POEMS 2019

Titles and abstracts

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Keynote presentations

Thursday, 14h00

Polyhedral discretizations for industrial applications

Jérôme Bonelle, EDF R&D

This talk will be devoted to the usage of new discretization schemes on polyhedral meshes in an industrial context. These discretizations called CDO [1, 2] (Compatible Discrete Operator) or Hybrid High Order [3,4] (HHO) schemes have been recently implemented in Code_Saturne [5]. Code_Saturne is an open-source code developed at EDF R&D aiming at simulating single-phase flows.
First, the advantages of robust polyhedral discretizations will be recalled. Then, the underpinning principles of CDO schemes will be presented as well as some applications: diffusion equations, transport problems, groundwater flows or the discretization of the Stokes equations. High Performance Computing (HPC) aspects will be also discussed as it is an essential feature in an industrial context either to address complex and large computational domains or to get a quick answer. Some highlights on the main outlooks will be given to conclude.

[1] J. Bonelle, A. Ern, *Analysis of Compatible Discrete Operator schemes for elliptic problems on polyhedral meshes*, ESAIM: Mathematical Modelling and Numerical Analysis (2014)
[2] P. Cantin, J. Bonelle, E. Burman and A. Ern, *A vertex-based scheme on polyhedral meshes for advection-reaction equations with sub-mesh stabilization*, Computers & Mathematics with Applications (2016)
[3] D. A. Di Pietro, A. Ern, and S. Lemaire *An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators*, Comput. Meth. Appl. Math. (2014)
[4] D. A. Di Pietro, A. Ern, A. Linke and F. Shieweck, *A discontinuous skeletal method for the viscosity-dependent Stokes problem*, Comput. Meth. Appl. Mech. Engrg. (2016)
[5] Code_Saturne website: `https://www.code-saturne.org`

Tuesday, 9h00

Some New Estimates for Virtual Element Methods

SUSANNE C. BRENNER, LOUISIANA STATE UNIVERSITY

We will present some new estimates for virtual element methods for the model Poisson problem in two and three dimensions. They include maximum norm error estimates for the virtual element solution and for its projections into the space of piecewise polynomial functions.

Wednesday, 9h00

Combining cut element methods and hybridization

Erik Burman, Louisiana State University

Recently there has been a surge in interest in cut, or unfitted, finite element methods. In this class of methods typically the computational mesh is independent of the geometry. Interfaces and boundaries are allowed to cut through the mesh in a very general fashion. Constraints on the boundaries such as boundary or transmission conditions are typically imposed weakly using Nitsche's method.

In this talk we will discuss how these ideas can be combined in a fruitful way with the idea of hybridization, where additional degrees of freedom are added on the interfaces to further improve the decoupling of the systems, allowing for static condensation of interior unknowns.

In the first part of the talk we will discuss how hybridization can be combined with the classical cut finite element method, using standard H^1-conforming finite elements in each subdomain, leading to a robust method allowing for the integration of polytopal geometries, where the subdomains are independent of the underlying mesh. This leads to a framework where it is easy to integrate multiscale features such as strongly varying coefficients, or multidimensional coupling, as in flow in fractured domains. Some examples of such applications will be given.

In the second part of the talk we will focus on the Hybridized High Order Method (HHO) and show how cut techniques can be introduced in this context. The HHO is a recently introduced nonconforming method that allows for arbitrary order discretization of diffusive problems on polytopal meshes. HHO methods have hybrid unknowns, made of polynomials in the mesh elements and on the faces, without any continuity requirement. They rely on high-order local reconstructions, which are used to build consistent Galerkin contributions and appropriate stabilization terms designed to preserve the high-order approximation properties of the local reconstructions.

Here we will show how cut element techniques can be introduced as a tool for the handling of (possibly curved) interfaces or boundaries that are allowed to cut through the polytopal mesh. In this context the cut element method plays the role of a local interface model, where the associated degrees of freedom are eliminated in the static condensation step. Issues of robustness and accuracy will be discussed and illustrated by some numerical examples.

Wednesday, 14h00

Nonlinear free energy diminishing schemes for convection-diffusion equations: convergence and long time behaviour

Claire Chainais-Hillairet, Université de Lille

The aim of the talk is to introduce a nonlinear Discrete Duality Finite Volume scheme to approximate the solutions of drift-diffusion equations. The scheme is built to preserve at the discrete level even on severely distorted meshes the energy / energy dissipation relation. This relation is of paramount importance to capture the long-time behavior of the problem in an accurate way. To enforce it, the linear convection diffusion equation is rewritten in a nonlinear form before being discretized. This is a joint work with Clément Cancès (Lille) and Stella Krell (Nice).

