallée	An introduction to the pseudo skew Brownian motion
19:30–	<b>Dinner</b>	

## Friday 8 September

7:00–9:00	<b>Breakfast</b>	
9:00–9:45	<b>Éva Löcherbach</b> Université Paris 1	Conditional propagation of chaos for generalized Hawkes processes having alpha-stable jump heights.
9:45–10:30	<b>Etienne Tanré</b> Inria Université Côte d'Azur	Synchronization detection by permutation test
10:30–11:00	<b>Coffee</b>	
11:00–11:45	<b>Jean-François Jabir</b> HSE University, Moscow	A Kramer's type law for the first collision-time of two self-stabilizing diffusions
11:45–12:30	<b>Olivier Faugeras</b> Inria Université Côte d'Azur	Two examples of thermodynamic limits in neuroscience
12:30–14:00	<b>Lunch</b>	
14:00	<b>End of the conference</b>	

# List of Abstracts – Talks

## Monday 4th

### Large Random Matrices and PDE's

*Pierre-Louis Lions*

Collège de France

### How many inner simulations to compute conditional expectations with least-square Monte Carlo?

*Bernard Lapeyre*

Ecole des Ponts

The problem of computing the conditional expectation  $\mathbb{E}[f(Y)|X]$  with least-square Monte-Carlo is of general importance and has been widely studied. To solve this problem, it is usually assumed that one has as many samples of  $Y$  as of  $X$ . However, when samples are generated by computer simulation and the conditional law of  $Y$  given  $X$  can be simulated, it may be relevant to sample  $K \in N$  values of  $Y$  for each sample of  $X$ . The present work determines the optimal value of  $K$  for a given computational budget, as well as a way to estimate it. The main take away message is that the computational gain can be all the more important that the computational cost of sampling  $Y$  given  $X$  is small with respect to the computational cost of sampling  $X$ . Numerical illustrations on the optimal choice of  $K$  and on the computational gain are given on different examples including one inspired by risk management.



## **Perfect simulation of the invariant laws of Markovian load-balancing queueing networks**

***Carl Graham***

CNRS, École Polytechnique

We define a wide class of Markovian load balancing queueing networks, including classic networks studied in a large and lively literature on the subject. Each network has identical single-server infinite-buffer queues and implements a load balancing policy to allocate each task at its arrival and possibly reallocate it at a service completion. The purpose of the policy is to optimize server utilization under constraints such as limited information, real-time decision taking, and network topology, and the queue length process is possibly not exchangeable. The invariant law is in general not known even up to normalizing constant. We provide perfect simulation methods in view of Monte Carlo estimation of quantities of interest in equilibrium, for instance for performance evaluation. In this infinite multi-dimensional state space, we use an unusual preorder defining an order up to permutation of the coordinates, define a coupling in which networks in this class are dominated by the network with uniform routing, and implement dominated coupling from the past methods.

## **The Cox Construction of Stopping Times and Markov Processes**

***Philip Protter***

Columbia University

The Cox Construction of a totally inaccessible stopping time with a given compensator is ubiquitous in Mathematical Finance, and in particular in Credit Risk. On the other hand, as P.A. Meyer showed long ago, totally inaccessible stopping times arise naturally as the jump times of a strong Markov process. We relate the two ideas and propose a solution to a question posed by Monique Jeanblanc.

## **Gambling for resurrection and the heat equation on a triangle**

***Christophette Blanchet-Scalliet***

Ecole Centrale de Lyon

We consider the problem of controlling the diffusion coefficient of a diffusion with constant negative drift rate such that the probability of hitting a given lower barrier up to some finite time horizon is minimized. We assume that the diffusion rate can be chosen in a progressively measurable way with values in the interval  $[0,1]$ . We prove that the value function is regular, concave in the space variable, and that it solves the associated HJB equation. To do so, we show that the heat equation on a right triangle, with a boundary condition that is discontinuous in the corner, possesses a smooth solution.

