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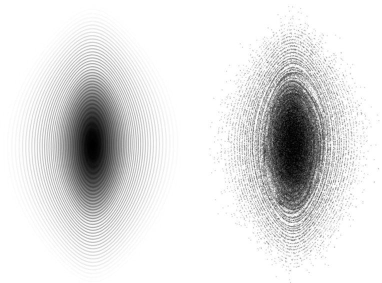
CIRM

Mathematical Justification for the Kinetic and Fluid Equations of Plasmas and Self-Gravitating Systems

July 24-28, 2023



Convergence to the Vlasov solution: N-body versus Vlasov simulations in 1D gravity. On the left, the "exact" evolved state of an initially Gaussian distribution function in phase space, with a Vlasov solver, on the right, the result obtained with a N-body code. The finite-N solution is subject to instabilities that disappear in the mean field limit. Images realized by S. Colombi, C. Alard and J. Touma (www.vlasix.org).

Sorbonne Université · PNCG Programme National de Cosmologie et Galaxies · Institut Archimède Mathématiques-Informatique Aix-Marseille Université · Aix-Marseille Université · Institut Sciences de la Fusion et de l'Instrumentation en Environnements Nucléaires Aix-Marseille Université · Doeblin · Sciences Fondamentales et Ingénierie Université Côte d'Azur · CNRS · Observatoire de la Côte d'Azur · IAP · CPT Centre de Physique Théorique · PIIM · Lagrange

CIRM: a breeding ground for talent, a mathematical melting pot!

CIRM is quite a unique place. At the heart of the Parc des Calanques, it is entirely dedicated to welcome researchers from around the world who meet here. Scientists can work together, exchange ideas, share their knowledge and advance the key issues of our discipline. They can also develop ambitious projects with the other sciences and pass on their knowledge and findings to young researchers and doctoral students.

CIRM is a breeding round for talent, a melting pot of mathematical cultures. The particularity of our center is its residential character. Here, the researcher lives in total immersion with his group. He sleeps, eats and works at Cirm. This proximity favors exchanges. The participants, who are entirely taken care of by Cirm's administrative and residential teams, can focus exclusively on their own scientific projects.

We wish all our visitors an excellent and enriching stay with us and enjoyable exchanges, whether physically present at CIRM or at a distance!

Pascal HUBERT, director of CIRM
(https://www.cirm-math.com/directors-welcome-note.html)

The aims of this conference

A primary objective, when studying a complex physical system from a theoretical perspective, is to describe it as simply as possible. With N-body systems such as plasmas, dark matter in the Universe, or even stars in galaxies, the first step in the simplification procedure often amounts to calculating kinetic limits at large N. More specifically, it involves showing that a system with a large number N of particles can be described by a kinetic equation governing the evolution of a one- (or possibly two-) particle distribution function when N tends to infinity, as, for example, the Boltzmann equation. In a second step, one can further simplify the kinetic equation by focusing on the evolution of the first moments of the distribution function. For instance, only considering the first two velocity moments, the Boltzmann equation can be reduced to the Navier-Stokes equations for hydrodynamics, provided the mean free path between two collisions is small compared to the size of the system. Thirdly, under the assumption that specific parameters remain small, a simplified (perturbative) approach to the resolution of the equations of motion makes it possible to reduce the problem to wave-particle interactions or wave-wave interactions. In cosmology for example, perturbation theory is widely used to precisely constrain models of large scale structure formation in the Universe.

The passage to the different limits, kinetic, fluid, or even the justification of the use of perturbation theory remains, because of the associated inherent difficulties and subtleties, the subject of intense discussions between mathematicians and physicists. The purpose of this multidisciplinary workshop is to review and critically evaluate the latest developments on specific aspects of these problems, with a particular emphasis on their mathematical justification.