Wednesday, 9h00

Superconvergence by M-decompositions

Bernardo Cockburn, University of Minnesota

We show how to systematically devise super convergent HDG and mixed methods by using the so-called technique of M-decompositions. We carry this out in the framework of steady-state diffusion and then show how to use the results as building blocks for the Navier-Stokes equations and the equations of linear elasticity.

Monday, 16h15

The Gradient Discretisation Method: Tools and Applications

Robert Eymard, Université Paris-Est Marne-la-Vallée

Some convergence properties for the approximation of second order elliptic problems with a variety of boundary conditions (homogeneous Dirichlet, homogeneous or non-homogeneous Neumann or Fourier boundary conditions), using a given discretisation method, can be obtained when this method is plugged into the Gradient Discretisation Method (GDM) framework.
Instead of defining one GDM framework for each of these boundary conditions, we show that these properties can be stated using the same abstract tools for all the above boundary conditions. Then these tools enable the application of the GDM to a larger class of elliptic problems.

Thursday, 16h15

On kinetic energy preserving convection operators on general

Jean-Claude Latché, Institut de Radioprotection et de Sûreté Nucléaire

We derive, by algebraic (and so "robust with respect to the cell geometry") arguments, kinetic ernergy preserving convection operators for staggered discretizations. Then we establish a Lax-Wendroff consistency result for the obtained operators.

Monday, 9h00

Virtual Element approximation of magnetostatic

Donatella Marini, University of Pavia

We present a lowest order Serendipity Virtual Element method, and show its use for the numerical solution of linear magneto-static problems in three dimensions. The method can be applied to very general decompositions of the computational domain (as is natural for Virtual Element Methods) and uses as unknowns the (constant) tangential component of the magnetic
eld $\mathbf{H}$ on each edge, and the vertex values of the Lagrange multiplier p (used to enforce the solenoidality of the magnetic induction $\mathbf{B} = \mu\mathbf{H}$). In this respect the method can be seen as the natural generalization of the lowest order Edge Finite Element Method (the so-called "fi
rst kind Nédélec" elements) to polyhedra of almost arbitrary shape, and as we show on some numerical examples it exhibits very good accuracy (for being a lowest order element) and excellent robustness with respect to distortions. Hints on a whole family of elements will also be given; these elements are the natural extension to polyhedra of the so-called "second kind Nédélec" elements.

Thursday, 9h00

Non standard virtual element methods for the Helmholtz problem

Ilaria Perugia, University of Vienna

The numerical approximation of time-harmonic wave propagation problems presents intrinsic difficulties. In fact, due to the oscillatory nature of analytical solutions, standard (polynomial-based) finite element methods deliver accurate approximation only at very high computational cost. Therefore, for these problems, finite element methods based on approximation spaces made of oscillatory functions with the same frequency as the original problem have become increasingly popular, as they allow to reach the same accuracy with less degrees of freedom, as compared to standard polynomial methods. The so-called Trefftz finite element methods, which use test and trial functions that are, element by element, in the kernel of the considered differential operator, belong to this class.
For time-harmonic wave problems, Trefftz approximation spaces have been used in combination with several variational frameworks (e.g. least-squares, ultra weak variational formulation / discontinuous Galerkin formulations, Lagrange multipliers, variational theory of complex rays, wave-based methods), giving rise to different Trefftz finite element methods.
In this talk, we present a method for the approximation of solutions to the 2D Helmholtz problem on general polygonal meshes, which combines the use of Trefftz basis functions with a nonconforming virtual element method (VEM) framework. The basis functions are defined in order to satisfy the Trefftz property, but are not not know in closed form, as typical in VEM. The degrees of freedom are associated with the mesh edges, and the interelement continuity constraints are imposed in a nonconforming sense à la Crouzeix-Raviart. Although the number of degrees of freedom in the definition of this new method is larger than that of other Trefftz methods, an edgewise orthogonalization-and-filtering procedure actually allows for the elimination of "almost" redundant basis functions, leading to a significant reduction of the number of degrees of freedom with no loss of accuracy. At the same time, this procedure has the beneficial effect of improving significantly the conditioning. Theoretical and numerical aspects of this Trefftz VEM will be discussed.

Friday, 09h00

The Virtual Element Method for polygons with curved edges

Alessandro Russo, University of Milano-Bicocca

In my talk I will present the theoretical and numerical results that we have accomplished so far regarding the possibility of using the Virtual Element Method in the presence of curved edges. The work is still ongoing and it is done in collaboration with L. Beirao da Veiga, F. Brezzi, L.D. Marini and G. Vacca.