Work in Collaboration with Stefan Ankirchner, Nabil Kazi-Tani, Chao Zhou

## Walking forward and backward in Euler schemes and random number generators

*Emmanuel Gobet*

École Polytechnique

I will present how to take advantage of the reversibility of some Random Number Generators to resampling the path of Euler approximations of a SDE. Doing so, this induces an additional error  $(1/N)$  which we prove to be smaller than the one for forward sampling  $(1/\sqrt{N})$ , with respect to the number of time steps  $N$ . The method allows in particular to retrieve sampled points with a minimal amount of information. It can be used in regression Monte Carlo methods for memory saving concerns.

## Tuesday 5th

### Exponent dynamics for branching processes

*Sylvie Méléard*

École Polytechnique

We consider a stochastic model for the evolution of a discrete population structured by a trait taking finitely many values on a grid of  $[0, 1]$ , with mutation and selection. We study of the dynamics of the population in logarithm size and time scales, under a large population assumption. In the first part of the talk, individual mutations are rare but the global mutation rate tends to infinity. Then negligible sub-populations may have a strong contribution to evolution. The traits can also be horizontally transferred, leading to a trade-off between natural evolution to higher birth rates and transfer which drives the population towards lower birth rates. We prove that the stochastic discrete exponent process converges to a piecewise affine continuous function, which can be described along successive phases determined by dominant traits. In the second part of the talk, the individual mutations are small but not rare, we don't have any transfer and we assume the grid mesh for the trait values becoming smaller and smaller. We establish that under our rescaling, the stochastic discrete exponent process converges to the viscosity solution of a Hamilton-Jacobi equation, filling the gap between individual-based evolutionary models and Hamilton-Jacobi equations.

Joint works with N. Champagnat and V.C. Tran, and S. Mirrahimi for the second part.

### Wasserstein convergence of penalized Markov processes

*Nicolas Champagnat*

Inria Nancy

We consider a Markov process living in some space  $E$ , and killed (penalized) at a rate depending on its position. In the last decade, several conditions have been given ensuring that the law of the process conditioned on survival converges to a quasi-stationary distribution exponentially fast in total variation distance. In this talk, we will present very simple examples of penalized Markov process whose conditional law cannot converge in total variation, and we will give a sufficient condition implying contraction and convergence of the conditional law in Wasserstein distance to a unique quasi-stationary distribution. Our criterion also imply a first-order expansion of the probability of survival, the ergodicity in Wasserstein distance of the Q-process, i.e. the process conditioned to never be killed, and quasi-ergodicity in Wasserstein distance. We then apply this criterion to several examples, including Bernoulli convolutions and piecewise deterministic Markov processes of the form of switched dynamical systems, for which convergence in total variation is not possible.

This is joint work with Edouard Strickler (CNRS, Université de Lorraine) and Denis Villemonais (Université de Lorraine).

## Asymptotically unbiased approximation of the QSD of diffusion processes

*Fabien Panloup*

Université Angers

We build and study a recursive algorithm based on the occupation measure of an Euler scheme with decreasing step for the numerical approximation of the quasistationary distribution (QSD) of an elliptic diffusion in a bounded domain. We prove the almost sure convergence of the procedure for a family of redistributions and show that we can also recover the approximation of the rate of survival and the convergence in distribution of the algorithm. This last point follows from some new bounds on the weak error related to diffusion dynamics with renewal.

This is a joint work with Julien Reygner.

## Stability and metastability in mean-field equations

*Quentin Cormier*

Inria Saclay

Consider the following mean-field equation on  $\mathbb{R}^d$ :

$$dX_t = V(X_t, \mu_t)dt + dB_t$$

, where  $\mu_t$  is the law of  $X_t$ , the drift  $V(x, \mu)$  is smooth and confining, and  $(B_t)$  is a standard Brownian motion. This McKean-Vlasov equation may admit multiple invariant probability measures. I will discuss the (local) stability of one of these equilibria. Using Lions derivatives, a stability criterion is derived, analogous to the Jacobian stability criterion for ODEs. Under this spectral condition, the equilibrium is shown to be attractive for the Wasserstein metric  $W_1$ . In addition, I will discuss a metastable behavior of the associated particule system, around a stable equilibrium of the mean-field equation.

