Mutifractal analysis and self-similarity

Program and abstracts

June 29, 2023

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Monday, June 26

- 9h-10h François Ledrappier
- 10h-11h Henna Koivusalo
- 11h-11h30 Break
- 11h30-12h Polina Vytnova
- 12h-12h30 Edouard Daviaud

- 16h-17h Katrin Gelfert
- 17h-17h30 Break
- 17h30-18h30 Matthieu Astorg
- 18h30-19h00 Jungwon Lee
- 19h00-19h30 Ayreena Bakhtawar

Tuesday, June 27

- 9h-10h De-Jun Feng
- 10h-11h Faustin Adiceam
- 11h-11h30 Break
- 11h30-12h Ai Hua Fan
- 12h-12h30 Arnaud Durand

- 16h-17h Céline Lacaux
- 17h-17h30 Break
- 17h30-18h30 Denis Grebenkov

- 18h30-19h00 Stéphane Jaffard

Wednesday, June 28

- 9h-10h Vincent Vargas
- 10h-10h30 Laurent Loosveldt
- 10h30-11h Mark Pollicott
- 11h-11h30 Break
- 11h30-12h Alex Rutar
- 12h-12h30 Bertrand Duplantier

Thursday, June 29

- 9h-10h Hermine Biermé
- 10h-11h Marcel Filoche
- 11h-11h30 Break
- 11h30-12h Antoine Ayache
- 12h-12h30 Jasson Vindas

- 16h-17h Ariel Rapaport
- 17h-17h30 Break
- 17h30-18h00 Yiltekin-Karatas Ayse
- 18h00-18h30 Caroline Wormell
- 18h30-19h Srijanani Anurag Prasad

Friday, June 30

- 9h-10h Kornélia Héra
- 10h-11h Ian Morris
- 11h-11h30 Break
- 11h30-12h Amlan Banaji
- 12h-12h30 Prasuna Bandi

Abstracts–Invited lectures

- Faustin Adiceam

  Title : Some Algebraic Tools in Fractal Geometry

  Abstract : The talk will present some recent advances at the crossroads between Number Theory and Fractal Geometry requiring the input of algebraic theories to estimate the measure and/or the factal dimension of sets emerging naturally in Diophantine Approximation. Examples include the proof of metric, uniform and quantitative versions of the Oppenheim conjecture generalised to the case of any homogeneous form and also the determination of the Hausdorff dimension of the set of well approximable points lying on polynomially defined manifolds (i.e. on algebraic varieties).

- Matthieu Astorg

  Title: Bifurcations of higher order in families of polynomial skew-products

  Abstract: Given a holomorphic family of endomorphisms of a complex projective space CP^k, there is a notion of bifurcation locus, which describes the set of parameters for which the dynamics is not robust under perturbation. This set is the support of a (1,1) positive closed current T. In the case of dimension k=1, the supports of the auto-intersections of T form a strictly decreasing sequence of closed sets, which describes higher codimensional bifurcation phenomena.

  By contrast with the case of dimension 1, we will show that in simple families of endomorphisms of CP^2 (polynomial skew-products), all these supports coincide. The proof uses, among other things, a recent Hausdorff dimension estimate of Julia sets due to Przytycki and Zdunik.

  Joint work with Fabrizio Bianchi.

- Hermine Biermé (IDP, Tours)

  Title: Monogenic representation for self-similar random fields and color images

  We consider the monogenic representation for self-similar random fields. This approach is based on the monogenic representation of a greyscale image, using Riesz transform, and is particularly well-adapted to detect directionality of self-similar Gaussian fields. In particular, we focus on distributions of monogenic parameters defined as amplitude, orientation and phase of the spherical coordinates of the wavelet monogenic representation. This allows us to define estimators for some anisotropic fractional fields. We then consider the elliptical monogenic model to define vector-valued random fields according to natural colors, using the RGB color model.

  Joint work with Philippe Carré (XLIM, Poitiers), Céline Lacaux (LMA, Avignon) and Claire Launay (IDP, Tours)

- De-Jun Feng

  Title: One-sided multifractal analysis of Gibbs measures on the line

  Abstract: Let μ be a self-similar measure on the line satisfying the strong separation condition, or more generally, a Gibbs measure supported on a nonlinear Cantor set on the line. We conduct the one-sided multifractal analysis of μ. This is based on joint work with Cai-Yun Ma.