Scientific Committee

Julien Devriendt (Oxford University)
Isabelle Gallagher (ENS – PSL)
Pierre Emmanuel Jabin (PennState University)
Laure Saint-Raymond (IHES)

Organizing Committee

Nicolas Besse (Observatoire de la Côte d'Azur)
Stéphane Colombi (CNRS Institut d'Astrophysique de Paris)
Yves Elskens (Aix-Marseille Université)
Roland Triay (Aix-Marseille Université)

Mathematical Justification for the Kinetic and Fluid Equations of Plasmas and Self-Gravitating Systems

24 – 28 July, 2023 — CIRM

Schedule

Sunday July 23rd	Monday July 24th	Tuesday July 25th	Wednesday July 26th	Thursday July 27th	Friday July 28th
	7:00-9:00 : breakfast	7:00-9:00 : breakfast	7:00-9:00 : breakfast	7:00-9:00 : breakfast	7:00-9:00 : breakfast
	9:00-10:45 : welcome at CIRM	9:00 : Golse	9:00 : White	9:00 : Cheverry	9:00 : Morrison
	10:15 : coffee break	9:45 : Fouvry	9:45 : Duerinckx	9:45 : Kiessling	9:45 : Barre
	10:45 : welcome address: Besse	10:30 : coffee break	10:30 : coffee break	10:30 : coffee break	10:30 : coffee break
	11:00 : Collot	11:00 : Bardos	11:00 : Tremblin	11:00 : Ricci	11:00 : Giachetti
	11:45 : Rocha Filho	11:35 : Escande	11:45 : Pegoraro	11:45 : Delos	11:45 : round table : Triay
	12:30 : lunch	12:30 : lunch	12:30 : lunch	12:30 : lunch	12:30 : lunch
	14:00 : Feistl	14:00 : Mouhot		14:00 : Vu	14:00 : discussions
				14:20 : Devriendt	
	14:45 : Feliachi	14:45 : Rampf		14:55 : Colombi	
	15:30 : coffee break	15:30 : coffee break		15:30 : coffee break	15:30 : coffee break
	16:00 : Chavanis	16:00 : Nota		16:00 : Munschy	
17:00-23:00 : welcome at CIRM	16:45 : discussions	16:45 : discussions		16:45 : discussions	
19:00-21:45 : cold dinner	19h30 : dinner	19h30 : dinner	19h30 : dinner	19h30 : conference dinner	19h30 : dinner

Homogenous Approximations for solutions of the Vlasov Equation from Quasi Linear to Balescu Lenhard equation

Claude Bardos
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Abstract

Consider the Vlasov equation in a space periodic domain:

$$\partial_t F + v \cdot \nabla_x F \pm E(f) \cdot \nabla_v F = 0 \quad (1)$$

$$(x, v) \in ((\mathbf{T}^d = (\mathbf{R} \backslash \mathbf{Z})_x)^d \times (\mathbf{R}_v)^d)\,, \quad (2)$$

$$G = \langle F \rangle = \int_{\mathbf{T}^d} F(x, v, t)dx\,, f = F - \langle F \rangle\,, \quad (3)$$

$$-\Delta V = f \quad E(f) = -\nabla_x V \quad (4)$$

with $+$ for electrostatic and $-$ for gravitational.

Then integration on $\mathbf{T}^d$ gives:

$$\partial_t G \pm \langle E(f) f \rangle = 0 \quad (5)$$

which is a space homogenous equation. The issue is to prove the validity of approximations for the term $\langle E(f) f \rangle$.

Such approximation depend on the stability \ unstablility of the tangent linearized dynamic. Hence one and consider two scenarios.

1. In the presence of unstable modes an approximation (basically valid as long as such modes exist, hence for "short" time) called the Quasi Linear approximation is the following non linear parabolic equation:

$$\partial_t G - \nabla_v (D(v, \nabla_v G(v), t) \nabla_v G = 0\,. \quad (6)$$

   where the diffusion is determined in term of the unstable modes.

2. In the absence of unstable mode one would rather consider the large time behavior.

Hence I intend to describe with some details, following some ongoing work with Nicolas Besse, the analysis and the validity of the equation (6) and then shortly compare it with what is known for the stable scenario which of course is closely related to the Landau Damping as studied by Mouhot and Villani or to the Balescu Lenhard (and other avatars like the Landau equation).

Hydrodynamic limits, macroscopic fluctuations and large deviations.

Julien Barré
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Abstract

To derive a macroscopic description of a system (in terms of hydrodynamical fields), starting from a microscopic one (in terms of interacting particles), a standard route introduces an intermediate kinetic equation, and takes advantage of the difference of time scales between fast and slow modes to set up a Chapman-Enskog expansion. When finite size effects are important at the macroscopic level, they are taken into account by adding a noise on the hydrodynamical equations, sometimes in an empirical way. We will explain how this whole procedure can be carried out at the level of large deviations functionals, with classical examples such as incompressible Navier-Stokes equations, and less classical ones. We will emphasize in particular the specific challenges of ballistic macroscopic equations.