Monday, 14h00

Virtual Elements for Finite Strain Problems in Plasticity

Peter Wriggers, Leibiz University

Virtual elements (VEM) were developed during the last decade and applied to various problems in elasticity. Due to the fact that the element shape of virtual elements can be arbitrary including even non convex shapes these elements are more exible when the geometry of the element is considered. The success of VEM discretizations in the linear range using different polynomial orders leads directly to the question whether these elements can also be applied successfully to nonlinear situations. This contribution is concerned with a simple low order virtual element formulation and its extension to different nonlinear problems that include inelastic material behaviour. Especially finite strain plasticity and phase field approaches are discussed in detail. Several possible formulations and discretizations are introduced and compared by means of examples.In order to show the applicability of the virtual element method to multi-field problems, a phase field formulation for VEM is developed that allows the investigation of fracturing solids.

Contributed talks

Monday, 17h00

Asymptotic homogenization of random fibre-reinforced composites: a numerical approach based on virtual element technology and a posteriori error estimation

Edoardo Artioli, University of Rome - Tor Vergata

The presentation will cover recent Virtual Element Method developments within the framework of homogenization of random fibre-reinforced composites. In this context, non-homogeneous composite constituents require a numerical approach for determining overall material properties, while use of long cylindrical circular fibre-like inclusions as a reinforcement implies curved elements to grant an accurate geometry approximation. With a focus on statistical homogenization for randomly distributed fibers, the proposed procedure is based on a fine adaptive tuning of the optimal mesh resolution through a residual-based a-posteriori error estimator, and, subsequently, on Monte Carlo simulations to determine representative unit cell size and homogenized material moduli. A gallery of numerical tests will prove the numerical efficiency of the presented methodology.

Monday, 11h45

Virtual Element Methods in subsurface simulations

Stefano Berrone, Politecnico di Torino

Underground simulations in poro-fractured media are particularly challenging as they are involving very complex domains characterized by several geometrical complexities, yielding in particular mesh generation problems.
Subsurface hydro-mechanical simulations in fractured media are commonly required in the analysis of enhanced Oil&Gas simulations, CO2 geological storage, water resources management and preservation, nuclear waste geological storage and high enthalpy geothermal applications. Since the domains are usually stochastically generated, numerical methods should display robustness with respect to arbitrary geometrical complexities. Virtual Element Methods display such robustness, in the talk several VEM approaches to deal with complex geometries will be discussed. The talk will focus on the following issues: construction of a suitable mesh for the problem, mesh refinement and adaptivity, stability, robustness and reliability of simulations. Other issues like VEM implementation and large scale parallel simulations will be marginally tackled.

Monday, 10h45

Building geometrically robust stabilisation term for VEM

Sivia Bertoluzza, CNR

By using a residual basis approach we provide a cheap way to construct a rough approximation of the basis functions for the local VEM space, which we use to define a stabilisation term for the virtual element method, which turns out to be very robust for polygonal elements failing the usual shape regularity criteria.

Friday, 11h45

Displacement-based formulations over star convex polytopes: strain smoothing and scaled boundary finite elements

Stéphane Bordas, University of Luxembourg

In this talk, we discuss two different displacement based formulations over arbitrary star convex polytopes, viz., the cell-based smoothed polygonal finite element method (SPFEM) and the scaled boundary FEM (SBFEM). The essence of the cell-based SPFEM is to approximate the compatible strain by a linear smoothing function and evaluate the smoothed nodal derivatives by the discrete form of the divergence theorem either at the geometric center or at quadrature points within the polytope. This reduces the computational burden in computing the terms in the bilinear and linear form. On the other hand, the SBFEM relies on transforming the cartesian coordinate in to a scaled boundary coordinate system with the origin at the geometric centre of the polytope. This helps in re-writing the governing partial differential equations into a set of ordinary differential equations. Analytical solutions are sought within the polytope and finite element like solution is sought on the boundary. Like the BEM, only the boundary needs to be discretised and like the FEM, the SBFEM does not require Greens functions. Within the SBFEM, an adaptive refinement strategy on quadtree/octree meshes for linear elasticity problems will also be discussed. The adaptive refinement is supplemented with a novel error indicator, which is estimated directly from the solution of the scaled boundary governing equations. The accuracy and the convergence properties of these formulations are studied with a few benchmark problems in the context of linear elasticity. Work in collaboration with Sundararajan Natarajan.

Wednesday, 10h45

Multilevel solution strategies for dG and HHO discretizations

Lorenzo Botti, Università degli studi di Bergamo

In this talk I will consider h-p-hp multilevel solution strategies as an effective mean of solving Hybrid High-Order and discontinuous Galerkin discretizations of Poisson and linear elasticity problems. The final ambitious goal is the numerical simulation of the blow-molding manufacturing process.

Thursday, 11h15

A posteriori error analysis and mesh adaptivity for Virtual Element Methods

Andrea Cangiani, University of Nottingham

An obvious motivation for polytopal element methods (POEMs) is in their application within automatically adaptive simulations. However, little has been done so far to exploit the endless possibilities offered by the availability of very general meshes in the context of mesh adaptive algorithms driven by reliable a posteriori error estimators.