- Marcel Filoche

  Title: Gas and surfactant delivery into the lung airway system

  Abstract: The pulmonary airway system is a highly hierarchical tree-like 3D network in charge of transferring gases between the trachea and the alveolar region where oxygen and carbon dioxide are exchanged between air and blood. It is also the place of numerous fluid transport processes. Given the complexity of the anatomy and of the physical processes involved, understanding its working and predicting the outcome of therapies is especially tricky. The range of scales represents here a huge computational and modeling challenge: from the meter scale of the entire organ size down to the micron scale of the mucus layer, about 5 or 6 orders of magnitude of length scale are crossed. In this talk, we will present mathematical models able to capture the various aspects of the lung airway system, from the simplest approach of gas transport to more advanced computational simulations of surfactant delivery. We will show, in particular, how the geometrical structure of the lung airway system influences the homogeneity and the efficiency of the distribution of fluid in the terminal regions, and what is the impact on patient-specific therapies.

- Katrin Gelfert

  Title: Multifractal analysis of fiber-Lyapunov exponents in circle-fiber skew products

  Abstract: We study skew products of circle diffeomorphisms over a shift space. Our primary motivation is the fact that they capture some key mechanisms of nonhyperbolic behavior of robustly transitive dynamical systems. We perform a multifractal analysis of fiber-Lyapunov exponents studying the topological entropy of fibers with equal exponent. This includes the study of restricted variational principles of the entropy of ergodic measures with given fiber-exponent, in particular, with exponent zero. This enables to understand transitive dynamical systems in which hyperbolicities of different type are intermingled. Moreover, it enables to "quantify of the amount of non-hyperbolicity" in a context where any other tools presently available fail. This is joint work with L.J. Díaz and M. Rams.

- Denis Grebenkov

  Title: Diffusion towards fractal boundaries: fresh insights onto an old problem

  Abstract: In this talk, I will first review several mathematical and numerical results on diffusion towards fractal boundaries such as von Koch curves or surfaces, including the harmonic measure distribution and its multifractal spectrum, an extension to partially reactive boundaries characterized by Robin boundary condition, and the distribution of the first-passage times. In the second part, more recent results on the statistics of diffusive encounters and related open problems will be presented.

- Kornélia Héra

  Title: Hausdorff dimension of Besicovitch sets of Cantor graphs

  Abstract: It is well known that planar Besicovitch sets sets containing a unit line segment in every direction have Hausdorff dimension 2. In a joint work with Iqra Altaf and Marianna Csörnyei we consider Besicovitch sets of Cantor graphs in the plane sets containing a rotated (and translated) copy of a fixed Cantor graph in every direction, and prove lower bounds for their Hausdorff dimension.

- Henna Koivusalo

Title: Shrinking targets on self-affine sets

Abstract: The classical shrinking target problem concerns the following set-up: Given a dynamical system (T, X) and a sequence of targets (B_n) of X, we investigate the size of the set of points x of X for which $T^n(x)$ hits the target B_n for infinitely many n. In this talk I will discuss shrinking target problems in the context of iterated function systems, where 'size' is studied both from the perspective of dimension and measure. I will give an overview of the topic, aiming also to cover some of my own results, joint with Simon Baker, Thomas Jordan, Lingmin Liao and Michal Rams.

Time permitting, and if the results are ready on time, further results of similar flavour might also be explored (joint with Balazs Barany and Sascha Troscheit) .

- Céline Lacaux

  Title: Fractional Gaussian and Stable randoms fields on fractals

  Abstract: In this talk, we adopt the viewpoint about fractional fields which is given in *Lodhia and al. Fractional Gaussian fields: a survey, Probab. Surv. 13 (2016), 1–56.* As example, we focus on random fields defined on the Sierpiński gasket but random fields defined on fractional metric spaces can also be considered. Hence, for $s \geq 0$, we consider the random measure $X = (-\Delta)^{-s}W$ where Δ is a Laplacian on the Sierpiński gasket K equipped with its Hausdorff measure μ and where W is a Gaussian random measure with intensity μ. For a range of values of the parameter s, the random measure X admits a Gaussian random field $(X(x))_{x\in K}$ as density with respect to μ. Moreover, using entropy method, an upper bound of the modulus of continuity of $(X(x))_{x\in K}$ is obtained, which leads to the existence of a modification with Hölder sample paths. Along the way we prove sharp global Hölder regularity estimates for the fractional Riesz kernels on the gasket. In addition, the fractional Gaussian random field X is invariant by the symmetries of the gasket. If time allows, some extension to α-stable random fields will also be presented. Especially, for $s \geq s_0$ there still exists a modification of the α-stable field X with Hölder sample paths whereas for $s < s_0$, such modification does not exist. This is a joint work with Fabrice Baudoin (University of Connecticut).