References

[1] Feliachi, O., Besse, M., Nardini, C., and Barré, J. (2022). Fluctuating kinetic theory and fluctuating hydrodynamics of aligning active particles: the dilute limit. Journal of Statistical Mechanics: Theory and Experiment, 2022(11), 113207.

[2] Feliachi, O., and Bouchet, F. (2021). Dynamical large deviations for plasmas below the Debye length and the Landau equation. Journal of Statistical Physics, 183(3), 42.

[3] O. Feliachi, From Particles to Fluids: A Large Deviation Theory Approach to Kinetic and Hydrodynamical Limits, PhD dissertation, submitted.

A heuristic wave equation parameterizing BEC dark matter halos with a quantum core and an isothermal atmosphere

Pierre-Henri Chavanis
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Abstract

The Gross-Pitaevskii-Poisson equations that govern the evolution of self-gravitating Bose-Einstein condensates, possibly representing dark matter halos, experience a process of gravitational cooling and violent relaxation. We propose a heuristic parametrization of this complicated process in the spirit of Lynden-Bell's theory of violent relaxation for collisionless stellar systems. We derive a generalized wave equation [1, 2, 3] involving a logarithmic nonlinearity associated with an effective temperature $T_{\rm eff}$ and a damping term associated with a friction ξ. The wave equation relaxes towards a stable equilibrium state which is a maximum of entropy at fixed mass and energy. This equilibrium state represents the most probable state of a Bose-Einstein condensate dark matter halo. It generically has a core-halo structure. The quantum core prevents gravitational collapse and may solve the core-cusp problem. The isothermal halo leads to flat rotation curves in agreement with the observations. These results are consistent with the phenomenology of dark matter halos.

References

[1] P.H. Chavanis, *Dissipative self-gravitating Bose-Einstein condensates with arbitrary nonlinearity as a model of dark matter halos*, Eur. Phys. J. Plus 132 (2017) 248.

[2] P.H. Chavanis, *Predictive model of BEC dark matter halos with a solitonic core and an isothermal atmosphere*, Phys. Rev. D 100 (2019) 083022.

[3] P.H. Chavanis, *A heuristic wave equation parameterizing BEC dark matter halos with a quantum core and an isothermal atmosphere*, Eur. Phys. J. B 95 (2022) 48.

The Relativistic Vlasov-Maxwell system
Local smooth solvability for a weak topology

Christophe Cheverry
Université de Rennes

Résumé

The aim of this talk is to explain how to construct smooth solutions to the three dimensional Relativistic Vlasov-Maxwell system in the regime of dense, hot and strongly magnetized plasmas. This result is the consequence of a property of local smooth solvability for the "weak" topology associated with $L^\infty \times H^1 \times H^1$. We can prove the local existence and uniqueness of solutions for initial data $(\mathrm{f}_0, \mathrm{E}_0, \mathrm{B}_0)$ in $L^\infty \times H^1 \times H^1$, with f_0 compactly supported in momentum. This is a joint work with S. Ibrahim (Victoria university, Canada).

On the derivation of the kinetic wave equation of wave turbulence

Charles Collot
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Abstract

This talk will be made accessible to a large audience. The mathematical problem is to describe the dynamics of small (weakly nonlinear) and random solutions to semilinear dispersive equations such as the nonlinear Schrödinger equation, over timescales where nonlinear effects are non-perturbative. Physicists, since the works of Peierls, Hasselman, Zakharov and collaborators, have identified an effective equation to describe statistically the evolution of the amplitude of the Fourier modes. This is the so-called kinetic wave equation, and we will present results and ideas that lead to its recent rigorous derivation in certain regimes. This talk will be based on the works [1, 2, 3] of the author.

References

[1] C. Collot and P. Germain, *On the derivation of the homogeneous kinetic wave equation*, to appear in Communications on Pure and Applied Mathematics.

[2] C. Collot and P. Germain, *Derivation of the homogeneous kinetic wave equation: longer time scales* arXiv preprint 2007.03508, submitted.

[3] I. Ampatzoglou, C. Collot and P. Germain, *Derivation of the kinetic wave equation for quadratic dispersive problems in the inhomogeneous setting* arXiv preprint 2107.11819, submitted.