I will present some background work on a posteriori analysis for the Virtual Element Method (VEM) in primal and mixed form. I will show that well established techniques from the FEM can be adapted to the VEM framework while ensuring that the derived a posteriori error estimators are only depending on VEM-computable quantities. In the case of parabolic problems, under mesh change the VEM itself must be based on computable mesh-transfer operators which I will discuss. In the case of mixed VEM, the analysis relies on a suitable Helmholtz decomposition depending on the appropriate primal VE space, much in the same way as in mixed FEM. Moreover, a fully computable and local post-processing technique can be used to provide an approximation of the flux variable for which we are able to prove optimal order a posteriori error bounds in the broken H(div)-norm.

I will present polygonal mesh adaptive simulations driven by such a posteriori error estimators and use them to discuss the above mentioned issue of exploiting polytopal meshed within adaptive algorithms.

Tuesday, 09h45

Skeletal schemes for eigenvalue localisation?

Carsten Carstensen, Humboldt University Berlin

The localisation of eigenvalue is one of the fundamental tasks in numerical analysis even for a Laplace operator and Dirichlet boundary conditions on a polyhedral bounded Lipschitz domain. The Rayleigh-Ritz principles for those symmetric PDE eigenvalue problems lead to (guaranteed) upper eigenvalue bounds with conforming finite elements even in view of inexact solve. A simple post-processing of nonconforming schemes like the Crouzeix-Raviart (for the Laplace) or Morley (for the bi-Laplace) finite element methods leads to guaranteed lower eigenvalue bounds. The improved version of this result from [1,2] will be presented and an attempt is made to generalise it to skeletal methods. This paper suggests the eigenvalue computation with a scheme that, for the source problem, belongs to the class of hybridizable discontinuous Galerkin schemes with Lehrenfeld-Schöberl stabilisation and is also called a weak Galerkin scheme in the literature. The presentation discusses the appropriate choice of a stabilisation parameter and the computation of guaranteed eigenvalue bounds. The presentation also discusses new proofs of the asymptotic lower bound properties in [3].
The topics reflect joint work with Joscha Gedicke (U Vienna) and Dietmar Gallistl (U Twente) and recently with Qilong Zhai and Ran Zhang (Jilin U, China)

[1] C. Carstensen and J. Gedicke, *Guaranteed lower bounds for eigenvalues*, Math. Comp. **83** (2014) 2605-2629.
[2] C. Carstensen and D. Gallistl, *Guaranteed lower eigenvalue bounds for the biharmonic equation*, Numer. Math. **126** (2014) 33-51.
[3] Qilong Zhai and Ran Zhang, *Lower and upper bounds of Laplacian eigenvalue problem by weak Galerkin method on triangular meshes*, Discrete & Continuous Dynamical Systems - B **24** (2019) 403-413.

Thursday, 09h45

Topology optimization with VEM

Heng Chi, Georgia Tech

In many practical applications, topology optimization is a powerful computational tool for improved structural design. This presentation introduces two tailored VEM-based topology optimization formulations considering both single and multiple candidate materials. In the first part, we propose a novel and efficient single-material topology optimization formulation on general polyhedral discretizations by synergistically employing the nodal VEM spaces to construct both the design and state spaces.

Tuesday, 14h45

On the development of an efficient order-adaptive DG method for the simulation of turbulent flows

Alessandro Colombo, Université de Bergamo

Wednesday, 11h45

Divergence-free Virtual Element Method for Stokes and Navier-Stokes problems in three dimensions

Franco Dassi, university Milano - Bicocca

In this talk we are interested in the resolution of Stokes and Navier-Stokes problems in three dimensions. To achieve this goal we use the Virtual Element Method. More specifically, we extend the two-dimensional divergence-free VEM scheme already proposed in a two dimensional setting by Beirão L. et al. in 2017. Other than verifing the theoretical trend of the error for both Stokes and Navier-Stokes equations, we will numerically show that the proposed scheme is divergence-free.

Friday, 09h45

Hybrid High-Order methods for elasticity

Daniele Di Pietro, Université de Montpellier

Originally introduced in [1], Hybrid High-Order (HHO) methods provide a framework for the discretisation of models based on PDEs with features that set it apart from traditional ones. The construction hinges on discrete unknowns that are broken polynomials on the mesh and on its skeleton, from which two key ingredients are devised:
(i) Local reconstructions obtained by solving small, trivially parallel problems inside each element, and conceived so that their composition with the natural interpolator of sufficiently smooth functions yields a physics- and problem-dependent projector on local polynomial spaces;
(ii) Stabilisation terms that penalise residuals designed at the element level so as to ensure stability while preserving the approximation properties of the reconstruction. These ingredients are combined to formulate local contributions, which are then assembled as in standard Finite Element methods. From this construction, several appealing features ensue: the support of polytopal meshes and arbitrary approximation orders in any space dimension; an enhanced compliance with the physics; a reduced computational cost thanks to the compact stencil along with the possibility to locally eliminate a large portion of the unknowns. In this talk we review some recent advances in the application of HHO methods to linear and nonlinear problems in solid mechanics [2,3].