- François Ledrappier

  Title: Exact dimension of the Oseledets distribution

  Abstract: We consider a random walk on the group of dxd matrices. We assume that the directing probability measure has a first moment and finite entropy. We show that the distributions of the Oseledets splitting, of the unstable flag and of the stable flag are exact dimensional. This is a joint work with Pablo Lessa (Montevideo).

- Ian Morris

  Title: A variational principle relating self-affine measures and self-affine sets

  Abstract: A breakthrough result of Bárány, Rapaport and Hochman published in 2019 showed that if a two dimensional affine iterated function system is strongly irreducible and satisfies the strong open set condition, then the Hausdorff dimension of its attractor is equal to a value defined by Falconer in 1988. Their result applied a Ledrappier-Young formula established by Bárány and Käenmäki and a variational principle due to Morris and Shmerkin in combination with deep new results on projections of self-affine measures. This Ledrappier-Young formula has since been extended to higher dimensions by Feng, and projections of self-affine measures in higher dimensions are currently being studied by

Rapaport. In this talk I will describe an extension of the variational principle to higher dimensions. In combination with a recent preprint of Rapaport this implies that the above theorem of Bárány, Rapaport and Hochman is also valid in dimension three. This is joint work with Çağri Sert.

- Ariel Rapaport

  Title: On the Rajchman property for self-similar measures on $\mathbb{R}^d$

  Abstract: I'll present a complete algebraic characterization of self-similar iterated function systems on $\mathbb{R}^d$, for which there exists a positive probability vector so that the Fourier transform of the corresponding self-similar measure does not tend to 0 at infinity.

- Vincent Vargas

  Title: A rigorous construction of the 2d sinh model

  Abstract: 2d quantum field theories with an exponential type interaction have received a lot of attention recently du to their importance in the study of statistical physics. In this talk, I will present a rigorous probabilistic construction of the finite volume sinh model. Despite the fact that the model is not a conformal field theory, it is expected to exhibit an integrable structure. Based on joint work with C. Guillarmou and T. Gunaratnam.

Abstracts-Contributed lectures

- Antoine Ayache

  Title: Harmonizable Fractional Stable Motion: simultaneous estimators for the both parameters

  Abstract: There are two classical very different extensions of the well-known Gaussian fractional Brownian motion to non-Gaussian frameworks of heavy-tailed stable distributions: the harmonizable fractional stable motion (HFSM) and the linear fractional stable motion (LFSM). As far as we know, while several articles in the literature, some of which appeared a long time ago, have proposed statistical estimators for the parameters of LFSM, no estimator has yet been proposed in the framework of HFSM. Among other things, what makes statistical estimation of parameters of HFSM to be a difficult problem is that, in contrast to LFSM, HFSM is not ergodic. The main goal of our talk is to propose a new strategy for dealing with this problem and obtaining solutions of it. The keystone of our new strategy consists in the construction of new transforms of HFSM which allow to obtain, at any dyadic level, a sequence of independent stable random variables.

- Ayreena Bakhtawar

  Title: Uniform approximation via continued fractions

  Abstract: Dirichlet's theorem (1842) is a fundamental result in Diophantine approximation that gives an optimal approximation rate of any irrational number. Recently, it has been shown that improvements to Dirichlet's theorem are concerned with the growth of the product of consecutive partial quotients. In this talk, I will describe metrical results for the sets associated with the product of an arbitrary block of consecutive partial quotients raised to different powers.

- Amlan Banaji

  Title : Multifractal analysis on Bedford-McMullen carpets

  Abstract : The multifractal analysis of self-affine measures supported on Bedford-McMullen carpets has been studied since the 1990s. Combining recent work of Rao et al. and B.-Kolossvary shows that the fine multifractal spectrum of the uniform self-affine measure is bi-Lipschitz invariant, which we will explain gives explicit necessary conditions for two carpets to be bi-Lipschitz equivalent. Moreover, we describe an intriguing connection between this multifractal spectrum and the intermediate dimensions of the support of the measure.