<https://arxiv.org/abs/2201.11612>

## **Construction of Boltzmann and McKean Vlasov type flows (the sewing lemma approach)**

***Vlad Bally***

Université Marne la Vallée

We are concerned with a mixture of Boltzmann and McKean-Vlasov type equations, this means (in probabilistic terms) equations with coefficients depending on the law of the solution itself, and driven by a Poisson point measure with the intensity depending also on the law of the solution. Both the analytical Boltzmann equation and the probabilistic interpretation initiated by Tanaka (1978) have intensively been discussed in the literature for specific models related to the behavior of gas molecules. In this paper, we consider general abstract coefficients that may include mean field effects and then we discuss the link with specific models as well. In contrast with the usual approach in which integral equations are used in order to state the problem, we employ here a new formulation of the problem in terms of flows of endomorphisms on the space of probability measure endowed with the Wasserstein distance. This point of view already appeared in the framework of rough differential equations. Our results concern existence and uniqueness of the solution, in the formulation of flows, but we also prove that the "flow solution" is a solution of the classical integral weak equation and admits a probabilistic interpretation. Moreover, we obtain stability results and regularity with respect to the time for such solutions. Finally we prove the convergence of empirical measures based on particle systems to the solution of our problem, and we obtain the rate of convergence. We discuss as examples the homogeneous and the inhomogeneous Boltzmann (Enskog) equation with hard potentials.

Joint work with Aurélien Alfonsi.

## **Systèmes de particules pour l'équation de Keller-Segel dans le plan.**

***Nicolas Fournier***

Sorbonne Université

L'équation de Keller-Segel décrit le mouvement de cellules par chimiotaxie. Les cellules diffusent dans le plan, et émettent un produit chimique. Ce produit, qui diffuse aussi, attire les cellules. Ceci conduit à une interaction relativement singulière entre les cellules (via le produit). Cette interaction est critique au sens où, selon les valeurs des constantes, il peut y avoir existence globale d'une solution, où formation d'un amas de cellules en temps fini. On parlera de l'approximation de cette équation par des systèmes de particules stochastiques, dans le cas elliptique, où le produit diffuse instantanément, et dans le cas parabolique, où le produit diffuse à une vitesse finie. La difficulté provient de la singularité de l'attraction.

Issu de travaux avec B. Jourdain, avec Y. Tardy et avec M. Tomasevic.

## **Propagation of chaos for stochastic particle systems with singular mean-field interaction of $L^q - L^p$ type**

**Milica Tomašević**

CNRS, École Polytechnique

In this work, we prove the well-posedness and propagation of chaos for a stochastic particle system in mean-field interaction under the assumption that the interacting kernel belongs to a suitable  $L_t^q - L_x^p$  space. Contrary to the large deviation principle approach recently proposed in the literature (Hoeksma et al, 2020), the main ingredient of the proof here are the Partial Girsanov transformations introduced by (Jabir-Talay-Tomašević.,2018) and developed in a general setting here.

## **Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with $L^q - L^p$ drift coefficient and additive noise**

**Benjamin Jourdain**

École des Ponts

We are interested in the discretization of stochastic differential equations with additive  $d$ -dimensional Brownian noise and  $L^q$ - $L^p$  drift coefficient when the condition  $d/p+2/q < 1$ , under which Krylov and Röckner proved existence of a unique strong solution, is met. We consider the Euler scheme with randomized time variable and cutoffed drift coefficient so that its contribution on each time-step does not dominate the Brownian contribution. We prove that both the diffusion and this Euler scheme admit transition densities and that the difference between these densities is bounded from above by some centered Gaussian density multiplied by the time-step to the order  $(1-(d/p+2/q))/2$  which corresponds to half the distance to the threshold.

(Joint work with Stéphane Menozzi)

## **Regularisation by noise for SDEs driven by fractional Brownian motion**

**Alexandre Richard**

CentraleSupélec

In this talk, we consider the SDE  $dX_t = b(X_t) + dB_t$ , where  $b$  is a distribution and  $B$  is a fractional Brownian motion of Hurst parameter  $H \leq 1/2$ . We present several results about this equation, starting from a well-posedness criterion that relates the regularity of  $b$  and the parameter  $H$ , then we present an approximation by a tamed Euler scheme, and we discuss the regularity of the law of the solution. Finally we will mention extensions towards McKean-Vlasov equations driven by fBm.

