TITLES AND ABSTRACTS OF THE CORTIPOM WORKSHOP IN LUMINY JULY 10-14 2023

Philippe Biane

Title: Free cumulants and the quantum exclusion process

Abstract: I will recall what are classical and free cumulants of random variables (commutative or not). Free cumulants occur, for example, in the theory of random matrices or in the asymptotics of characters of large symmetric groups. Then I will define the quantum symmetric exclusion process and explain how the large size fluctuations of this process can be described using free cumulants.

Mattia Cafasso

Title: Multiplicative statistics and integrable equations

Abstract: The discrete Bessel point process appears naturally when studying the (Poissonized) Plancherel measure on the space of Young diagrams. After defining the multiplicative statistics associated with it, I will show some applications of combinatorial flavor, and show that these quantities satisfy some famous integrable equations, called 2D-Toda equations. Time permitting, I will also explain a "continuous" analog of this result, linking multiplicative statistics of the Airy process and the Korteweg-de-Vries equation.

Guillaume Chapuy

Title: Coxeter factorizations with generalized Jucys-Murphy weights and Matrix Tree theorems for reflection groups

Abstract: I will talk about the paper arXiv:2012.04519 written a few years ago with Theo Douvropoulos. This is about a generalization of the magic number n^{n-2}, which counts Cayley trees with n vertices, or equivalently, factorizations of a long cycle in the symmetric group S_n into a product of $(n-1)$ transpositions. There are three "natural" generalizations of this formula: 1/ replace S_n by any finite Coxeter group; 2/ consider "higher genus" factorizations; 3/ put weights on transpositions, this is the matrix-tree theorem. The generalizations $1+2$ and $2+3$ are due respectively to myself and Stump (2014) and Burman+Zvonkine (2010). My talk will present the $1+2+3$ generalization. There is a price to pay to do that: we can't be too ambitious in the system of weights we allow: we can only consider weights inherited from a "parabolic tower" decomposition, a kind of generalization to all W of the well-known Jucys-Murphy elements in S_n. Beyond the formula, the introduction of these new JM-elements and of the W-Laplacian have other consequences in the world of Coxeter combinatorics. I'll try to explain all the words – and that's more or less all I understand, since the proofs are not very illuminating and are computer-assisted. Again, everything is joint with Theo Douvropoulos.

Maciej Dołęga

Title: Universality of global asymptotics of Jack-deformed random Young diagrams at varying temperatures

Abstract: We discuss discrete analogues of a one-dimensional log-gas system of N particles in a potential V at inverse temperature $\beta > 0$. We establish universal formulas describing the global asymptotics of two different models of discrete β-ensembles in high, low and fixed temperature regimes, which affirmatively answers previous questions posed in the literature. These formulas have surprising positivity properties and are expressed in terms of weighted lattice paths. Finally, we discuss the limit shape in the high/low-temperature regimes and show that, contrary to the continuous case of β-ensembles, there is a phase transition phenomenon in passing from the fixed temperature regime to the high/low temperature regimes.

Victor Dubach

Title: Increasing subsequences of linear size in random permutations and the Robinson-Schensted tableaux of permutons

Abstract: The study of longest increasing subsequences of permutations is intimately linked to that of Young diagrams via Robinson-Schensted's correspondence. Kerov and Vershik proved a limit theorem for such diagrams under Plancherel measure and deduced the $2\sqrt{n}$ estimate for the length of longest increasing subsequences in a uniformly random permutation of size n. Our goal is to establish similar results for another model of random permutations, obtained by sampling from objects called permutons. We explain how to define a notion of Robinson-Schensted shape on the space of permutons, generalizing in a sense the RS shape of permutations. When the RS shape of a permuton is non-zero we establish a linear behavior for the first row lengths and column lengths of the sampled permutations' RS shape. We can go further by defining a notion of RS tableaux on the space of permutons, satisfying the same type of convergence. Finally we study Fomin's algorithm for constructing Robinson-Schensted's correspondence on permutations and show that it leads to a differential equation satisfied by the RS tableaux of a permuton.

Valentin Feray

Title: Random Young tableaux via determinantal point processes

Abstract: Given a Young diagram λ, we consider a uniform random Young tableau T of shape λ, i.e. a uniform filling of the cells of λ conditioned to have increasing rows and columns. Letting λ go to infinity with some "limit shape", it is known that the random tableau T converges to some limiting surface. This problem has been investigated through various angles: combinatorics, representation theory, statistical physics, ...

In this talk, we will use a recent determinantal representation of random tableau (or more precisely Poissonized random tableaux) due to Gorin and Rahman to find a new description of the limit surface for tableaux of multirectangular shape. In particular, we discover that this limiting surface might be discontinuous and characterize exactly when such discontinuities occur.

Joint work with Jacopo Borga, Cedric Boutillier and Pierre-Loic Meliot.

