

Talks: Titles and abstracts
“Automorphic forms, endoscopy and trace formulas”
a conference in honor of Jean-Pierre Labesse
CIRM, September 18-22, 2023

James Arthur

Title: Automorphic representations and motives.

Abstract: Motives represent hidden building blocks for both number theory and geometry. Automorphic representations are spectral objects with the analytic power to resolve some of the deepest questions in modern harmonic analysis. It has long been thought that there were fundamental relations between these very different sides of mathematics. We shall describe conjectures on the explicit nature of some of these relations, as expressed in terms of the automorphic and motivic Galois groups. If time permits, we shall comment on how these universal groups might extend to the broader theories of mixed motives and exponential motives.

Anne-Marie Aubert

Title : Endoscopic identities for classical groups over non Archimedean local fields of positive characteristic.

Abstract : If G is a split reductive group over \mathbb{Z} , and F and F' are non Archimedean local fields that are close in a suitable sense, then there is a bijection, defined by Kazhdan, between irreducible smooth representations of $G(F)$ and $G(F')$ that have nonzero vectors fixed under appropriate compact open subgroups. This transfer of representations preserves several important properties of arithmetic interest.

We will consider the case where G is a classical group and study the behavior with respect to this transfer of certain identities of characters coming from twisted endoscopy.

We will apply our result to build tempered L-packets for $G(F)$ with F of positive characteristic from the tempered L-packets for $G(F')$, with F' of characteristic zero, that have been constructed by Arthur.

This is joint work with Sandeep Varma.

Laurent Clozel

Title: On the central value of Rankin L -functions for self-dual algebraic representations of linear groups over totally real fields

Abstract: Let F be a totally real number field and π, ρ self-dual, cuspidal representations, respectively, of $GL(m)$ and $GL(n)$ over F . We assume m odd, n even. The Rankin L -function $L(s, \pi \times \rho)$ has a critical value, according to Deligne's definition, at $s = 1/2$. If we assume π, ρ algebraic regular, they can be conjugated by automorphisms of the complex numbers \mathbb{C} . According to Deligne's conjecture, the existence of a zero at $s = 1/2$ is then invariant by these automorphisms. We prove this by a simple argument relying on Zucker's 'conjecture'. In most cases the result follows from calculations of periods (resp. intertwining operators) by Grobner and Raghuram (resp. Harder and Raghuram). However a 'singular' case is new. This is common work with Arno Kret.

Pierre Henri Chaudouard

Title : On periods of some Eisenstein series

Abstract : We will present a statement about the non-vanishing and the computation of periods of some Eisenstein series of unitary groups. As such, it is an extension of the conjectures of Gan-Gross-Prasad and Ichino-Ikeda about periods of cuspidal automorphic forms. As an application we get the Gan-Gross-Prasad conjecture and the Ichino-Ikeda conjecture for Bessel periods of cuspidal automorphic forms. (Based on a joint work with Raphaël Beuzart-Plessis).

Jean-François Dat

Title : Finiteness properties of Hecke algebras of p -adic groups.

Abstract : if K is a compact open subgroup of a p -adic group G , the fact that any double K -coset in G is the union of finitely many left K -cosets allows one to define the Hecke ring $H(G, K)$ of the pair (G, K) . In the most crucial case where K is hyperspecial, this ring is commutative and finitely generated. In the next most interesting cases (Iwahori subgroup and its pro-radical), several presentations of this ring are known, from which one can prove that it is finitely generated as a module over its center and that the latter is a finitely generated ring. It is very plausible that this property remains true for any K , although it seems hard to deduce from the explicit generators and relations that are known e.g. for congruence subgroups of Iwahori subgroups. On the other hand, Bernstein proved such a result after extending scalars to \mathbb{C} , using representation theoretic methods. Unfortunately, Bernstein's approach does not work if one replaces \mathbb{C} by a field of positive (non banal) characteristic. In this talk, I will explain a proof of these finiteness properties of $H(G, K)$ after "only" inverting p , using Fargues and Scholze geometric constructions, and some of Bernstein's old ideas. This is joint with Helm, Kurinczuk and Moss.

Jessica Fintzen

Title: Representations of p -adic groups and Hecke algebras

Abstract: An explicit understanding of the category of all (smooth, complex) repre-

representations of p -adic groups provides an important tool in the construction of an explicit or categorical local Langlands correspondence. The category of representations of p -adic groups decomposes into subcategories, called Bernstein blocks, which are indexed by equivalence classes of so called supercuspidal representations of Levi subgroups. In this talk, I will give an overview of what we know about an explicit construction of supercuspidal representations and about the structure of the Bernstein blocks. In particular, I will discuss a joint project in progress with Jeffrey Adler, Manish Mishra and Kazuma Ohara in which we show that general Bernstein blocks are equivalent to much better understood depth-zero Bernstein blocks. This is achieved via an isomorphism of Hecke algebras and allows to reduce a lot of problems about the (category of) representations of p -adic groups to problems about representations of finite groups of Lie type, where answers might already be known or are easier to achieve.