[1] D. A. Di Pietro and A. Ern, A hybrid high-order locking-free method for linear elasticity on general meshes, Comput. Meth. Appl. Mech. Engrg., 2015, 283:1-21. DOI: 10.1016/j.cma.2014.09.009
[2] M. Botti, D. A. Di Pietro, and P. Sochala, A Hybrid High-Order method for nonlinear elasticity, SIAM J. Numer. Anal., 2017, 55(6):2687-2717. DOI: 10.1137/16M1105943
[3] M. Botti, D. A. Di Pietro, and P. Sochala, Analysis of a Hybrid High-Order-discontinuous Galerkin discretization method for nonlinear poroelasticity, HAL preprint hal-01785810, May 2018

Tuesday, 15h45

Discontinuous Galerkin methods for curved elements

Emmanuil Georgoulis, University of Leicester

An number of polytopic approaches exist for dealing with polytopic element shapes. Most classes of methods restrict the number of faces per element to a globally uniform bound. This hinders the extension of such methods to essentially arbitrary curved element shapes, which would naturally arise as geometrical limits of many-face polytopes. In this talk, I will present and analyse an interior penalty formulation able to be used on essentially arbitrary, yet shape-regular in some sense, curved elements. The key challenge of proof of stability of the method for practical choices of penalty parameter will be addressed. The distinctive feature of stability of the basis functions upon small perturbations of the element shape, which has put interior penalty DG methods at the the forefront of mesh-generality capabilities over other polytopic formulations, will be an essential element for the analysis. The range of PDEs for which this theory is valid includes linear elliptic, parabolic and first order hyperbolic problems.

Tuesday, 11h45

Basis construction techniques for serendipity-type spaces

Andrew Gillette, University of Arizona

Serendipity-type finite element methods aim to use as few degrees of freedom as possible to ensure 'a priori' estimates of error decay of a given order. Traditionally associated with scalar elements on squares and cubes, their use has been expanded to vector differential form elements as well as more generic quad and hex geometries in recent years. In this talk, I will discuss recent progress in the development of basis functions for serendipity spaces suited for computation and efforts to seamlessly incorporate these functions into existing software packages, including deal.ii and Firedrake. I will discuss the state of the art in this area in the context of my recent work with Tyler Kloefkorn and undergraduate math major Victoria Sanders.

Thursday, 14h45

Impact of robust discretizations on linear solvers

Frank Huelsemann, EDF R&D

Robust discretization schemes on polytopal meshes, such as Compatible Discrete Operators (CDO) or Hybrid High-Order methods (HHO), are capable of obtaining acceptable approximation precision on geometrically complicated meshes. In the context of an industrial use of such schemes, the challenge is now to render these schemes available to the users ideally without increasing the overall computation time. To this end, fast, reliable linear solvers are crucial. This talk will report on first experiences with different linear solvers, such as algebraic multigrid, and in particular highlight the challenges that remain to be overcome.

Thursday, 15h45

TrioCFD: code & numerical schemes

Erell Jamelot, CEA France

The TrioCFD code [1,2] is a CFD freeware developed in CEA Saclay at the Thermohydraulics Department. It is provided with different numerical methods for solving local problems within a reactor, such as the flow of water around an assembly, or in a primary circuit line. We will present its current numerical schemes and main capacities, as well as our needs and developments perspectives.
More precisely, the TrioCFD code [1, 2] allows the solving of Navier-Stokes equations by a finite volume-element method [4, 5, 6] based on Crouzeix-Raviart finite elements [3]. The velocity is approached with order 1 polynomials. In order to model the turbulence phenomena (vortices) more finely, it is necessary to use a polynomial approximation of at least order 2. Moreover, TrioCFD supports Cartesian and tetrahedral meshes, but some simulations need more flexibility in the meshes choice, as for instance while doing near wall computations. Thus, studies are underway to design high order numerical schemes on polyhedral meshes, which is a challenging topic in terms of numerical analysis as well as memory footprint and CPU time.
We will also present some realistic simulations showing the performance of TrioCFD code.