The early evolution of cosmic structures

Stéphane Colombi
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Abstract

The first nonlinear structures to form in the universe are expected to be pancakes. I will discuss the approximate modeling of these objects in the context of the classic cold dark matter scenario and their subsequent evolution in the form of microhalos.

References

[1] Colombi S., 2021, A&A, 647, A66.

[2] Saga S., Taruya A., Colombi S., 2022, A&A, 664

Prompt cusp formation from the cosmological initial conditions

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Abstract

The first cosmic structures formed by the gravitational collapse of smooth peaks in the initial density field. Numerical simulations show that this process creates prompt density cusps that scale approximately as $\rho \propto r^{-1.5}$, which persist largely unaltered through cosmic time. I will review the cosmological initial conditions and how they give rise to smooth density peaks. Next, I will show how prompt cusps form and persist at the sites of these peaks and how the properties of a cusp connect to those of the peak. In the collisionless dark matter paradigm, a prompt cusp is expected to reside at the center of every dark matter halo; I will discuss how this impacts our understanding of the nature of dark matter.

On the ability of quadrature-based moment methods to follow cold dark matter dynamics

Julien Devriendt
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Abstract

An inertial system of particles can be described by a velocity distribution function (VDF) which satisfies a kinetic equation. Solving such an equation requires either a discrete sampling of phase space through a Lagrangian approach or taking velocity moments to obtain a Eulerian system of conservation laws. In the latter case and for systems of particles whose VDF can be far away from equilibrium, the main difficulty is the closure of the moment hierarchy. In this talk, I will focus on quadrature-based moment methods where the VDF is approximated by a sum of Dirac delta functions (better known as the Quadrature Method Of Moments) with an hyperbolic closure of arbitrary order for the moment hierarchy, such as discussed in e.g. [1] and [2]. After introducing the general framework in one spatial dimension, I will show a few examples of classical Riemann problems solved using the method, before moving on to an application of fundamental interest to cosmologists: the dynamics of a self-gravitating, collapsing, cold dark matter perturbation.

References

[1] R. O. Fox, F. Laurent, A. Vié, *Conditional hyperbolic quadrature method of moments for kinetic equations*, J. Comput. Phys. 365 (2018) 269-293.

[2] M. van Cappellen, M. R. Vetrano, D. Laboureur, *Higher order hyperbolic quadrature method of moments for solving kinetic equations*, J. Comput. Phys. 436 (2021) 110280.

Dynamics of point-vortex systems around equilibrium

Mitia Duerinckx
Université Libre de Bruxelles, Belgique

Abstract

In this talk, based on an ongoing joint work with P.-E. Jabin, we discuss the long-time dynamics of point vortices around equilibrium and we investigate the emergence of collisional relaxation — so-called point-vortex diffusion. More precisely, we consider a tagged point vortex coupled to a large number of background point vortices that are initially at equilibrium, and we analyze its long-time dynamics. In the spirit of our previous work on the Lenard–Balescu relaxation in plasmas [1, 2, 3], we obtain in a generic setting some partial results compatible with the validity of a slow relaxation of the tagged particle. Yet, we further show that a completely different phenomenology is also possible in some exceptional cases where the leading dynamics of the tagged particle happens to remain oscillatory.

References

[1] M. Duerinckx and L. Saint-Raymond. *Lenard–Balescu correction to mean-field theory.* Prob. Math. Phys., 2(1):27–69, 2021.

[2] M. Duerinckx. *On the size of chaos via Glauber calculus in the classical mean-field dynamics.* Commun. Math. Phys. 382:613–653 (2021).

[3] M. Duerinckx and R. Winter. *Well-posedness of the Lenard–Balescu equation with smooth interactions.* Arch. Ration. Mech. Anal., to appear (2023).

The Vlasovian limit: a singular and renormalized description of the actual N-body dynamics

Dominique Franck Escande
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Abstract

Classical N-body mechanics describes non-trivial aspects of the macroscopic dynamics of a plasma usually described in the Vlasovian limit [1, 2, 3]. The singularity of this limit shows up in various ways. In particular, (i) the dielectric function of a N-body plasma does not converge toward the Vlasovian one, when particle density increases; (ii) phase mixing is important for Landau growth in the N-body approach, while it is absent in the Vlasovian calculation; (iii) adding a test particle to the N-body system does not provide its shielded potential, in contrast with the Vlasovian case. Indeed, the latter includes a part of collisions in its dielectric function, a signature of renormalization.