Based on joint works with L. Anzeletti, L. Goudenège, E. Haress and E. Tanré.

## Wednesday 6th

### Systematic Jump Risk

*Jean Jacod*

Sorbonne Université

In a factor model for a large panel of  $N$  asset prices, a random time  $S$  is called a "systematic jump time" if it is not a jump time of any of the factors, but nevertheless is a jump time for a significant number of prices: one might for example think that those  $S$ 's are jump times of some hidden or unspecified factors. Our aim is to test whether such systematic jumps exist and, if they do, to estimate a suitably defined "aggregated measure" of their sizes. The setting is the usual high frequency setting with a finite time horizon  $T$  and observations of all prices and factors at the times  $iT/n$  for  $i = 0, \dots, n$ . We suppose that both  $n$  and  $N$  are large, and the asymptotic results (including feasible estimation of the above aggregate measure) are given when both go to  $\infty$ , without imposing restrictions on their relative size.

(joint work with Huidi Lin and Viktor Todorov)

### Regularity results in optimal stopping: a probabilistic approach

*Damien Lambertson*

Université Marne la Vallée

We discuss regularity results for optimal stopping of one-dimensional diffusions using probabilistic methods. In particular, we address the differentiability of the free boundary. We also derive estimates for the derivatives of the value function.

This talk is based on joint work with Tiziano De Angelis (University of Torino).

## **Numerical Analysis Of Degenerate Kolmogorov Equations of Constrained Stochastic Hamiltonian Systems**

***Olivier Pironneau***

Sorbonne Université

In this work, we propose a method to compute numerical approximations of the invariant measures and Rice's formula (frequency of threshold crossings) for a certain type of stochastic Hamiltonian system constrained by an obstacle and subjected to white or colored noise. As an alternative to probabilistic Monte-Carlo simulations, our approach relies on solving a class of degenerate partial differential equations with non-local Dirichlet boundary conditions, as derived in [Mertz, Stadler, Wylie; 2018]. A functional analysis framework is presented; regularisation and approximation by the finite element method is applied; numerical experiments on these are performed and show good agreement with probabilistic simulations.

(Joint work with L. Mertz)

## **Stochastic control for medical treatment optimization**

***Benoîte de Saporta***

Université Montpellier

We are interested in monitoring patients in remission from cancer. Our aim is to detect their relapses as soon as possible, as well as detect the type of relapse, to decide on the appropriate treatment to be given. Available data are some marker level of the rate of cancerous cells in the blood which evolves continuously but is measured at discrete (large) intervals and through noise. The patient's state of health is modeled by a piecewise deterministic Markov process (PDMP). Several decisions must be taken from these incomplete observations: what treatment to give, and when to schedule the next medical visit. After presenting a suitable class of controlled PDMPs to model this situation, I will describe the corresponding stochastic control problem and will present the resolution strategy that we adopted. The objective is to obtain an approximation of the value function (optimal performance) as well as build an explicit policy applicable in practice and as close to optimality as possible. The results will be illustrated by simulations calibrated on a cohort of a clinical trial on multiple myeloma provided by the Center of Cancer Research in Toulouse.