[1] http://www-trio-u.cea.fr/
[2] P.-E. Angeli, M.-A. Puscas, G. Fauchet and A. Cartalade. FVCA8 benchmark for the Stokes and Navier-Stokes equations with the TrioCFD code. In C. Cancès and P. Omnes, editor, Finite Volumes for Complex Applications VIII - Methods and Theoretical Aspects, volume 199, pages 181-302. Springer Proceedings in Mathematics & Statistics, 2017.
[3] M. Crouzeix, P.-A. Raviart, Conforming and nonconforming finite element methods for solving the stationary Stokes equations. RAIRO, Sér. Anal. Numer., 33 (1973)
[4] P. Emonot. Méthodes de Volumes Eléments Finis. Application aux équations de Navier-Stokes et résultats de convergence. PhD thesis, Université Claude Bernard, Lyon I, 1992.
[5] S. Heib. Nouvelles discrétisations non structurées pour des écoulements de fluides à incompressibilité renforcée. PhD thesis, Université Paris VI, 2003.
[6] T. Fortin. Une méthode d'éléments finis à décomposition L2 d'ordre élevé motivée par la simulation d'écoulement diphasique bas Mach. PhD thesis, Université Paris VI, 2006.

Wednesday, 09h45

A unified formulation and analysis of HHO and VE methods

Simon Lemaire, Inria Lille - Nord Europe

After some insight on the differences between the conforming and nonconforming cases, we present a unifying viewpoint at Hybrid High-Order (HHO) and Virtual Element (VE) methods, both in terms of formulation and analysis. Departing from the usual VE framework and taking inspiration from the HHO one, our key ingredient is the definition of a local polynomial reconstruction operator, that maps any local vector of degrees of freedom to a polynomial of degree k (for the method of order k) and that enjoys, when composed with the local reduction of a function, optimal approximation properties. We construct such a local reconstruction operator for the VE method, and we show how it enables, on a toy problem, to lead a completely unified VE/HHO analysis in broken H^1-seminorm.

Monday, 15h45

On high-order pressure-robust space discretisations, their advantages for incompressible high Reynolds number generalised Beltrami flows and beyond

Alexander Linke, Weierstrass Institute

An improved understanding of the divergence constraint for high Reynolds number incompressible Navier-Stokes equations (iNSE) leads to the observation that a semi-norm and corresponding equivalence classes of forces play a fundamental role for the nonlinear dynamics of the iNSE. Two forces in the momentum balance are *velocity-equivalent*, if they lead to the same velocity field, i.e., if they differ only by an (arbitrary) gradient field such that their Helmholtz-Leray projectors are the same. The recent concept of *pressure-robustness* allows to discriminate between space discretisations that discretize these equivalence classes of forces in an accurate manner or not. Mathematically, pressure-robustness is based on the L^2-orthogonality of discretely divergence velocity test (!) functions and arbitrary gradient fields, exploiting vector-valued $H(\mathrm{div})$-conforming finite element spaces. It is shown that non-pressure-robust mixed methods, which are only inf-sup stable, may heavily suffer from dominant gradient fields in the iNSE momentum balance. For example, it is shown that non-pressure-robust methods will lose half of its (formal) convergence order in high Reynolds number vortex-dominated flows (without external forcing), because the material derivative will be approximately a gradient field, i.e., the material derivative is velocity-equivalent to the zero vector! Thus, pressure-robust methods outperform non-pressure-robust methods dramatically in high-Reynolds number flows. For the numerical analysis, the concept of a discrete Helmholtz-Leray projector is emphasized, and novel-short-time a-priori error estimates are derived. Further, it is shown how the concept of pressure-robustness allows to construct novel well-balanced schemes for the compressible Navier-Stokes equations.

Reference: Nicolas R. Gauger, Alexander Linke, Philipp W. Schroeder: On high-order pressure-robust space discretisations, their advantages for incompressible high Reynolds number generalised Beltrami flows and beyond. arXiv:1808.10711, 2018.
Volker John, Alexander Linke, Christian Merdon, Michael Neilan, Leo Rebholz: On the divergence constraint in mixed finite element methods for incompressible flows. SIAM Review, 2017.

Monday, 09h45

High-order remap methods on curvilinear meshes

Konstantin Lipnikov, Los Alamos National Laboratory

A remap of physical fields between two meshes is an important step of arbitrary Lagrangian-Eulerian simulations. This step is challenging for high-order discontinuous Galerkin schemes since the Lagrangian flow motion leads to high-order meshes with curved faces. It becomes even more challenging for unstructured polygonal meshes that do not have a polynomial map from the reference to a current cell. We propose and analyze a new framework to create remap schemes on curvilinear polygonal meshes based on the theory of virtual element projectors. We derive a conservative remap scheme that is high-order accurate in space and time. The properties of this scheme are studied numerically for smooth and discontinuous fields on unstructured quadrilateral and polygonal meshes.

