Complex Geometry, Dynamical Systems and Foliation Theory, CIRM, October 17–21, 2022 – ABSTRACTS

Masanori Adachi
A residue formula for meromorphic connections and applications to stable sets of foliations

Abstract: We discuss a proof for Brunella's conjecture: a codimension one holomorphic foliation on a compact complex manifold of dimension > 2 has no exceptional minimal set if its normal bundle is ample. The main idea is the localization of the first Chern class of the normal bundle of the foliation via a holomorphic connection. Although this localization was done via that of the first Atiyah class in our previous proof, we shall explain that this can be shown more directly by a residue formula. This talk is based on joint works with S. Biard and J. Brinkschulte.

Hugues Auvray
Bergman kernels on punctured Riemann surfaces

Abstract: In joint works with X. Ma (Paris 7) and G. Marinescu (Cologne) we obtain refined asymptotics for Bergman kernels computed form singular data. We work on the complement of a finite number of points, seen as punctures, on a compact Riemann surface, that we endow with a metric extending Poincaré's cusp metric around the puncture points. We moreover fix a holomorphic line bundle polarizing such a metric. I'll explain how an advanced description of the model (on the punctured unit disc) and weighted techniques in a singular context allow to describe the Bergman kernels associated to such Riemann surfaces, up to the singularities.

If time permits, I'll also mention geometric, or even arithmetic, interpretations of such results.

Turgay Bayraktar
Expected Topology of Random Real Algebraic Plane Curves

Abstract: Broadly speaking Hilbert's sixteenth problem suggests studying possible number and arrangement of connected components of a smooth real algebraic plane curve. One can also approach this problem from a probabilistic point-view and try to obtain statistical results such as computing the expected number of components, depth of nested ovals etc. In this talk, I'll review some recent probabilistic results on this problem. In particular, I will give a Kac-Rice type formula computing the expected number of nested ovals winding around a fixed point.

Fabrizio Bianchi
Holomorphic motions of Julia sets: dynamical stability in one and several complex variables

Abstract: We discuss the stability of holomorphic dynamical systems under perturbation. In dimension 1, the theory is now classical and is based on works by Lyubich, Mané-Sad-Sullivan and DeMarco. I will review this theory and present a recent generalisation valid in any dimension. Since classical 1-dimensional techniques no longer apply in higher dimensions, our approach is based on ergodic and pluripotential methods.

Junyan Cao
Infinitesimal extension of adjoint forms in Kahler setting and applications, II

Abstract (Junyan Cao + Mihai Păun): We will report on a recent joint work. In the first first part (MP lecture) we present a version of M. Levine's approach for the extension of pluricanonical forms in Kaehler setting. The second lecture (by JC) will mainly be focused on an application: the techniques we are developing can be applied to obtain an "injectivity" result conjectured by O. Fujino.

Dan Coman
Extension of quasiplurisubharmonic functions

Abstract: Let (V, ω) be a compact Kähler manifold and X be an analytic subvariety of V. We address the problem of extending ω-plurisubharmonic functions on X to ω-plurisubharmonic functions on V. Our results are joint with Vincent Guedj and Ahmed Zeriahi.

Ya Deng
Hyperbolicity and fundamental groups of complex quasi-projective varieties

Abstract: Non-abelian Hodge theories in both Archimedean and non-Archimedean settings are important tools in studying fundamental groups of quasi-projective varieties. In this talk I will explain some progress on these techniques and their application to the hyperbolicity of quasi-projective varieties whose fundamental groups admit linear representations. This talk is based on a joint work with Benoit Cadorel and Katsutoshi Yamanoi.

Tien-Cuong Dinh
On the automorphisms of compact Kähler manifolds

Abstract: The automorphism group of a compact Kähler manifold satisfies Tits alternative: any subgroup either admits a solvable subgroup of finite index or contains a free non-abelian group of two generators (Campana-Wang-Zhang). In the first case, this group cannot be too big. Some algebraic (rational) manifolds with special automorphisms admit infinitely many non-equivalent real forms. This talk is based on my (old and recent) works with F. Hu, H.-Y. Lin, V.-A. Nguyen, K. Oguiso, N. Sibony, X. Yu, D.-Q. Zhang.