## Thursday 7th

### Forward Convex random fields with Applications to Convex pricing.

Nicole El Karoui

Sorbonne Université

Concave and convex functions are basic functions in economy and finance. In derivatives market, options pay-offs as Call and Put are in general convex functions of their underlying  $((x - K)^+$ , or  $(K - x)^+$ ) and their Black-Scholes Prices are also convex. This property can be maintained in a random universe, (without reference to finance). Here, we are looking for the pricing point of view. The data is an underlying random field,  $\{X_t(x)\}$ , non negative with  $X_t(0) = 0, X_t(+\infty) = \infty$ , and a pricing (strictly) convex function  $\Phi(0, z)$  whose the right-derivative is denoted  $\phi$ , given the price today of convex European derivative. The problem is to characterize a convex pricing rule  $\{\Phi(t, z)\}$  in the future, optimal in the sense that  $\{\Phi(t, X_t(x))\}$  is a martingale. Obviously, without additional constraint, the problem has many solutions. So, thanks to convexity assumptions, it is natural to introduce the convex conjugate random field  $\Psi(t, y)$ . By the Fenchel theory, the Gap function  $G_\Phi(t, z, y) = \Phi(t, z) + \Psi(t, y) - zy \geq 0, = 0$  if  $\phi(t, z) = y$ .

Put  $Y_t(\phi(z)) := \Phi_z(t, X_t(z))$ . The problem is to solve a revealed problem find a pair of conjugate convex random fields  $(\Phi(t, z), \Psi(t, y))$  such that  $\Phi(t, X_t(x))$  and  $\Psi(t, Y_t(y))$  are martingales. The Legendre formula implies that  $X_t(z)Y_t(\phi(z))$  is a martingale. As for revealed utility, the problem has at least a solution if and only if there exists an equivalent intrinsic framework, where necessarily the processes " $\{X_t(x)\}, \{Y_t(y)\}, \{\Phi(t, z)\}$ " are supermartingales, and  $\{X_t(x)Y_t(\phi(x))\}$  is a martingale. The family  $\{Y_t(\phi(x))\}$  is a family of pricing kernel for  $X_t(x)$ . The relation  $Y_t(\phi(z)) := \Phi_z(t, X_t(z))$ , and the monotony of  $X_t(z)$  gives the way to obtain  $\Phi_z(t, z) = Y_t(\phi(X_t^{-1}(z)))$  by a pathwise procedure. The convexity of the pricing kernel reduced the arbitrage problems. Itô's semimartingale framework is used to illustrate this characterization. The revealed pricing kernel  $y$  is solution of a non-linear SPDE. Many properties can be deduced of this pathwise construction.

Joint work Mohamed Mrad.

## Functional convex order for stochastic processes: a constructive (and simulable) approach

*Gilles Pagès*

Sorbonne Université

After a few reminders on the convex order  $\leq_{cv}$  between two random vectors  $U$  and  $V$  defined by

$$U \leq_{cv} V \text{ if } \mathbb{E}f(U) \leq \mathbb{E}f(V)$$

for every convex function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$ , (with some variants like monotonic convex order) and their first applications in finance, we will explain how to extend this order in a functional way to stochastic processes, in particular to diffusions (Brownian, with jumps, McKean Vlasov type), even to non-Markovian processes, such as the solutions of Volterra equations with singular kernels like those appearing in rough volatility modeling in Finance. We systematically establish our comparison results by an approximation procedure of Euler scheme type, generally simulable. Thus, among other virtues, this approach makes it possible in finance to ensure that the prices of derivative products computed by simulation cannot give rise to arbitrages by lack of convexity. As a by-product we will also establish the convexity of functionals  $x \rightarrow \mathbb{E}F(X^x)$  of such stochastic processes  $X^x$  when  $F$  is convex and  $x$  is the starting value of  $X^x$ .

(includes some joint works with B. Jourdain and Y. Liu).

## Walking in abrupt lands

*Antoine Lejay*

Inria Nancy

There are many situations in which the land supporting the random walk present some sharp variations, which require appropriate techniques at the intersection between probability and analysis.

In this talk, we present an overview of several approaches to tackle the problem of simulating diffusion processes in discontinuous media.

## **Multidimensional Stable driven McKean-Vlasov SDEs with distributional interaction kernel : existence, uniqueness and propagation of chaos**

***Stéphane Menozzi***

Université Evry

We are interested in establishing weak and strong well-posedness for McKean-Vlasov SDEs with additive stable noise and a convolution type non-linear drift with singular interaction kernel in the framework of Lebesgue-Besov spaces. In particular, we characterize quantitatively how the non-linearity allows to go beyond the thresholds obtained for linear SDEs with singular interaction kernels. We prove that the thresholds deriving from the scaling of the noise can be achieved and that the corresponding SDE can be understood in the classical sense. We also specifically characterize in function of the stability index of the driving noise and the parameters of the drift when the dichotomy between weak and strong uniqueness occurs.