Monday, 14h45

Virtual Element Methods for Linear Elasticity Problems

Carlo Lovadina, University of Milan

The Virtual Element Method (VEM) is an emerging methodology for the approximation of partial differential equation problems. VEM was born in 2012, see [1], and shares the same variational background of the Finite Element Method (FEM).
In the framework of 2D elasticity problems, we first present a family of Virtual Element schemes based on the Hellinger-Reissner variational principle, see [2,3]. As it is well-known, imposing both the symmetry of the stress tensor and the continuity of the tractions at the inter-element is typically a great source of troubles in the framework of classical Galerkin schemes. We exploit the great flexibility of VEM to present alternative methods, which provide symmetric stresses, continuous tractions and are reasonably cheap with respect to the delivered accuracy. VEMs reach this goal by abandoning the local polynomial approximation concept, a feature originally used to design conforming Galerkin schemes on general polytopal meshes, see [1]. We then present a new VEM scheme based on a dual hybrid formulation of the linear elastic problem, see [4]. Here, *a priori* locally self-equilibrated stresses are employed, while the displacement field is essentially required only on the mesh skeleton and serves as a Lagrange multiplier for the traction continuity.
In this talk, we detail the ideas which led to the design of our schemes, we state the theoretical results, and we present several numerical tests to assess the actual computational performance of the our approach. Finally, we discuss some possible future extensions.

[1] Beirào da Veiga, L., Brezzi, F., Cangiani, A., Manzini, G., Marini, L. D., Russo, A., Basic principles of Virtual Element Methods, Math. Models Methods Appl. Sci., 23, pp. 119-214, (2013).
[2] Artioli, E., de Miranda, S., Lovadina, C., Patruno, L., A stress/displacement Virtual Element method for plane elasticity problems, Computer Methods in Applied Mechanics and Engineering, 325, pp. 155-174, (2017).
[3] Artioli, E., de Miranda, S., Lovadina, C., Patruno, L., A Family of Virtual Element Methods for Plane Elasticity Problems Based on the Hellinger-Reissner Principle, Computer Methods in Applied Mechanics and Engineering, 340, pp. 978-999, (2018).
[4] Artioli, E., de Miranda, S., Lovadina, C., Patruno, L., A dual hybrid Virtual Element Method for plane elasticity problems, ESAIM: Mathematical Modelling and Numerical Analysis, submitted.

Tuesday, 10h45

A primal discontinuous Galerkin method with static condensation on very general meshes

Alexei Lozinski, Univ. de Franche-Comte

We present an efficient variant of a primal Discontinuous Galerkin method with interior penalty for the second order elliptic equations on very general meshes (polytopes with eventually curved boundaries). Efficiency, especially when higher order polynomials are used, is achieved by static condensation, i.e. a local elimination of certain degrees of freedom cell by cell. This alters the original method in a way that preserves the optimal error estimates. Numerical experiments confirm that the solutions produced by the new method are indeed very close to that produced by the classical one.

Monday, 10h45

A unified framework for conforming and nonconforming virtual element methods.

Marco Manzini, Los Alamos National Laboratory

We present a unified framework for conforming and nonconforming Virtual Element Method (VEM) for the numerical treatment of elliptic problems. This formulation works on very general unstructured meshes in 2D and 3D for arbitrary order of accuracy. Examples of applications are for Poisson problem as well as general diffusion problems (advection-diffusion-reaction equations). Numerical experiments verify the theory and validate the performance of the proposed method.

Thursday, 11h45

Numerical modeling of elasto-acoustic coupling by a discontinuous Galerkin method on general meshes

Ilario Mazzieri, Politecnico di Milano

Coupled elasto-acoustic wave propagation arises in several scientific and engineering contexts such earthquake ground motion near coastal environments, geophysical detection of underground cavities, exploration seismology, structural acoustics and medical ultrasonics. In practical applications, the underlying geometry one has to deal with is remarkably complicated and irregular. For this reason, considering a conforming triangulation would be computationally very expensive. We are thus led to consider a space discretization capable to reproduce the geometrical constraints under consideration to a reasonable extent of accuracy, without being at the same time too much demanding. Such discretization is then performed using general polygonal/polyhedral (polytopic) elements, with no restriction on the number of faces each element can possess, and possibly allowing for face degeneration in mesh refinement. This work is focused on the development and analysis of a discontinuous Galerkin (dG) method on polygonal and polyhedral grids for the space discretization of an evolution problem modeling the coupled propagation of (visco)elastic and acoustic waves. We state and prove a well-posedness result for the strong formulation of the problem, present a stability result for the semi-discrete formulation, and finally prove an a priori energy error estimate for the resulting formulation. The convergence results are verified by two and three dimensional numerical experiments.

Wednesday, 11h15

Finite volume level-set methods on polyhedral meshes

Karol Mikula, Slovak University of Technology Bratislava

Inspired by industrial cooperation with AVL List GmbH Graz (Jooyoung Hahn and Branislav Basara) we outline our recent common work on a construction of finite volume schemes on general polyhedron meshes for solving level-set equations for motion of phase interfaces in combustion engine simulations. This is also common work with Peter Frolkovic and Matej Medla from Slovak University of Technology in Bratislava.