References

[1] D. F. Escande, D. Bénisti, Y. Elskens, D. Zarzoso, and F. Doveil, *Basic microscopic plasma physics from N-body mechanics: A tribute to Pierre-Simon de Laplace*, Rev. Mod. Plasma Phys. 2 (2018) 9.

[2] Y. Elskens and D. F. Escande, *Microscopic dynamics of plasmas and chaos*, Institute of Physics, Bristol, (2003), 328 pages.

[3] D. F. Escande, F. Doveil, and Y. Elskens, *N-body description of Debye shielding and Landau damping*, Plasma Phys. Control. Fusion 58 (2016) 014040.

Microscopic Derivation of Vlasov type equations

Manuela Feistl
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Abstract

I present a probabilistic technique for the proof of the mean-field limit and propagation of chaos of a N-particle system in three dimensions with highly singular interactions. The two recent results lead to a derivation of the Vlasov-Poisson equation one the one hand and to a derivation of the Vlasov-Dirac-Benney equation on the other hand, depending on the pair interaction potential. In the first application we prove for typical initial data convergence of the empirical distributions to solutions of the Vlasov-Poisson system for Coulomb interaction and cut-off size much smaller than the typical inter particle distance. More precisely, the interaction fulfils $f^N(q) = \pm \frac{q}{|q|^3}$ for $|q| > N^{-\frac{5}{12}+\sigma}$ and has a cut-off at $|q| = N^{-\frac{5}{12}+\sigma}$ where $\sigma > 0$ can be chosen arbitrarily small.
In the second application we prove also for typical initial data, convergence of the empirical distributions to solutions of the Vlasov-Dirac-Benney system with compactly supported pair potentials of the form $N^{3\beta-1}\phi(N^\beta x)$ for $\beta \in \left[0, \frac{1}{7}\right)$ and $\phi \in L^\infty(R^3) \cap L^1(R^3)$. Thus our result leads to a derivation of the Vlasov-Dirac-Benney equation from the microscopic N-particle dynamics with a strong short range force.
In particular, for typical initial data, we show convergence of the Newtonian trajectories to the characteristics of the Vlasov type equations.

References

[1] M. Feistl, P. Pickl, *Microscopic derivation of Vlasov equation with compactly supported pair potentials.*

[2] M. Feistl, P. Pickl, *On the mean-field limit for the Vlasov-Poisson system.*

Dynamical large deviations for homogeneous system with long-range interactions

Ouassim Feliachi
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Université de Tours and CNRS, 45067 Orléans, France

Abstract

This talk is based on [1]. The Balescu-Lenard (BL) equation describes the relaxation to equilibrium of N-particle system that interact through a long-range pairwise potential. The BL equation is a deterministic; and of course, when the particle number N is large but finite, we expect fluctuations from this deterministic limit. I will first discuss the derivation of the BL equation from the particle dynamics from the so-called Klimontovich approach. Then, I will explain how to assess the probability of any spatially homogeneous evolution path for the empirical measure of the particles, beyond the most probable one which is described by the kinetic BL equation, using the formalism of large deviations for slow-fast systems. The main result is a dynamical large deviation principle that quantifies explicitly the probability of any evolution path for the empirical measure in the large N limit. This derivation is done under the same assumptions formulated in theoretical physics textbook that lead to the BL equation. Joint work with Freddy Bouchet.

References

[1] Feliachi, O., Bouchet, F. *Dynamical Large Deviations for Homogeneous Systems with Long Range Interactions and the Balescu–Guernsey–Lenard Equation,* J Stat Phys 186, 22 (2022).

Kinetic theory of one-dimensional long-range interacting N-body systems at order $1/N^2$

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Abstract

Long-range interacting systems irreversibly relax as a result of their finite number of particles, N. At order $1/N$, this process is described by the inhomogeneous Balescu–Lenard equation [1, 2]. Yet, this equation exactly vanishes in one-dimensional inhomogeneous systems with a monotonic frequency profile and sustaining only 1:1 resonances. In the limit where collective effects can be neglected, I will present a closed and explicit $1/N^2$ collision operator for such systems [3]. I will detail its properties, and compare its predictions with direct N-body simulations. Finally, I will exhibit a generic class of long-range interaction potentials for which this $1/N^2$ collision operator exactly vanishes.