The associated convergence rate for the propagation of Chaos, based on the previous a priori controls, will be discussed as well.

## Recent results on epidemic models

*Étienne Pardoux*

Université Aix-Marseille

In 1927, two Scottish epidemiologists, Kermack and McKendrick, published a paper on a SIR epidemic model, where each infectious individual has an age of infection dependent infectivity, and a random infectious period whose law is very general. This paper was quoted a huge number of times, but almost all authors who quoted it considered the simple case of a constant infectivity, and a duration of infection following the exponential distribution, in which case the integral equation model of Kermack and McKendrick reduces to an ODE.

It is classical that an ODE epidemic model is the Law of Large Numbers limits, as the size of the population tends to infinity, of finite population stochastic Markovian epidemic models.

One of our main contributions in recent years has been to show that the integral equation epidemic model of Kermack and McKendrick is the law of large numbers limit of stochastic non Markovian epidemic models. It is not surprising that the model of Kermack and Mc Kendrick, unlike ODE models, has a memory, like non Markovian stochastic processes. One can also write the model as a PDE, where the additional variable is the age of infection of each infected individual.

Similar PDE models have been introduced by Kermack and Mc Kendrick in their 1932 and 1933 papers, where they add a progressive loss of immunity. We have also shown that this 1932-33 model is the Law of Large Numbers limit of appropriate finite population non Markovian models.

Joint work with R. Forien (INRAE Avignon, France), G. Pang (Rice Univ., Houston, Texas, USA) and A.B. Zotsa-Ngoufack (AMU and Univ. Yaoundé 1)

### Bibliography

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## Rearranged stochastic heat equation

**François Delarue**

Université Côte d'Azur

We provide an explicit construction of a strong Feller semigroup on the space of 1d probability measures that maps bounded measurable functions into Lipschitz continuous functions, with a Lipschitz constant that blows up in an integrable manner in small time. The construction relies on a rearranged version of the stochastic heat equation on the circle driven by a coloured noise. Under the action of the rearrangement, the solution is forced to live in a space of quantile functions that is isometric to the space of probability measures on the real line. As an application, we show that the noise resulting from this approach can be used to perturb, in an ergodic manner, gradient flows on the space of 1d probability measures. We also show that the same noise can be used to enforce uniqueness to some types of mean field games.

Based on joint works with William Hammersley (Nice) and Youssef Ouknine (Marrakech).

## Mean-field Optimization regularized by Fisher Information

**Julien Claisse**

Université Paris-Dauphine

Recently there is a rising interest in the research of mean-field optimization, in particular because of its role in analyzing the training of neural networks. In this talk, by adding the Fisher Information (in other word, the Schrodinger kinetic energy) as the regularizer, we relate the mean-field optimization problem with a so-called mean field Schrodinger (MFS) dynamics. We develop a free energy method to show that the marginal distributions of the MFS dynamics converge exponentially quickly towards the unique minimizer of the regularized optimization problem. We shall see that the MFS is a gradient flow on the probability measure space with respect to the relative entropy. Finally we propose a Monte Carlo method to sample the marginal distributions of the MFS dynamics.

This is a joint work with Giovanni Conforti, Zhenjie Ren and Songbo Wang.

## An introduction to the pseudo skew Brownian motion

**Miguel Martinez**

Université Marne la Vallée

We will present a definition and first results regarding the pseudo skew Brownian motion : a Brownian motion skewed at zero but whose *probability* of having positive or negative excursions is governed by a pseudo random variable with Rademacher distribution whose parameter may possibly take a negative value. Therefore, we are moving away from the realm of classical probability.