Thomas Gobet

Title: On some lattices arising in combinatorial group theory

Abstract: In this talk, we will illustrate how interesting families of finite lattices naturally arise in questions from combinatorial group theory. Given an infinite monoid or group defined by a finite presentation, we say that the word problem for this group or monoid is solvable if there is an algorithm allowing one to determine in finite time if any word in the generators represents the identity or not. For certain classes of groups with a solvable word problem, the solution to the word problem involves a finite lattice obtained from a particular element of the group called a "Garside element", and calculating normal forms for elements of the group reduces to calculating meets and joins in this lattice. Well-known lattices arising in this way include for instance the noncrossing partition lattice, or the lattice of permutations ordered by the weak Bruhat order. Without introducing all the formal definitions, we will give several examples of such groups, and introduce a family of group presentations giving rise to presumably new family of lattices, whose elements are enumerated by the even Fibonacci numbers (joint work with B. Rognerud).

Charlie Herent

Title: An inverse Pitman's theorem for a space-time brownian motion in a type A_1^1 Weyl chamber

Abstract: We present an inverse Pitman's theorem for a space-time Brownian motion conditioned in Doob's sense to remain in an affine Weyl chamber. Our theorem provides a way to recover an unconditioned spacetime Brownian motion from a conditioned one applying a sequence of path transformations.

Pierre-Loïc Méliot

Title : Large deviations of the major index of a random permutation

Abstract : We consider the sum of the descents of a permutation: this statistics is called the major index. When the permutation is chosen uniformly among those with size n, the distribution of the major index is a convolution of uniform discrete laws, and the computation of the large deviations is easy. We shall see that this large deviation principle still holds if one consider much smaller parts of the symmetric group, namely, the Robinson-Schensted classes. Then, the computation of the rate functions rely on many ingredients from combinatorics and probability theory: Schur functions, polynomial functions of Young diagrams, change of measures, etc.

Salim Rostam

Title : Constructing Young diagrams by vertical and horizontal rectangles via regular partitions

Abstract : One can naturally tile a Young diagram by horizontal rectangles, and the dual partition gives a tiling by vertical rectangles. Both points of view are used with Frobenius coordinates, which give a tiling of the Young diagram with horizontal (resp. vertical) rectangles above (resp. below) the diagonal. In this talk, we propose a way to tile a Young diagram where horizontal and vertical tiles are mixed. The tiling for an $(e-1)$-tuple of partitions is given by a certain e-regular partition, via a "caterpillar" reading of the e-abacus. One can recover the e-regular partition from the $(e-1)$-tuple of partitions via nested e'-regularisations for $e' \in \{2, \dots, e\}$.

Sofia Tarricone

Title: Toeplitz determinants related to the discrete Painlevé II hierarchy.

Abstract: The aim of this talk is to give a recursion formula for certain Toeplitz determinants, related to random partitions models, in terms of solutions of the discrete Painlevé II hierarchy. This result is a non-trivial generalization of Borodin's formula found for the classical random partitions model when the measure is the Poissonized Plancherel measure. To obtain the result, we use the Riemann-Hilbert problem (due to Baik-Deift-Johansson) associated with the family of orthogonal polynomials on the unit circle related to the Toeplitz determinants of interest. The talk is based on the work with Thomas Chouteau (available here https://www.emis.de/journals/SIGMA/2023/030/sigma23-030.pdf).

Jacinta Torres

Title: Atoms and charge beyond type A

Abstract: In this talk I will describe a general philosophy, due to Patimo, for constructing positive combinatorial formulas for Kostka-Foulkes polynomials beyond type A. This amounts to constructing atomic decompositions for crystals as well as swapping functions which allow to define charge statistics. Then I will explain such a construction for crystals of type C_2 and point out future directions to follow. This is joint work with Leonardo Patimo.

David Wahiche

Title: An introduction to the Littlewood decomposition (and beyond)

Abstract : In 1951, Littlewood introduced the so-called Littlewood decomposition, which maps an integer partitions to a partition, called a t-core together with a tuple of partition, called the t-quotient. This decomposition can be thought of as an arithmetic operation on integer partitions. In this talk, I will try to introduce different points of view on how to compute this decomposition with notions such as beta-sets, Maya diagrams, ...

Harriet Walsh

Title: Multicritical random partitions

Abstract: I will discuss a family of probability laws on integer partitions which give rise to non-generic edge fluctuations. They are in the same universality classes as certain free fermions models previously studied by Le Doussal, Majumdar and Schehr, characterised by critical exponents of the form $1/(2m+1)$ and asymptotic distributions given by Fredholm determinants constructed from higher-order Airy kernels, which encode solutions of higher-order Painlevé equations. We show an exact mapping from these laws to the multicritical unitary matrix models previously encountered by Periwal and Shevitz, and to Toeplitz determinants more recently shown by Thomas Chouteau and Sofia Tarricone to satisfy discrete analogues of the higher-order Painlevé equations. Based on joint work with Dan Betea and Jérémie Bouttier.