Mathilde Gerbelli-Gauthier

Title: Counting non-tempered automorphic forms using endoscopy

Abstract: In this talk, we consider the limit multiplicity question (and some variants): how many automorphic forms of fixed infinity-type and level N are there as N grows? The question is well-understood when the archimedean representation is a discrete series, and we focus on non-tempered cohomological representations on unitary groups. Using an inductive argument which relies on the stabilization of the trace formula and the endoscopic classification, we give asymptotic counts of multiplicities, and prove the Sarnak-Xue conjecture at split level for (almost!) all cohomological representations of unitary groups. Additionally, for some representations, we derive an average Sato-Tate result in which the measure is the one predicted by functoriality. This is joint work with Rahul Dalal.

Michael Harris

Title: Construction of supercuspidal L-packets

Abstract: This is a report on work in progress with Beuzart-Plessis and Thorne. Let G be a quasi-split group over a local field K of positive characteristic p . We show, conditionally on the existence of a version of the twisted trace formula adequate for stable cyclic base change, that every irreducible parameter of the Weil group of K with values in the L-group of G is the image of a (necessarily) supercuspidal representation of $G(K)$ under the local parametrization constructed by Genestier-Lafforgue and Fargues-Scholze. The proof is based on the potential automorphy theorem of the speaker's paper with Böckle, Khare, and Thorne and on a new multiplicity theorem due to Gaitsgory and Raskin, which is proved using the categorical (geometric) Langlands program. At several points the current proof needs to assume that p is prime to the order of the Weyl group of G .

Guy Henniart

Title : Simple cuspidals for classical groups and the local Langlands correspondence

Abstract: Let p be a prime number, and F a finite extension of \mathbb{Q}_p . Let G be a split classical group Sp_{2n} , SO_{2n+1} or SO_{2n} over F , and let π be simple (smooth, irreducible, complex) cuspidal representation of $G(F)$ (the simple cuspidals \tilde{A} la Gross-Reeder are indeed simple to construct). By the local Langlands correspondence proved by Arthur, π has a corresponding Langlands parameter, a morphism ϕ of the Weil-Deligne group of F into the dual group $\hat{G} = SO_{2n+1}(\mathbb{C}), Sp_{2n}(\mathbb{C}), SO_{2n}(\mathbb{C})$ respectively. Seeing ϕ as a self-dual representation of the required dimension N , the question arises to describe the smooth irreducible representation Π of $GL_N(F)$ corresponding to ϕ . It turns out that the answer, obtained in work of Adrian, Kaplan, Oi and I, is quite different in the two cases p odd or p even. In the talk I shall concentrate on the case where $G = Sp_{2n}$.

Tasho Kaletha

Title: Covers of reductive groups and functoriality

Abstract: When studying problems arising from Langlands' functoriality principle, one often encounters groups that are extensions of complex reductive groups by Galois groups, but that do not necessarily satisfy all properties to be L-groups of reductive groups. We will show, in the case of a local base field FF , that such group can be understood as L-groups of covers of reductive groups. This generalizes to the case of arbitrary local fields work of Adams-Vogan for real groups.

These covers, for a fixed connected reductive group G , can be understood as arising from a certain "universal" cover of the topological group $G(F)$ by a certain "fundamental" group $\tilde{\pi}_1(G)$.

We will present two concrete applications of this, one that gives a characterization of the local Langlands correspondence for supercuspidal L-parameters when p is sufficiently large, and one to the construction of transfer factors in the theory of endoscopy.

Wen-Wei Li

Title: An intertwining relation via Takeda-Wood isomorphism

Abstract: For p -adic local fields of characteristic not equal to 2, but with no constraint on p , Takeda and Wood obtained an isomorphism between the Iwahori-Hecke algebra of $SO(2n+1)$ and the Hecke algebra of $Mp(2n)$ for the Bernstein component containing the even Weil representation; for the odd component one takes the non-split inner form of $SO(2n+1)$ instead. I will try to explain how their isomorphism behaves under parabolic induction, whose proof is not entirely trivial. Then I will sketch a Gindikin-Karpelevich formula for $Mp(2n)$ that applies to dyadic local fields as well. The motivation comes from Arthur's local intertwining relation for $Mp(2n)$, which becomes "wild" when $p = 2$. This is a joint work with Fei Chen.