Tuesday, 11h15

Virtual Element Methods for Elastodynamic Problems with Explicit Time Integrations

Kyungsoo Park, Yonsei University

To handle general convex and nonconvex elements for elastodynamic problems in 2D and 3D, the virtual element method (VEM) is utilized with the explicit time integration. Convergence and stability of VEM solution is demonstrated in relation to time increment, mass lumping scheme, element size and distortion for arbitrary element shapes. The critical time step is approximated using two approaches, i.e., maximum eigenvalue of a local system of mass and stiffness matrices, and an effective element length. Computational results demonstrate that small edges on convex polygonal elements do not significantly affect the critical time step, while convergence of the VEM solution is observed regardless of the stability term and element shape. Thus, the VEM is able to consistently handle general convex and nonconvex elements in both 2D and 3D.

Monday, 11h15

The conforming virtual element method for polyharmonic problems

Marco Verani, Politecnico di Milano

In this talk, we present the capability of virtual element methods in accommodating approximation spaces featuring high-order continuity to numerically approximate differential problems of the form $\Delta^p u = f$, $p \geq 1$. More specifically, we develop and analyze the conforming virtual element method for the numerical approximation of polyharmonic boundary value problems, and prove an abstract result that states the convergence of the method in the energy norm.

Friday, 10h45

A simple a posteriori estimate on general polytopal meshes with applications to complex porous media flows

Martin Vohralik, Inria Paris

We develop an a posteriori error estimate for lowest-order locally conservative methods on meshes consisting of general polytopal elements. The evaluation of our estimates merely consists in some local matrix-vector multiplications, where, on each mesh element, the matrices are either directly inherited from the given numerical method, or easily constructed from the element geometry, while the vectors are the flux and potential values on the given element. Applications to unsteady coupled degenerate problems describing complex multiphase flows in porous media are presented. It is a joint work with Soleiman Yousef (IFPEN).

Friday, 11h15

Higher order multipoint flux mixed finite element methods on quadrilaterals and hexahedra

Ivan Yotov, University of Pittsburgh

We develop higher order multipoint flux mixed finite element (MFMFE) methods for solving elliptic problems on quadrilateral and hexahedral grids that reduce to cell-based pressure systems. The methods are based on a new family of mixed finite elements, which are enhanced Raviart-Thomas spaces with bubbles that are curls of specially chosen polynomials. The velocity degrees of freedom of the new spaces can be associated with the points of tensor-product Gauss-Lobatto quadrature rules, which allows for local velocity elimination and leads to a symmetric and positive definite cell-based system for the pressures. We prove optimal k-th order convergence for the velocity and pressure in their natural norms, as well as $(k+1)$-st order superconvergence for the pressure at the Gauss points. Moreover, local postprocessing gives a pressure that is superconvergent of order $k+1$ in the full L^2-norm. Numerical results illustrating the validity of our theoretical results are included.
This is joint work with Ilona Ambartsumyan, Eldar Khattatov, and Jeonghun Lee.

Posters

Monday, 17h30

Posters

Dibyendu Adak, Indian Institute of Technology Madras

Laurence Beaude, Université Nice Sophia-Antipolis

Michele Botti, Politecnico di Milano

Konstantin Brenner, Université Nice Sophia-Antipolis

Daniel Castanon Quiroz, Université de Montpellier

Florent Chave, INRIA Lille-Nord Europe

Andrea Chiozzi, University of Ferrara

Julien Coulet, IFP Energies Nouvelles

Alessandro D'Auria, Politecnico di Torino

Alessandra Guglielmana, Université de Montpellier

André Harnist, Université de Montpellier

Julian Hennicker, Université de Genève

Rekha Khot, Indian Institute of Technology Bombay

Hyeongtae Kim, Yonsei University

Giulia Lissoni, Université Nice Sophia-Antipolis

Sebastian Minjeaud, Université Nice Sophia-Antipolis

Sundararajan Natarajan, Indian Institute of Technology Madras

Luca Patruno, University of Bologna

Alexander Pichler, University of Vienna

Daniele Prada, IMATI - CNR

Houssaine Quenjel, Université Nice Sophia-Antipolis

Michele Visinoni, University of Milano-Bicocca

List of participants

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Lourenço Bierao Da Veiga, University of Milano-Bicocca
Stefano Berrone, Politecnico di Torino
Silvia Bertoluzza, CNRS CNR
Jérôme Bonelle, EDF R&D
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Carsten Carstensen, Humboldt University Berlin
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Claire Chainais, Université de Lille
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Andrea Chiozzi, University of Ferrara
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Jérôme Droniou, Monash University
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Thierry Gallouet, Aix-Marseille Université
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