Siarhei Finski

On the metric structure of section ring

Abstract: We study the relationship between metric and algebraic structures on the section ring of a projective manifold and an ample line bundle over it. More precisely, we prove that once the kernel is factored out, the multiplication operator of the section ring becomes an approximate isometry (up to normalization) with respect to the L^2-norm. We then show that, in fact, those algebraic properties characterise L^2-norms and describe some applications of this classification. The semiclassical version of Ohsawa-Takegoshi theorem lies at the heart of our approach.

Chin-Yu Hsiao

Semi-classical spectral asymptotics of Toeplitz operators on CR manifolds

Abstract: Let X be a compact strongly pseudoconvex CR manifold and let T_P be the Toeplitz operator on X, where P is a first order pseudodifferential operator. We consider $\chi_k(T_P)$ the function calculus of T_P by rescaled a cut-off function χ. In this work, we show that $\chi_k(T_P)$ admits a full asymptotic expansion in k. As applications, we obtain several CR analogous of Theorems concerning high power of line bundles in complex geometry but without any group action assumptions on the CR manifold. This is a joint work with Hendrik Herrmann, George Marineacu and Wei-Chuan Shen.

Jun-Muk Hwang

Formal principle for rational curves in complex threefolds

Abstract: A complex submanifold in a complex manifold satisfies the formal principle if its formal neighborhood determines its biholomorphic germ. A smooth rational curve in a complex manifold satisfies the formal principle if its normal bundle is positive. It is unknown whether a rational curve with semi-positive normal bundle satisfies the formal principle. We discuss the simplest unknown case of a smooth rational curve in a threefold whose normal bundle is the direct sum of a trivial line bundle and a line bundle of degree 1.

Sung-Yeon Kim
Proper holomorphic maps between bounded symmetric domains with small rank differences

Abstract: In this talk, we study the rigidity of proper holomorphic maps $f\colon \Omega \to \Omega'$ between irreducible bounded symmetric domains Ω and Ω' with small rank differences: $2 \leq \operatorname{rank}(\Omega') < 2\operatorname{rank}(\Omega) - 1$. More precisely, if either Ω and Ω' have the same type or Ω is of type III and Ω' is of type I, then up to automorphisms, f is of the form $f = \imath \circ F$, where $F = F_1 \times F_2 \colon \Omega \to \Omega'_1 \times \Omega'_2$. Here Ω'_1, Ω'_2 are bounded symmetric domains, the map $F_1 \colon \Omega \to \Omega'_1$ is a standard embedding, $F_2 : \Omega \to \Omega'_2$, and $\imath \colon \Omega'_1 \times \Omega'_2 \to \Omega'$ is a totally geodesic holomorphic isometric embedding. As a consequence, $f : \Omega \to \Omega'$ is a holomorphic totally geodesic isometric embedding with respect to Kobayashi metrics. Moreover we show that, under the rank condition above, there exists no proper holomorphic map $f : \Omega \to \Omega'$ if Ω is of type I and Ω' is of type III, or Ω is of type II and Ω' is either of type I or III. This is a joint work with N. Mok and A. Seo.

Lucas Kaufmann
Complex analytic methods in random matrix theory

Abstract: In the theory of products of random matrices, several deep results can be achieved via spectral gap theorems, including equidistribution results, fine limit theorems and Fourier decay properties of natural measures. However, spectral gap theorems are often hard to obtain.

In this talk, I will show how methods from complex analysis and analogies with holomorphic dynamics offer us good tools and yield new and often optimal results for random 2 x 2 matrices. This is based on joint works with T.-C. Dinh and H. Wu.