Bao Chau Ngô

Title: On the Fourier and Hankel kernels for symmetric powers functoriality of $GL(2)$

An essential ingredient in the Braverman-Kazhdan program towards the functional equation of automorphic L -functions is a generalized Fourier transform depending on a reductive group G and a representation of its Langlands dual group. We expect this Fourier transform to be the convolution with a stably invariant function defined on a Zariski open subset of G . We present a formula for this function in the case of $GL(2)$ and the symmetric power representations of its dual group. We present another formula for the effect of this transformation on orbital integrals. This is a joint work with Zhilin Luo.

Yiannis Sakellaridis

Title: From the trace formula to the Kuznetsov formula for GL_n .

I will report on ongoing joint work with Chen Wan. We prove a direct comparison between the (non-invariant) Arthur–Selberg trace formula for GL_n and a nonstandard form of the Kuznetsov formula, with “boundary terms”. This answers the question of separating the non-tempered contributions from the trace formula, which was addressed by Frenkel–Langlands–Ngô and Altuğ for GL_2 , and can be seen as a comparison of relative trace formulas within the “beyond endoscopy” paradigm. The comparison is possible via a local “transfer operator” of test measures which, however, is implicitly defined; we explicitly express this operator in terms of additive Fourier transforms when $n \leq 5$.

Marie France Vignéras

Title : Representations of $SL_2(F)$ near the identity.

Abstract : Let F be a local non-archimedean field of residual characteristic p and R an algebraically closed field of characteristic $\neq p$. We study with Peyi Cui and Guy Henniart the irreducible admissible R -representations of $G = SL_2(F)$ near the identity (work in progress). “There is every reason to believe that the results will have general analogues” (Labesse-Langlands 1971).

Jean-Loup Waldspurger

Titre. Equivalence presque partout de données endoscopiques.

Résumé. Soient G un groupe réductif connexe défini sur un corps de nombres k et \tilde{G} un espace tordu sous G . Considérons deux données endoscopiques de \tilde{G} (relatives à deux caractères automorphes de G). Supposons qu’elles soient équivalentes en presque toutes les places de k . Sous certaines hypothèses portant sur le groupe dual \hat{G} , on prouve que les deux données sont alors globalement équivalentes. Nous expliquerons la preuve de ce résultat, laquelle passe par une description combinatoire simple des données endoscopiques quand \hat{G} est adjoint. Ce travail est en partie commun avec B. Lemaire.

Yujie Xu

Title: Hecke algebras for p -adic groups, the explicit Local Langlands Correspondence and stability

I will talk about my joint work with Aubert where we prove the Local Langlands Conjec-

ture for G_2 (explicitly). This uses our earlier results on Hecke algebras attached to Bernstein components of (arbitrary) reductive p -adic groups, as well as an expected property on cuspidal support, along with a list of characterizing properties (including stability). In particular, we obtain "mixed" L -packets containing F -singular supercuspidals and non-supercuspidals. Our methods are inspired by the Langlands-Shahidi method, Deligne-Lusztig and Lusztig theories etc. If time permits, I will explain how to characterize our correspondence using stability of L -packets, by computing character formulae in terms of (generalized) Green functions; one key input is a homogeneity result due to Waldspurger and DeBacker. Moreover, I will mention how to adapt our general strategy to construct LLC for other reductive groups, such as $GSp(4)$, $Sp(4)$, etc. The latter parts are based on recent joint work with Suzuki.

Cong Xue

Title : Cohomology sheaves of stacks of shtukas

Abstract : I will recall the cohomology sheaves of stacks of shtukas and the smoothness property outside the level. I will also talk about what happens at the level, which is a joint work with Arnaud Eteve.

Hongjie Yu

Title : Counting l -adic local systems over a curve over a finite field.

Abstract : In 1981, Drinfeld enumerated the number of irreducible l -adic local systems of rank two on a projective smooth curve fixed by the Frobenius endomorphism. Interestingly, this number looks like the number of points on a variety over a finite field. Deligne proposed conjectures to extend and comprehend Drinfeld's result. By the Langlands correspondence, it is equivalent to count certain cuspidal automorphic representations over a function field. In this talk, I will present some counting results where we connect counting to the number of stable Higgs bundles using Arthur's non-invariant trace formula.

Yihang Zhu

Title: Zeta Functions of Shimura Varieties: Past, Present, and the Near Future

I will first recall the general expectations of Shimura, Langlands, and Kottwitz on the shape of the zeta function of a Shimura variety, or more generally its étale cohomology. I will then report on some recent progress which partially fulfills these expectations, for Shimura varieties of unitary groups and special orthogonal groups. Finally, I will give a preview of some foreseeable developments in the near future.