- Prasuna Bandi

  Title : Exact simultaneous approximation

  Abstract : In Diophantine approximation, it is a classical problem to determine the size of the sets related to ψ-approximable set for a given non-increasing function ψ. Jarnìk showed that the Exact ψ approximable set i.e. the set of vectors which are ψ approximable but not any better is non-empty. Bugeaud determined the Hausdorff dimension of the exact set in reals using continued fractions. We extend this result to higher dimensions by translating the problem to dynamics using Dani's correspondence. This is joint work with Nicolas de Saxcé.

- Edouard Daviaud

Title : Mass transference principle for inhomogeneous measures

Abstract : In Diophantine aaproximation, mass transference is a key tool to estimate Hausdorff dimensions of sets approximated at a certain speed rate by particular points (such as rationals). Many of the known results, such has Jaffard's Theorem, Beresnevich-Velani's Theorem and Koivuslo-Rams's Theorem, requires that the ambiant space is Alfors-regular. In this talk, we present comparable results when the measure involved is not Alfhors regular. As an application to his result, we will present some result about the approximation by weakly-conformal IFS with overlaps and an application to rational approximation among the set of points of [0,1] whose expansion in base 3 is such that the digit 1 appears with frequence arbitrarly close to 0 infinitely many often.

- Bertrand Duplantier

  Title: Complex Generalized Integral Means Spectrum for Drifted Whole-Plane SLE

  Abstract: We study the random conformal map f(z) from the unit disk to the slit plane, associated with whole-plane Schramm-Loewner evolution with SLE parameter kappa and with a Brownian drift a. We consider the joint complex moments of respective orders p and q of the derivative f'(z) and of f(z) itself. The exact form of the generalized integral means spectrum beta(p,q,kappa,a) is derived from a Beliaev-Smirnov type PDE, together with the help of Liouville quantum gravity.

  Based on joint work with Yong Han, Chi Nguyen and Michel Zinsmeister.

- Arnaud Durand

  Title: Capacities and dimensions for random sets and measures in doubling metric spaces

  Abstract: We work in doubling metric spaces, and we introduce new classes of random sets that are bound to almost surely intersect any given deterministic set with positive capacity in some given gauge function. Using capacities, we express a lower bound on the size of the random sets in these classes, and of their intersections with arbitrary deterministic sets. We also show that the classes are closed under countable intersections and bilipschitz mappings. In the special case of Ahlfors regular spaces, this yields a connection with new large intersection classes that generalize those of Falconer. Our methods are based on tree percolation and an extension of Mandelbrot's fractal percolation process to general metric spaces.

  As an application, we study the size and large intersection properties of a class of random sets that are derived from random measures on the space of balls. We also study the complements of these sets, and their intersections with fixed deterministic sets. In particular, we estimate the dimension of these random sets in terms of the multifractal spectrum of an underlying intensity measure.

  When the random measure is a point process, the resulting sets are limsup sets that are derived from arbitrary systems of random balls. Our approach thus unifies and extends numerous results previously obtained in rather specific models (balls with independent centers, Poisson processes, orbits of exponentially/polynomially mixing dynamical systems).

- Ai Hua Fan

  Title: Multifractal analysis of weighted ergodic averages

  Abstract: The multifractal analysis of weighted ergodic averages is a new subject and new ideas are needed because the classical thermodynamical formalism and the classical ergodic

theory are no longer adapted. We will present some first works on the following cases: full shifts and subshifts, Markov interval dynamical systems and conformal dynamical systems etc.

- Stéphane Jaffard

  Title: Random wavelet series

  Abstract: We compare the boundedness and regularity properties of Fourier vs. wavelet series when the coefficients are randomized. Many results (e.g. by Marcus, Pisier, Kahane, etc) show that, for Fourier series, randomization has a smoothing effect. We show that it is the opposite for wavelet series. This is a joint work with Céline Esser and Béatrice Vedel.

- Jungwon Lee

  Title: Dynamics of Ostrowski skew-product: Limit laws and Hausdorff dimensions

  Abstract: We discuss dynamical and multifractal aspects of the Ostrowski map, which given by the skew-product extension of continued fraction transformation. We plan to outline the set up and strategy based on the transfer operator analysis, as well as application towards the arithmetic of number fields (joint with Valérie Berthé).