References

[1] Heyvaerts, J., 2010, MNRAS, 407, 355

[2] Chavanis, P.-H., 2012, Physica A, 391, 3680

[3] Fouvry, J.-B., 2022, Phys. Rev. E, 106, 054123

Coarse-grained collisionless dynamics with long-range interactions

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Abstract

The phase-space dynamics of long-range interacting systems of particles is described by the noncollisional Boltzmann, or Vlasov, equation. Even if the time-reversal symmetry is not broken, large scale observables are known to relax toward a stationary value. We propose [1] a (dissipative) deformation of the Vlasov equation for the effective dynamics of the system after a coarse-grain, foreseeing a finite scale-dependent damping time. The general form of such corrections can be foreseen through symmetry considerations only, while we derive an explicit form for one-dimensional systems. Numerical checks are presented for some one-dimensional systems, e.g. the Hamiltonian mean-field model, $1d$ gravity, and ring-gravity.

References

[1] G. Giachetti, A. Santini and L. Casetti, *Coarse-grained collisionless dynamics with long-range interactions*, Phys. Rev. Research 2, 023379 (2018)
.

The Regularity Problem for the Landau Equation

François Golse
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Abstract

It is well known that the dynamics of particles interacting through the Coulomb potential cannot be described by the Boltzmann equation. In this case, the Boltzmann collision integral must be replaced with the Landau operator. In the late 1990's, Villani defined a notion of global, space-homogeneous solutions to the Landau equation, called H-solutions (in view of the importance of Boltzmann's H-Theorem in the definition of such solutions). This talk will review some recent progress on the regularity of Villani solutions of the Landau equation. (Based in particular on joint work with M.P. Gualdani, C. Imbert and A. Vasseur [1, 2].)

References

[1] F. Golse, M.-P. Gualdani, C. Imbert, A. Vasseur, *Partial Regularity in Time for the Space-Homogeneous Landau Equation with Coulomb Potential*, Ann. Scient. Éc. Norm. Sup. 4e série, **55** (2022) 1575–1611.

[2] F. Golse, C. Imbert, A. Vasseur, *Local regularity for the space-homogeneous Landau equation with very soft potentials*, preprint arXiv:2206.05155 [math.AP].

On the microscopic foundations of the relativistic Vlasov-Maxwell equations

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Abstract

The talk reviews the state of affairs in the mathematically rigorous foundations of the special-relativistic Vlasov-Maxwell equations [1]. The progress is made possible by a recent formulation of a well-posed Lorentz co-variant initial value problem for the joint evolution of charged point particles and their electromagnetic Maxwell fields in a Bopp–Landé–Thomas–Podolsky (BLTP) vacuum [2], [3].

References

[1] Y. Elskens and M. K.-H. Kiessling, *Microscopic foundations of kinetic plasma theory: The relativistic Vlasov–Maxwell equations and their radiation-reaction-corrected generalization*, J. Statist. Phys. **180** (2020), 749–772.

[2] M.K.-H. Kiessling, *Force on a point charge source of the classical electromagnetic field*, Phys. Rev. D **100** (2019), art.065012, 19pgs. (2019); [Erratum, ibid. **101** (2020), 109901, 1pg.].

[3] M.K.-H. Kiessling and A.S. Tahvildar-Zadeh, *Bopp–Landé–Thomas–Podolsky electrodynamics as initial value problem*, $>$ 50pgs., (2023) (in preparation).

The metriplectic 4-bracket: a curvature-like framework for describing dissipation in joined Hamiltonian and dissipative fluid and plasma systems

Philip J. Morrison
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Abstract

An inclusive framework [1] for joined Hamiltonian and dissipative dynamical systems, which builds on early work [2, 3, 4], will be described. The framework describes dynamical systems that preserve energy and produce entropy. The dissipative dynamics of the framework is based on the metriplectic 4-bracket, a quantity like the Poisson bracket defined on phase space functions, but unlike the Poisson bracket has four slots with symmetries and properties motivated by Riemannian curvature. Metriplectic 4-bracket dynamics is generating using two generators, the Hamiltonian and the entropy, with the entropy being a Casimir of the Hamiltonian part of the system. The formalism includes all known previous binary bracket theories for dissipation or relaxation as special cases. Rich geometrical significance of the formalism and methods for constructing metriplectic 4-brackets are explored. Examples of both finite and infinite dimensions will be discussed including plasma, fluid, and kinetic descriptions.