Ursula Ludwig
Bismut-Zhang theorem for singular spaces

Abstract: The famous theorem of Cheeger and Müller states the equality between the analytic (or Ray-Singer) torsion and the topological torsion of a smooth compact manifold equipped with a unitary flat vector bundle. Using local index techniques and the Witten deformation Bismut and Zhang gave the most general comparison theorem of torsions for a smooth compact manifold. The aim of this talk is the generalisation of the Bismut-Zhang theorem to the context of isolated conical singularities: We first establish a comparison formula between the analytic torsion and a torsion, which we call the Bismut-Zhang torsion. We also establish anomaly formulas for all three terms in the comparison formula, i.e. we study how the terms behave under variations of the Riemannian conical metric.

Xiaonan Ma

Geometric quantization on CR manifolds

Abstract: Let X be a compact connected orientable Cauchy-Riemann (CR) manifold with the action of a compact Lie group G. Then we can define a notion so-called "CR reduction". We try to understand the relation on G-invariant CR functions on the original CR manifold and the CR functions on its CR reduction. Under natural pseudoconvexity assumptions we show that the natural map induced by restriction is Fredholm at the level of Sobolev spaces of CR functions. As application we study this map for holomorphic line bundles which are positive near the inverse image of 0 by the momentum map. We also show that "quantization commutes with reduction" for Sasakian manifolds.

Stéphanie Nivoche

New solution of a problem of Kolmogorov on width asymptotics in holomorphic function spaces

Abstract: Given a domain D in $\mathbb{C}^n$ and K a compact subset of D, we denote $\mathcal{A}_K^D$ the compact set in $C(K)$, of all restrictions in K of holomorphic functions on D bounded by 1. The sequence $(d_m(\mathcal{A}_K^D))_{m\in\mathbb{N}}$ of Kolmogorov m-widths of $\mathcal{A}_K^D$ provides a measure of the degree of compactness of the set $\mathcal{A}_K^D$ in $C(K)$ and the study of its asymptotics has a long history, essentially going back to Kolmogorov's work on ϵ-entropy of compact sets in the 1950s. The precise statement of this problem is

$$\lim_{m\to\infty} \frac{-\log d_m(\mathcal{A}_K^D)}{m^{1/n}} = 2\pi \left(\frac{n!}{C(K,D)}\right)^{1/n}, \tag{1}$$

where $C(K,D)$ is the Bedford-Taylor relative capacity of K in D. This problem has already been proved in 2004 by S.N., using pluripotential theory technics.

Here, with O. Bandtlow, we give a totally new proof of the asymptotics (1) for D strictly hyperconvex and K non-pluripolar. We proceed by a two-pronged approach establishing sharp upper and lower bounds for the Kolmogorov widths. The lower bounds follow from concentration results for the eigenvalues of a certain family of Toeplitz operators, while the upper bounds follow from an application of the Bergman-Weil formula together with an exhaustion procedure by special holomorphic polyhedra.

Keiji Oguiso
Kawaguchi-Silverman Conjecture for birational automorphisms of smooth irrational varieties

Abstract: Kawaguchi–Silverman Conjecture (KSC) predicts an interesting relation between arithmetic and algebraic dynamics that for a birational self-map f of a smooth projective variety X defined over an algebraic closure L of the field of rational numbers, the arithmetic degree $\alpha_f(x)$ exists and coincides with the first dynamical degree δ_f for any L-point x of $X(L)$ with a Zariski dense orbit. In this talk, after a brief introduction of arithmetic degree and KSC , we show that KSC holds when X has Kodaira dimension zero and irregularity $q(X) \geq \dim X - 1$ or X is an irregular threefold (modulo one possible exception). We also study the existence of Zariski dense orbits, with explicit examples. This is a joint work with Professors Jungkai Chen and Hsueh-Yung Lin.

Mihai Păun
Infinitesimal extension of adjoint forms in Kahler setting and applications, I

Abstract (Junyan Cao + Mihai Păun): We will report on a recent joint work. In the first first part (MP lecture) we present a version of M. Levine's approach for the extension of pluricanonical forms in Kaehler setting. The second lecture (by JC) will mainly be focused on an application: the techniques we are developing can be applied to obtain an "injectivity" result conjectured by O. Fujino.