- Laurent Loosveldt

  Title: Wavelet-type approximation of Hermite processes

  Abstract: In recent years, wavelet analysis has proved to be an efficient tool to study stochastic processes. It allows to express some of them as a random series involving smooth functions. This feature is particularly appreciated when it comes to numerically simulate such processes or study paths regularity. For instance, one can cite the wavelet-type series of fractional Brownian motion or Rosenblatt process. These two processes belong to the class of Hermite processes, fractional Brownian motion being of order 1 and Rosenblatt process of order 2. As of today, there is no wavelet-type representation for Hermite processes of higher order. The aim of this talk is to fill this gap. It is a joint work with Antoine Ayache and Julien Hamonier from the University of Lille.

- Mark Pollicott

  Title: The dimension of the Laplacian spectrum of the Sierpinski triangle

  Abstract: The dimension of the Sierpinski triangle T is well known to be log(3)/log(2). However, one can also associate to T a suitable Lapalacian and consider its spectrum. This is a set of non-zero dimension in the real line whose value we can estimate. This is joint work with Julia Slipantschuk.

- Srijanani Anurag Prasad

  Title: Spectrum of a self-affine measure with four-element digit set

  Abstract: Let $D = \{(0,0)^t, (1,0)^t, (0,1)^t, (-1,-1)^t\}$ and $M \in M_2(\mathbb{Z})$ be an expansive matrix. In this paper, we have proved that there exists a Λ such that the set $E_\Lambda = \{e^{2\pi i\langle\lambda,x\rangle} : \lambda \in \Lambda\}$ is an orthonormal basis of $L_2(\mu_{M,D})$ for a matrix which has all even entries. Also, we have found a spectrum Λ of $L_2(\mu_{M,D})$ for some specific M and D.

- Alex Rutar

  Title: Multifractal Analysis of Self-affine Carpets

Abstract: I will present an interpretation of the multifractal formalism for self-similar measures using ideas from convex duality. I will then discuss how these ideas generalize to the multifractal analysis of planar self-affine measures without irreducibility or domination. Through some examples, we will see that the L^q and multifractal spectra of measures in this class exhibit a range of exotic behaviour. This is joint work with Thomas Jordan and István Kolossvary.

- Jasson Vindas

  Title : Pointwise behavior of fractional integrals of modular forms via complex analysis

  Abstract : Recently in 2019, Pastor has completed the computation of the pointwise Hölder exponents for fractional integrals of modular forms. For the analysis at irrational numbers, his proofs are indirect, making use of Tauberian theorems for the wavelet transform. In this talk we discuss an alternative direct approach that only employs basic complex analysis in order to determine the pointwise Hölder exponent at the irrationals from the boundary behavior of the modular forms.

- Polina Vytnova

  Title: 32 digits of the Hausdorff dimension of the Apollonian gasket

  Abstract: We combine the ideas developed in a recent joint work with M. Pollicott on Hausdorff dimension estimates, (Transactions of the AMS, Ser. B, Vol. 9, pp. 1102–1159) with the approach to systems with neutral fixed points by C. Wormell (Efficient computation of statistical properties of intermittent dynamics, arXiv: 2106.01498) to rigorously confirm an earlier estimate on Hausdorff dimension of the Apollonian gasket by Bai and Finch (Fractals 26(04) DOI: 10.1142/S0218348X18500500).

- Caroline Wormell

  Title: Chebyshev methods for contracting and parabolic IFS

  Abstract: Many important fractal quantities, such as Hausdorff dimensions and Gibbs measures, can be expressed as solutions of spectral problems involving transfer operators. I will present a simple, highly effective approach for strictly contractive IFS' that discretises these spectral problems in a Chebyshev basis, and show how the theory of Abel functions can be harnessed to also compute similar quantities for parabolic IFS. In both cases the convergence is exponential, and many dozens of digits may be computed.

- Ayshe Yiltekin-Karatas

  Title: Families of Markov Maps Associated to Cocompact Fuchsian Triangle Groups

  Abstract: In 1979, for each signature for Fuchsian groups of the first kind, Bowen and Series constructed an explicit fundamental domain, and from this a function on unit circle tightly associated with this group. In general, their fundamental domain enjoys what has since been called the 'extension property'. We determine the exact set of signatures for cocompact triangle groups for which this property can hold for any convex fundamental domain. To each Bowen-Series function in this corrected setting, we naturally associate four one-parameter deformation families of circle functions. We determine when these functions are finite Markov and find that not each function in the constructed family admits an ergodic invariant measure equivalent to Lebesgue.