References

[1] P. J. Morrison and M. Updike, *An inclusive curvature-like framework for describing dissipation: metriplectic 4-bracket dynamics*, arXiv:2306.06787v1 [math-ph] 11 Jun 2023.

[2] P. J. Morrison, *Bracket formulation for irreversible classical fields*, Phys. Lett. A 100, 423 (1984).

[3] P. J. Morrison, *Some Observations Regarding Brackets and Dissipation*, Tech. Rep. PAM–228, University of California at Berkeley (1984).

[4] P. J. Morrison, *A Paradigm for joined Hamiltonian and dissipative systems*, Physica D 18, 410 (1986).

Review of recent mathematical progresses on Landau damping without confinement

Clément Mouhot
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Abstract

We will review the recent mathematical results from various research groups about the stability and long-time scattering behaviour of the solutions to the Vlasov-Poisson equation in the whole space without boundary or external confinement and close to a spatially homogeneous equilibrium (with infinite mass and energy).

Kinetic plasma-wall interaction using immersed boundary conditions

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Abstract

Plasma-wall interaction is studied through a (1D-1V) kinetic code solving the Vlasov-Poisson system [1]. Immersed boundary conditions are used to model the wall. The main properties of the Debye sheath physics are recovered [2]. This non-neutral layer is accompanied by a drop of the electric potential that confines slow electrons and accelerates ions.
Discrepancies are found with respect to fluid predictions [3]. First, the non-vanishing conductive heat flux observed in kinetic simulations is usually neglected in fluid analyses. Its properties and impact on the overall dynamics reveal hardly tractable within the fluid framework. Second, we show that the expression of the plasma sound speed strongly depends on the chosen closure of the fluid hierarchy. The immediate consequence is that Bohm's criterion defining the Debye sheath entrance in terms of the Mach number becomes non-operational.

References

[1] Emily Bourne, Yann Munschy, Virginie Grandgirard, Michel Mehrenberger, Philippe Ghendrih, *Non-uniform splines for semi-Lagrangian kinetic simulations of the plasma sheath*, Journal of Computational Physics, Volume 488, 2023, 112229, ISSN 0021-9991.

[2] Yann Munschy, Emily Bourne, Guilhem Dif-Pradalier, Peter Donnel, Philippe Ghendrih, Virginie Grandgirard, and Yanick Sarazin *Kinetic plasma-wall interaction using immersed boundary conditions*, submitted Nuclear Fusion.

[3] Yann Munschy, Emily Bourne, Guilhem Dif-Pradalier, Peter Donnel, Philippe Ghendrih, Virginie Grandgirard, and Yanick Sarazin *Kinetic plasma-sheath self-organization*, submitted Nuclear Fusion.

Long-time asymptotics for Homoenergetic solutions of the Boltzmann equation.

Alessia Nota
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Abstract

In this talk I will consider a particular class of solutions of the Boltzmann equation, known as homoenergetic solutions, which were introduced by Galkin and Truesdell in the 1960s. These are a particular type of non-equilibrium solutions of the Boltzmann equation and they are useful to describe the dynamics of Boltzmann gases under shear, expansion or compression. Due to the fact that these solutions describe far-from-equilibrium phenomena their long-time asymptotics cannot always be described by Maxwellian distributions. For several collision kernels the asymptotics of homoenergetic solutions is given by particle distributions which do not satisfy the detailed balance condition. I will discuss different possible long-time asymptotics of homoenergetic solutions of the Boltzmann equation, as well as some conjectures and open problems in this direction.

A 1-dimensional electrostatic plasma model for testing the validity of kinetic theory

Francesco Pegoraro
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Abstract

A starting point for deriving the Vlasov equation with a collision operator is the BBGKY hierarchy that describes the dynamics of coupled marginal distribution functions. With a large plasma parameter (number of particles in a Debye sphere) one justifies dropping certain correlations and makes assumptions so as to eliminate 2-point correlations in terms of the 1-point function.

Because of the curse of dimensions, numerically testing the assumptions of the Vlasov-Landau-Lenard-Balescu theory is prohibitive.