The results presented have been obtained in collaboration with Eric Bonnetier (Institut Fourier) and Pierre Etoré (Laboratoire Jean Kuntzmann)

## Friday 8th

### Conditional propagation of chaos for generalized Hawkes processes having alpha-stable jump heights.

*Éva Löcherbach*

Université Paris 1

We study the convergence of systems of interacting particles driven by Poisson random measure, having mean field interactions and position dependent jump rate. Jumps are simultaneous, that is, at each jump time, all particles of the system are affected by this jump and receive a positive random jump height. This random kick is distributed according to a one-sided alpha-stable law and scaled in  $N^{-1/\alpha}$ , where  $N$  is the size of the system. This particular scaling implies that the limit of the empirical measures of the system is random, describing the conditional distribution of one particle in the limit system. Such limits are conditional McKean-Vlasov limits. The conditioning in the limit measure reflects the dependencies between coexisting particles in the limit system such that we are dealing with a conditional propagation of chaos property. I will spend some time to explain the explicit structure of the limit system which turns out to be the solution of a non-linear SDE driven by Poisson random measure and an independent alpha-stable subordinator. In a second part of the talk I discuss strong error bounds allowing us to control the rate of convergence of the finite particle system to the limit system.

This is a joint work with Dasha Loukianova (Université d'Evry)

### Synchronization detection by permutation test

*Etienne Tanré*

Inria Université Côte d'Azur

We observe simultaneous measures of the activity of neurons. We simply start with the activity of two selected neurons in a fixed area of the brain: their activity is fully described by the spiking times. We develop a statistical test to decide whether we can use models with independent noise or whether, on the contrary, we should consider (spatially) correlated noise. We illustrate with two examples: - a toy example based on Poisson processes; - Hawkes processes in interaction. We know in this case that chaos propagation occurs, as the number of processes in interaction goes to infinity. But, for a finite number of interacting Hawkes processes, how many observations are necessary to distinguish that they are not independent?

Joint work with J. Tchouanti-Fotso, P. Reynaud-Bouret and É. Löcherbach

## **A Kramer's type law for the first collision-time of two self-stabilizing diffusions**

*Jean-François Jabir*

HSE University, Moscow

The present work investigates the asymptotic regime at the zero-noise limit of the first (near) collision times and first collision locations related to a pair of Brownian-driven self-interacting diffusions and of their related particle approximation. Diffusions are considered in a peculiar setting evolving in a bistable self-stabilized landscape, and where collisions are only due to the combined action of the Brownian motions driving the diffusions. As the vanishing noise limit, we establish a Kramers' type law where the first collision time is shown to grow at an exponential rate and more interestingly that the related collision location persists at a given region in space. Similar results are established for related interacting-particle approximations and in the one one-dimensional case where "true collisions" can be directly studied.

This is a joint work with Julian Tugaut.

## **Two examples of thermodynamic limits in neuroscience**

*Olivier Faugeras*

Inria Université Côte d'Azur

The human brain contains billions of neurones and glial cells that are tightly interconnected. Describing their electrical and chemical activity is mind-boggling hence the idea of studying the thermodynamic limit of the equations that describe these activities, i.e. to look at what happens when the number of cells grows arbitrarily large. It turns out that under reasonable hypotheses the number of equations to deal with drops down sharply from millions to a handful, albeit more complex. There are many different approaches to this which are usually called mean-field analyses. I present two mathematical methods to illustrate these approaches. They both enjoy the feature that they propagate chaos, a notion I connect to physiological measurements of the correlations between neuronal activities. In the first method, the limit equations can be read off the network equations and methods "à la Sznitman" can be used to prove convergence and propagation of chaos as in the case of a network of biologically plausible neurone models. The second method requires more sophisticated tools such as large deviations to identify the limit and do the rest of the job, as in the case of networks of Hopfield neurones such as those present in the trendy deep neural networks.

# List of Posters

Ashot Aleksian	Université St Etienne
Lukas Anzeletti	Centralesupelec
Loic Bethencourt	Sorbonne Université
Michel Davydov	Ecole Normale Supérieure
Lev Fedorov	Vega Institute
Philipp Jettkant	University of Oxford
Lucas Journal	Sorbonne Université
Łukasz Mądry	Université Paris-Dauphine
Mattia Martini	Université Côte d'Azur
Amira Meddah	University Linz
Antonio Ocello	Sorbonne Université
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