Jean Ruppenthal
Canonical sheaves at isolated (canonical Gorenstein) singularities

Abstract: The canonical line bundle and the corresponding canonical sheaf belong to the most important geometric/analytic objects associated to a complex manifold. They play a crucial role e.g. in classification theory, Serre duality or vanishing theorems. If we consider singular varieties instead of smooth manifolds, then there exist various possibilities to generalize the canonical sheaf to that setting. One can consider for example the Grothendieck(-Barlet-Henkin-Passare) dualizing sheaf or the Grauert-Riemenschneider L^2-sheaf. In this talk, we will discuss another possible generalization, i.e., the sheaf of L^2 holomorphic n-forms with a certain boundary condition at the singular set. This sheaf is essential for L^2-$\overline{\partial}$-theory on singular spaces, but difficult to understand. We will describe it explicitly for isolated canonical Gorenstein singularities.

Nikhil Savale
Bochner Laplacians and Bergman kernels for families

Abstract: We generalize prior results of Marinescu-Savale to families of Bochner Laplacians. This particularly leads to the fiberwise expansion for Bergman kernels of horizontally semi-positive index bundles. The proof uses Ma-Zhang's description for the curvature of the index bundle as a fiberwise Toeplitz operator. Joint work with X. Ma and G. Marinescu.

Trung Tuyen Truong
Some interesting birational morphisms of smooth affine quadric 3-folds

Abstract: We study a family of birational maps of smooth affine quadric 3-folds $x_1x_4 - x_2x_3 =$ constant, over $\mathbb{C}$, which seems to have some (among many others) interesting/unexpected characters: a) they are cohomologically hyperbolic, b) their second dynamical degree is an algebraic number but not an algebraic integer, and c) the logarithmic growth of their periodic points is strictly smaller than their algebraic entropy. These maps are restrictions of a polynomial map on $\mathbb{C}^4$ preserving each of the quadrics. The study in this paper is a mixture of rigorous and experimental ones, where for the experimental study we rely on Bertini which is a reliable and fast software for expensive numerical calculations in complex algebraic geometry. This is joint work with Cinzia Bisi and Jonathan Hauenstein.

Gabriel Vigny
On a complex Sobolev space

Abstract: The usual Sobolev space H^1 plays a central role in potential theory as it is connected to the Laplacian operator. Nevertheless, it is not so convenient in (complex) pluripotential theory since it does not enjoy good invariance properties under changes of coordinates.

To overcome that fact, Dinh and Sibony defined fifteen years ago a complex Sobolev space W^* whose very definition involves the complex structure: it is the set of functions f on a complex manifold X such that $df \wedge d^c f$ is bounded from above by a positive closed current. They introduced the space W^* in order to show the exponential decay correlations for meromorphic maps of a compact Kähler manifolds. Since then, W^* has been used several times in complex dynamics and also in the study of the Monge-Ampère operator.

The purpose of this talk is to review new and old results on that space.

Duc-Viet Vu
Moser-Trudinger inequalities and complex Monge-Ampere equations

Abstract: I present a version of the Moser-Trudinger inequality in the setting of complex geometry. As a very particular case, our result gives a new Moser-Trudinger inequality for functions in the Sobolev space $W^{1,2}$ of a domain in $\mathbb{R}^2$. We also deduce a necessary condition for the complex Monge-Ampere equation for a given measure on a compact Kahler manifold to admit a Holder continuous solution. This is a joint-work with George Marinescu and Tien-Cuong Dinh.

De-Qi Zhang
Jordan property of compact complex varieties

Abstract: A group G has the Jordan property (like a complex general linear group) if every finite subgroup H of G has a finite-index abelian subgroup H_1 with index $[H : H_1]$ bounded by a so called Jordan constant $J(G)$ depending only on G. We report some recent results on the Jordan property of the full automorphism group $Aut(X)$, where X is a compact complex variety.