In this presentation, we will present a 1-dimensional Hamiltonian model [1] composed of electrostatically interacting aligned charged disks in order to address in a computable model the validity of the Bogoliubov assumption on the decay of correlations, a basic premise of plasma kinetic theory.

This presentation is dedicated to the derivation of the main properties of such a disk-plasma in order to clarify where and how the dynamics of such a model system can mimic the dynamics of a real plasma, at least as long as the longitudinal electric field limit is concerned.

References

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Vlasov–Poisson equations and shell-crossings

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Abstract

The gravitational evolution of collisionless matter, governed by Vlasov–Poisson equations, leads to so-called shell-crossing singularities, which are instances with large matter densities that arise when trajectories of fluid particles intersect. In this talk I provide an overview of recent advances, on both the analytical and numerical front, for improving the accuracy and efficiency of shell-crossing predictions.

References

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Mathematical modelling of magnetic reconnection

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Abstract

Magnetic reconnection phenomena collect a wide variety of processes that involve a drastic change of the magnetic field topology in plasmas. We shall give some detail about the mathematical modelling of this class of phenomena and discuss some simplified model, connected to [1]. The work is in collaboration with B.Coppi, in the framework of the IGNITOR project.

References

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Analytic Solution of the One-Dimensional Vlasov-Poisson System

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Abstract

We present an approach to determine analytic particular solutions of the Vlasov-Poisson system for one-dimensional self-gravitating systems. It is based on the determination of Lie and Non-Classical symmetries on the integro-differential system, using extensive computer algebra and the maple package SADE (Symmetry Analysis of Differential Equations) [1]. Non-classical symmetries computations require the solution of a set of non-linear partial differential equations and usually requires the introduction of some ansatze [2, 3], but are much more general that the symmetries. Invariant solutions are then determined from the obtained symmetry generators, and yield new stationary and time-dependent solutions.

References

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Non-ideal self-gravitating hydrodynamics

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Abstract

Inspired by the statistical mechanics of an ensemble of interacting particles (BBGKY hierarchy), we propose to account for small-scale inhomogeneities in self-gravitating astrophysical fluids by deriving a nonideal virial theorem and nonideal Navier-Stokes equations [1]. These equations involve the pair radial distribution function (similar to the two-point correlation function used to characterize the large-scale structures of the Universe), similarly to the interaction energy and equation of state in liquids. In order to solve the problem of the non-extensive nature of the gravitational interaction energy, we propose to use a decomposition of the gravitational potential into a near- and far-field component in order to account for the gravitational force and correlations in the thermodynamics properties of the fluid. Within this framework, small-scale correlations lead to a nonideal amplification of the gravitational interaction energy, as well as a non-ideal stress tensor, whose contribution cannot be neglected as soon as the Jeans length is not resolved at large scale. We suggest that small-scale gravitational interactions in bound structures (spiral arms or local clustering) could yield a transition to a viscous regime that can impact the rotation curve of galaxies. This transition could also explain the dichotomy between (Keplerian) low surface brightness elliptical galaxy and (nonkeplerian) spiral galaxy rotation profiles.

References

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Long time behavior for collisional strongly magnetized plasma in three space dimensions

Anh-Tuan Vu

Joint work with Mihaï Bostan

I2M, University of Aix-Marseille

Abstract

We consider the long time evolution of a population of charged particles, under strong magnetic fields and collision mechanisms. We derive a fluid model: the limit particle densities are Maxwellian equilibria, parameterized by the particle concentration. The concentration is advected along the electric cross field drift, magnetic gradient drift, magnetic curvature drift. In three space dimensions, a constraint ocurs along the parallel direction.

A universal density profile from hierarchical clustering

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Abstract

It has been known for 25 years that collisionless gravitational clustering of non-interacting matter from cosmologically realistic initial conditions produces nonlinear quasi-equilibrium objects, so-called dark matter halos, of near-universal structure [1]. This result has been established empirically through many generations of numerical simulations and seems, to a good approximation, to hold for objects over a very wide range of masses forming at different times in cosmologies with different global parameters and with initial linear density fluctuations with different power spectra. Halo structure does appear to be tightly linked to halo assembly history [2]. I will review the various aspects of this problem in order to provide some understanding of why such approximate univerality might hold, and of the situations where it can be expected to fail.

References

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Mathematical Justification for the Kinetic and Fluid Equations of Plasmas and Self-Gravitating Systems

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